Lesson 3.2: The Sine and Cosine Ratio

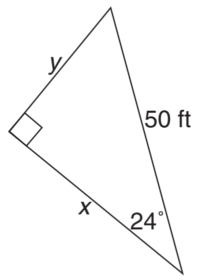
Practice – III

1. Although six different ratios can be produced for any triangle, this course only uses three of the ratios: the sine ratio, the cosine ratio, and the tangent ratio. Explain why only these three ratios are required to relate any pair of sides.

   Three of the ratios are the reciprocals of the other three. For example, $\frac{\text{length opposite } \theta}{\text{hypotenuse}}$ and $\frac{\text{hypotenuse}}{\text{length opposite } \theta}$ are reciprocals. Because the same pairs of sides are used in the reciprocal ratios, no new information is gained by the use of the reciprocal ratios (at this level of trigonometry).

   The three reciprocal trigonometric ratios not used in this course become important in more advanced trigonometry.

2. Determine the unknown side lengths, to the nearest tenth, in the diagram.



$$\cos 24^\circ = \frac{\text{length adjacent to } 24^\circ}{\text{hypotenuse}}$$

$$\cos 24^\circ = \frac{x}{50 \text{ ft}}$$

$$50 \text{ ft} \cdot \cos 24^\circ = \frac{x}{\cancel{50 \text{ ft}}} \cdot \cancel{50 \text{ ft}}$$

$$45.7 \text{ ft} \doteq x$$

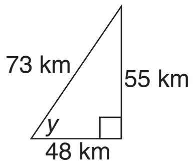
$$\sin 24^\circ = \frac{\text{length opposite } 24^\circ}{\text{hypotenuse}}$$

$$\sin 24^\circ = \frac{y}{50 \text{ ft}}$$

$$50 \text{ ft} \cdot \sin 24^\circ = \frac{y}{\cancel{50 \text{ ft}}} \cdot \cancel{50 \text{ ft}}$$

$$20.3 \text{ ft} \doteq y$$

3. Show that the sine ratio, cosine ratio, and tangent ratio can each be used to determine the measure of angle *y*, to the nearest tenth of a degree, in the following triangle.



$$\sin y = \frac{\text{length opposite } y}{\text{hypotenuse}} \qquad \cos y = \frac{\text{length adjacent to } y}{\text{hypotenuse}} \qquad \tan y = \frac{\text{length opposite } y}{\text{length adjacent to } y}$$

$$\sin y = \frac{55 \not{\text{km}}}{73 \not{\text{km}}} \qquad \cos y = \frac{48 \not{\text{km}}}{73 \not{\text{km}}} \qquad \tan y = \frac{55 \not{\text{km}}}{48 \not{\text{km}}}$$

$$y = \sin^{-1}\left(\frac{55}{73}\right) \qquad y = \cos^{-1}\left(\frac{48}{73}\right) \qquad y = \tan^{-1}\left(\frac{55}{48}\right)$$

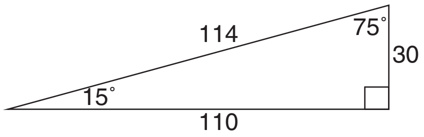
$$y \doteq 48.9^\circ \qquad y \doteq 48.9^\circ \qquad y \doteq 48.9^\circ$$

4. Look at the table in *Lesson 3.2* that shows the tangent, sine, and cosine ratio values for various angles. Notice that $\sin 5^\circ = \cos 85^\circ$, $\sin 10^\circ = \cos 80^\circ$, $\sin 15^\circ = \cos 75^\circ$, etc.

   a. What do the angles in each of the equalities add to?

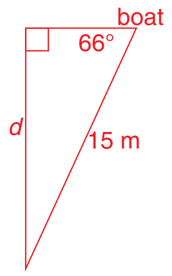
   90°

   b. Use the following triangle to explain the equalities. (Hint: What fraction represents both $\sin 15^\circ$ and $\cos 75^\circ$ in the diagram?)



In the diagram, the side length opposite 15° and the side length adjacent to 75° are both 30. The hypotenuse is 114 for both ratios. This means both sin 15° and cos 75° are equal to $\frac{30}{114}$. In a right triangle, the sine of one acute angle and the cosine of the other acute angle are always equal.

5. A boat is anchored in a river. If there are 15 m of rope between the boat and the bottom of the river and the rope makes an angle of 66° with the surface of the water. How deep is the river to the nearest tenth of a metre?



$$\sin 66^\circ = \frac{\text{length opposite } 66^\circ}{\text{hypotenuse}}$$

$$\sin 66^\circ = \frac{d}{15 \text{ m}}$$

$$15 \text{ m} \cdot \sin 66^\circ = \frac{d}{\cancel{15 \text{ m}}} \cdot \cancel{15 \text{ m}}$$

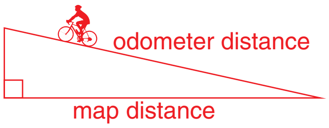
$$13.7 \text{ m} \doteq d$$

The river is approximately 13.7 m deep.

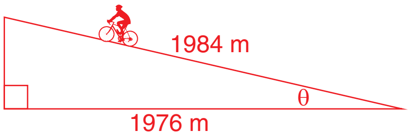
6. Chad cycled up a long straight hill. The odometer on his bike showed that the hill was 1984 m long, while a digital map showed that he was 1976 m from the base of the hill.

a. Assuming Chad rode in a straight line and that both the odometer and the digital map are accurate, why did the two instruments show different distances?

The odometer shows the distance traveled by the bicycle, while the map shows the horizontal distance from start to stop.



b. What is the average angle of elevation of the hill? Express your answer to the nearest degree.



$$\cos\theta = \frac{\text{length adjacent to } \theta}{\text{hypotenuse}}$$

$$\cos\theta = \frac{1976}{1984}$$

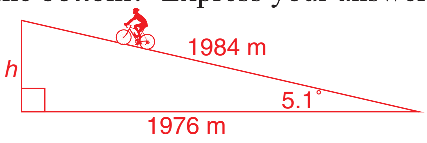
$$\theta = \cos^{-1}\left(\frac{1976}{1984}\right)$$

$$\theta = 5.147...^\circ$$

$$\theta \doteq 5^\circ$$

The average angle of elevation of the hill is approximately 5°.

c. How much higher is Chad when he is at the top of the hill compared to when he is at the bottom? Express your answer to the nearest metre.



$$\sin 5.147...^\circ = \frac{\text{length opposite } 5.147...^\circ}{\text{hypotenuse}}$$

$$\sin 5.147...^\circ = \frac{h}{1984}$$

$$1984 \cdot \sin 5.147...^\circ = \frac{h}{\cancel{1984}} \cdot \cancel{1984}$$

$$178 \text{ m} \doteq h$$

Chad is approximately 178 m higher when he is at the top of the hill.

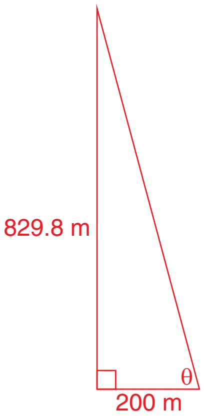
The Pythagorean theorem could have also been used to solve this problem.

Please complete *Lesson 3.2 Explore Your Understanding Assignment* located in *Workbook 3.2* before proceeding to *Lesson 3.3*.

Lesson 3.3: Solving Problems with Triangles

Practice – IV

1. At the time of this writing, the Burj Khalifa (formerly the Burj Dubai) is the world's tallest building at 829.8 m. If you were to stand 200 m from the centre of the building, what would the angle of elevation be to the top? Express your answer to the nearest degree.



$$\tan \theta = \frac{\text{length opposite } \theta}{\text{length adjacent to } \theta}$$

$$\tan \theta = \frac{829.8 \cancel{\text{ m}}}{200 \cancel{\text{ m}}}$$

$$\tan \theta = \frac{829.8}{200}$$

$$\theta = \tan^{-1}\left(\frac{829.8}{200}\right)$$

$$\theta \doteq 76^\circ$$


© *Thinkstock*

The angle of elevation to the top is approximately 76°.