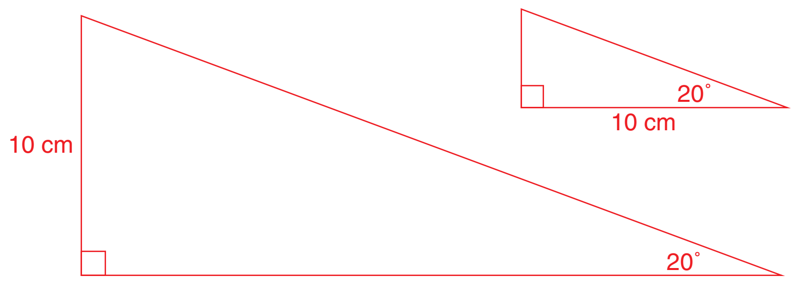
Lesson 3.1: The Tangent Ratio

Practice – II

1. Explain the meaning of the expression $\tan 32^\circ = 0.624...$.

   In a triangle with an acute angle of 32°, dividing the length of the side opposite the 32° angle by the length of the side adjacent to the 32° angle gives you $0.624\ldots$.

2. Show that two different right angles can be drawn with an acute angle of 20° and a leg length of 10 cm.



3. Use a calculator to determine the value of each of the following.

   a. $\tan 43^\circ$

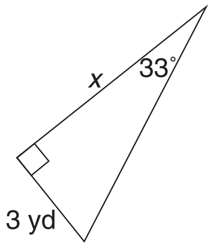
   0.932...

   b. $\tan 17^\circ$

   $0.305\ldots$

4. After looking at the triangle shown, Terry wrote the following solution for x.

$$\tan 33^\circ = \frac{3\text{ yd}}{x}$$
$$x \bullet \tan 33^\circ = 3\text{ yd}$$
$$x = \frac{3\text{ yd}}{\tan 33^\circ}$$
$$x \doteq 4.6\text{ yd}$$



A more explicit version of the calculation follows.

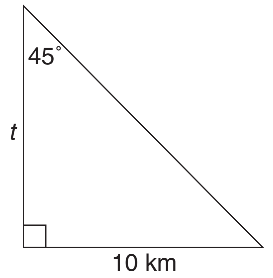
$$\tan 33^\circ = \frac{\text{length opposite } 33^\circ}{\text{length adjacent to } 33^\circ}$$
$$\tan 33^\circ = \frac{3\text{ yd}}{x}$$
$$x \bullet \tan 33^\circ = \frac{3\text{ yd}}{\cancel{x}} \bullet \cancel{x}$$
$$x \tan 33^\circ = 3\text{ yd}$$
$$\frac{x\cancel{\tan 33^\circ}}{\cancel{\tan 33^\circ}} = \frac{3\text{ yd}}{\tan 33^\circ}$$
$$x = \frac{3\text{ yd}}{\tan 33^\circ}$$
$$x \doteq 4.6\text{ yd}$$

a. Explain the steps Terry used in his solution.
Terry set up the tangent ratio using the lengths of the sides opposite and adjacent to the given angle. Terry multiplied both sides of the equation by x and then divided both sides of the equation by tan 33°. Finally, Terry evaluated the expression $\frac{3\text{ yd}}{\tan 33^\circ}$.

b. Explain why Terry might have chosen not to evaluate $\tan 33^\circ$ until the last step.
tan 33° is 0.649407593…, a non-repeating, non-terminating decimal. By entering 3 ÷ tan 33 into his calculator, Terry does not need to record a large number of digits, he doesn't need to use the memory key on his calculator to store the exact value, and he doesn't end up rounding tan 33° to get a less accurate final answer.

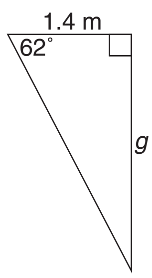
5. Determine the length of the unknown variable, to the nearest tenth, in each of the following diagrams.

a.



$$\tan 45^\circ = \frac{\text{length opposite } 45^\circ}{\text{length adjacent to } 45^\circ}$$
$$\tan 45^\circ = \frac{10\text{ km}}{t}$$
$$t \bullet \tan 45^\circ = \frac{10\text{ km}}{\cancel{t}} \bullet \cancel{t}$$
$$\frac{t \bullet \cancel{\tan 45^\circ}}{\cancel{\tan 45^\circ}} = \frac{10\text{ km}}{\tan 45^\circ}$$
$$t = 10.0\text{ km}$$

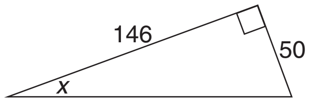
b.



$$\tan 62^\circ = \frac{\text{length opposite } 62^\circ}{\text{length adjacent to } 62^\circ}$$
$$\tan 62^\circ = \frac{g}{1.4\text{ m}}$$
$$1.4 \bullet \tan 62^\circ = \frac{g}{\cancel{1.4}\text{ m}} \bullet \cancel{1.4}$$
$$2.6\text{ m} \doteq g$$

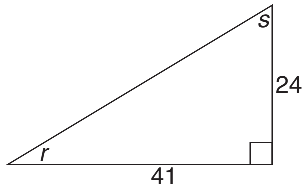
6. Determine the measure of the unknown angle(s), to the nearest degree, in each of the following diagrams.

a.



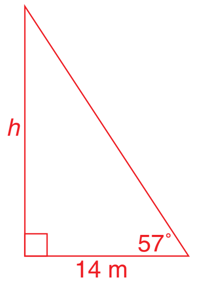
$$\tan x = \frac{50}{146}$$
$$x = \tan^{-1}\left(\frac{50}{146}\right)$$
$$x \doteq 19^\circ$$

b.



$$\tan r = \frac{24}{41} \qquad \tan s = \frac{41}{24}$$
$$r = \tan^{-1}\left(\frac{24}{41}\right) \qquad s = \tan^{-1}\left(\frac{41}{24}\right)$$
$$r \doteq 30^\circ \qquad s \doteq 60^\circ$$

7. A wire supporting a radio tower is secured to the ground 14 m from the base of the tower. If the angle between the ground and the wire is 57°, what is the height of the tower, to the nearest tenth of a metre?

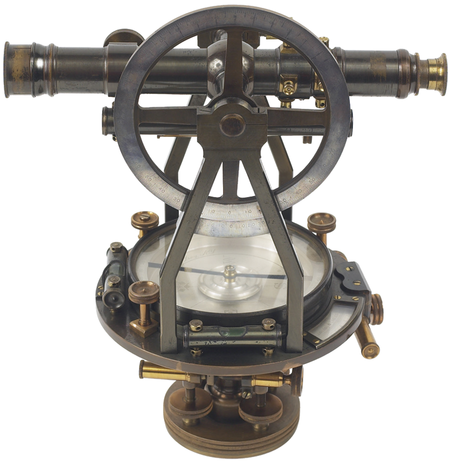


$$\tan 57^\circ = \frac{\text{length opposite } 57^\circ}{\text{length adjacent to } 57^\circ}$$
$$\tan 57^\circ = \frac{h}{14 \text{ m}}$$
$$14 \text{ m} \cdot \tan 57^\circ = \frac{h}{\cancel{14 \text{ m}}} \cdot \cancel{14 \text{ m}}$$
$$21.6 \text{ m} \doteq h$$

The height of the tower is approximately 21.6 m.

8. A transit is a tool that can be used to measure angles and is often used for surveying. A simplified version of a transit can be built using a protractor.

a. Use a protractor and the diagram on p. 113 of *Mathematics 10* to build a simple transit. If you do not have the appropriate materials or if you have difficulty building the transit, contact your teacher.



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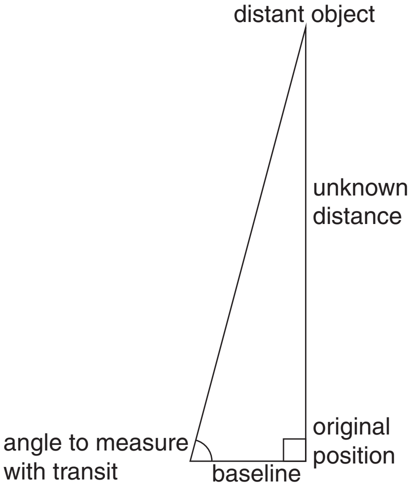
b. Your transit will be easiest to work with if you place it on a table or a stool. To determine an angle using your transit, point one of the zeros on the protractor along one leg of the angle. Next, look through the straw and aim it along the other leg of the angle. When both are lined up, you can read the angle beneath the straw on the protractor. Try measuring a couple angles with your transit.

c. Now, you will use your transit and the tangent ratio to determine an unknown distance. Choose a moderately distant object. Mark your current location and then move in a direction perpendicular to the object to a new position. Measure the distance traveled with a measuring tape. This distance is called a 'baseline'. At your new location, use your transit to measure the angle between your original position and the distant object.



d. Use the baseline length, the transit angle, and the tangent ratio to determine the unknown distance.
Responses will vary, but all should include the following.

$$\tan\theta = \frac{\text{unknown distance}}{\text{baseline}}$$

$$\text{baseline} \cdot \tan\theta = \text{unknown distance}$$

e. Measure the unknown distance (if possible). How close was your calculation?
Responses will vary.

f. Suggest a way you could improve your transit or your measurement process.
Improvements will vary, but may include a more careful construction of the transit or using a longer baseline.

Please complete *Lesson 3.1 Explore Your Understanding Assignment* located in *Workbook 3.1* before proceeding to *Lesson 3.2*.