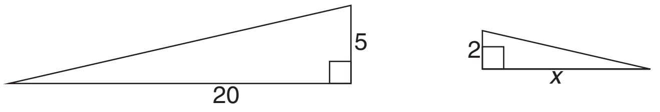
Appendix

Lesson 3.1: The Tangent Ratio

Practice – I

1. The following two triangles are similar.



a. Write the two pairs of corresponding side lengths.
5 and 2, 20 and x

b. In the first triangle, the ratio of sides can be written as $\frac{20}{5} = 4$. What is the corresponding ratio for the second triangle? Explain.
The two triangles are similar, so the ratio of corresponding side lengths must be equal. This means $\frac{x}{2} = 4$.

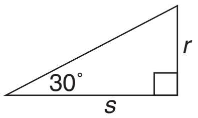
c. Solve for x.

$$\frac{x}{2} = 4$$

$$\frac{x}{\cancel{2}} \cdot \cancel{2} = 4 \cdot 2$$

$$x = 8$$

2. A triangle with sides p and q is similar to the triangle shown below, where side p corresponds to side r, and side q corresponds to side s.



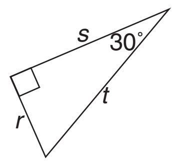
a. If $\frac{p}{q} = 0.58$, what must $\frac{r}{s}$ equal?

Similar triangles have the same ratio of corresponding side lengths, so $\frac{r}{s} = 0.58$.

b. If $\frac{p}{q} = 0.58$, and $s = 7$, what must r equal?

$$\frac{r}{s} = 0.58$$
$$\frac{r}{7} = 0.58$$
$$\frac{r}{\cancel{7}} \cdot \cancel{7} = 0.58 \cdot 7$$
$$r = 4.06$$

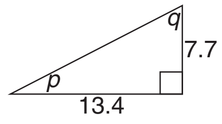
3. Use the table to state a ratio of sides for the following triangle. Explain what the ratio represents.



0.58 is the value produced by dividing the length of side *r* (opposite) by the length of side *s* (adjacent).

θ	$\dfrac{\text{length opposite } \theta}{\text{length adjacent to } \theta}$ ratio (approximate values, rounded to the nearest hundredth)
5°	0.09
10°	0.18
15°	0.27
20°	0.36
25°	0.47
30°	0.58
35°	0.70
40°	0.84
45°	1
50°	1.19
55°	1.43
60°	1.73
65°	2.14
70°	2.75
75°	3.73
80°	5.67
85°	11.43

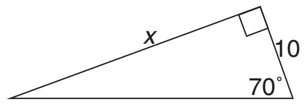
4. Use the table to state the value of each variable.



$$\frac{7.7}{13.4} = 0.58\text{, so } \quad p \doteq 30^\circ$$

$$\frac{13.4}{7.7} = 1.74\text{, so } \quad q \doteq 60^\circ$$

5. Use the table to determine the unknown length, *x*, to the nearest tenth, in the diagram.



Look for the ratio that corresponds to 70° in the table.

$$\frac{\text{length opposite } 70^\circ}{\text{length adjacent to } 70^\circ} = 2.75$$

$$\frac{x}{10} = 2.75$$

$$\frac{x}{\cancel{10}} \cdot \cancel{10} = 2.75 \cdot 10$$

$$x = 27.5$$

Please return to *Unit 3: Trigonometry Lesson 3.1* in the *Module* to continue your exploration.

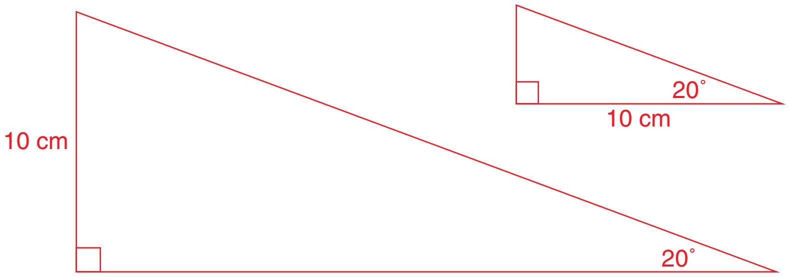
Lesson 3.1: The Tangent Ratio

Practice – II

1. Explain the meaning of the expression $\tan 32^\circ = 0.624...$.

   In a triangle with an acute angle of 32°, dividing the length of the side opposite the 32° angle by the length of the side adjacent to the 32° angle gives you $0.624...$.

2. Show that two different right angles can be drawn with an acute angle of 20° and a leg length of 10 cm.



3. Use a calculator to determine the value of each of the following.

   a. $\tan 43^\circ$

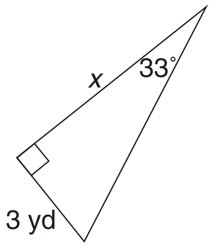
   0.932...

   b. $\tan 17^\circ$

   $0.305\ldots$

4. After looking at the triangle shown, Terry wrote the following solution for x.

$$\tan 33^\circ = \frac{3\text{ yd}}{x}$$
$$x \cdot \tan 33^\circ = 3\text{ yd}$$
$$x = \frac{3\text{ yd}}{\tan 33^\circ}$$
$$x \doteq 4.6\text{ yd}$$



A more explicit version of the calculation follows.

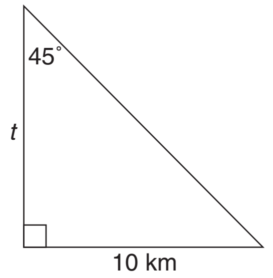
$$\tan 33^\circ = \frac{\text{length opposite } 33^\circ}{\text{length adjacent to } 33^\circ}$$
$$\tan 33^\circ = \frac{3\text{ yd}}{x}$$
$$x \cdot \tan 33^\circ = \frac{3\text{ yd}}{\cancel{x}} \cdot \cancel{x}$$
$$x \tan 33^\circ = 3\text{ yd}$$
$$\frac{x\cancel{\tan 33^\circ}}{\cancel{\tan 33^\circ}} = \frac{3\text{ yd}}{\tan 33^\circ}$$
$$x = \frac{3\text{ yd}}{\tan 33^\circ}$$
$$x \doteq 4.6\text{ yd}$$

a. Explain the steps Terry used in his solution.
Terry set up the tangent ratio using the lengths of the sides opposite and adjacent to the given angle. Terry multiplied both sides of the equation by x and then divided both sides of the equation by tan 33°. Finally, Terry evaluated the expression $\frac{3\text{ yd}}{\tan 33^\circ}$.

b. Explain why Terry might have chosen not to evaluate $\tan 33^\circ$ until the last step.
tan 33° is 0.649407593…, a non-repeating, non-terminating decimal. By entering 3 ÷ tan 33 into his calculator, Terry does not need to record a large number of digits, he doesn't need to use the memory key on his calculator to store the exact value, and he doesn't end up rounding tan 33° to get a less accurate final answer.

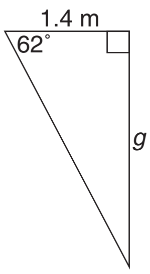
5. Determine the length of the unknown variable, to the nearest tenth, in each of the following diagrams.

a.



$$\tan 45^\circ = \frac{\text{length opposite } 45^\circ}{\text{length adjacent to } 45^\circ}$$
$$\tan 45^\circ = \frac{10\text{ km}}{t}$$
$$t \cdot \tan 45^\circ = \frac{10\text{ km}}{\cancel{t}} \cdot \cancel{t}$$
$$\frac{t \cdot \cancel{\tan 45^\circ}}{\cancel{\tan 45^\circ}} = \frac{10\text{ km}}{\tan 45^\circ}$$
$$t = 10.0\text{ km}$$

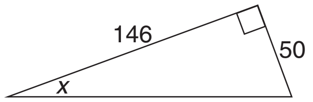
b.



$$\tan 62^\circ = \frac{\text{length opposite } 62^\circ}{\text{length adjacent to } 62^\circ}$$
$$\tan 62^\circ = \frac{g}{1.4\text{ m}}$$
$$1.4 \cdot \tan 62^\circ = \frac{g}{\cancel{1.4}\text{ m}} \cdot \cancel{1.4}$$
$$2.6\text{ m} \doteq g$$

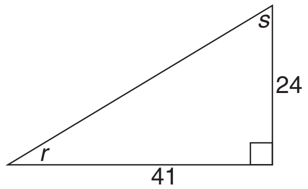
6. Determine the measure of the unknown angle(s), to the nearest degree, in each of the following diagrams.

a.



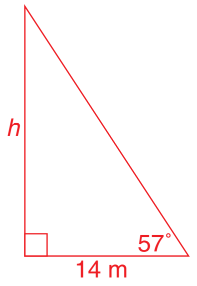
$$\tan x = \frac{50}{146}$$
$$x = \tan^{-1}\left(\frac{50}{146}\right)$$
$$x \doteq 19^\circ$$

b.



$$\tan r = \frac{24}{41} \qquad \tan s = \frac{41}{24}$$
$$r = \tan^{-1}\left(\frac{24}{41}\right) \qquad s = \tan^{-1}\left(\frac{41}{24}\right)$$
$$r \doteq 30^\circ \qquad s \doteq 60^\circ$$

7. A wire supporting a radio tower is secured to the ground 14 m from the base of the tower. If the angle between the ground and the wire is 57°, what is the height of the tower, to the nearest tenth of a metre?

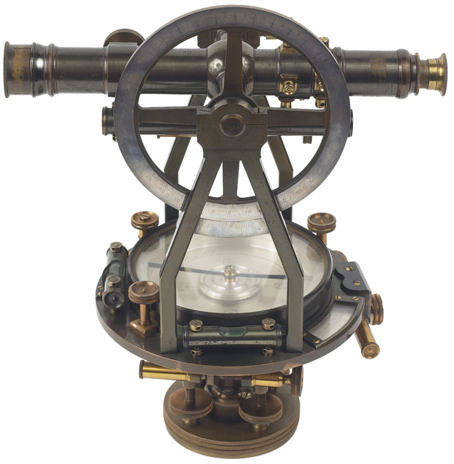


$$\tan 57^\circ = \frac{\text{length opposite } 57^\circ}{\text{length adjacent to } 57^\circ}$$
$$\tan 57^\circ = \frac{h}{14\text{ m}}$$
$$14\text{ m} \cdot \tan 57^\circ = \frac{h}{\cancel{14\text{ m}}} \cdot \cancel{14\text{ m}}$$
$$21.6\text{ m} \doteq h$$

The height of the tower is approximately 21.6 m.

8. A transit is a tool that can be used to measure angles and is often used for surveying. A simplified version of a transit can be built using a protractor.

   a. Use a protractor and the diagram on p. 113 of *Mathematics 10* to build a simple transit. If you do not have the appropriate materials or if you have difficulty building the transit, contact your teacher.



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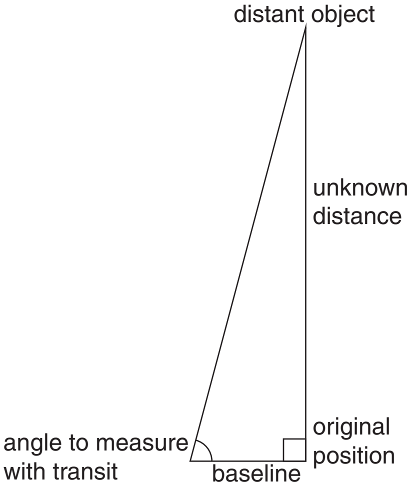
   b. Your transit will be easiest to work with if you place it on a table or a stool. To determine an angle using your transit, point one of the zeros on the protractor along one leg of the angle. Next, look through the straw and aim it along the other leg of the angle. When both are lined up, you can read the angle beneath the straw on the protractor. Try measuring a couple angles with your transit.



   c. Now, you will use your transit and the tangent ratio to determine an unknown distance. Choose a moderately distant object. Mark your current location and then move in a direction perpendicular to the object to a new position. Measure the distance traveled with a measuring tape. This distance is called a 'baseline'. At your new location, use your transit to measure the angle between your original position and the distant object.

   d. Use the baseline length, the transit angle, and the tangent ratio to determine the unknown distance.
   Responses will vary, but all should include the following.

$$\tan\theta = \frac{\text{unknown distance}}{\text{baseline}}$$

$$\text{baseline} \cdot \tan\theta = \text{unknown distance}$$

   e. Measure the unknown distance (if possible). How close was your calculation?
   Responses will vary.

   f. Suggest a way you could improve your transit or your measurement process.
   Improvements will vary, but may include a more careful construction of the transit or using a longer baseline.

Please complete *Lesson 3.1 Explore Your Understanding Assignment* located in *Workbook 3.1* before proceeding to *Lesson 3.2*.

Lesson 3.2: The Sine and Cosine Ratio

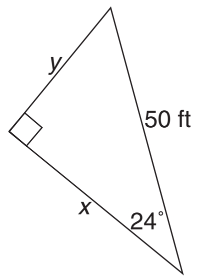
Practice – III

1. Although six different ratios can be produced for any triangle, this course only uses three of the ratios: the sine ratio, the cosine ratio, and the tangent ratio. Explain why only these three ratios are required to relate any pair of sides.

Three of the ratios are the reciprocals of the other three. For example, $\frac{\text{length opposite }\theta}{\text{hypotenuse}}$ and $\frac{\text{hypotenuse}}{\text{length opposite }\theta}$ are reciprocals. Because the same pairs of sides are used in the reciprocal ratios, no new information is gained by the use of the reciprocal ratios (at this level of trigonometry).

The three reciprocal trigonometric ratios not used in this course become important in more advanced trigonometry.

2. Determine the unknown side lengths, to the nearest tenth, in the diagram.



$$\cos 24^\circ = \frac{\text{length adjacent to }24^\circ}{\text{hypotenuse}}$$

$$\cos 24^\circ = \frac{x}{50\text{ ft}}$$

$$50\text{ ft} \bullet \cos 24^\circ = \frac{x}{\cancel{50\text{ ft}}} \bullet \cancel{50\text{ ft}}$$

$$45.7\text{ ft} \doteq x$$

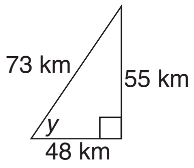
$$\sin 24^\circ = \frac{\text{length opposite }24^\circ}{\text{hypotenuse}}$$

$$\sin 24^\circ = \frac{y}{50\text{ ft}}$$

$$50\text{ ft} \bullet \sin 24^\circ = \frac{y}{\cancel{50\text{ ft}}} \bullet \cancel{50\text{ ft}}$$

$$20.3\text{ ft} \doteq y$$

3. Show that the sine ratio, cosine ratio, and tangent ratio can each be used to determine the measure of angle *y*, to the nearest tenth of a degree, in the following triangle.



$$\sin y = \frac{\text{length opposite } y}{\text{hypotenuse}} \qquad \cos y = \frac{\text{length adjacent to } y}{\text{hypotenuse}} \qquad \tan y = \frac{\text{length opposite } y}{\text{length adjacent to } y}$$

$$\sin y = \frac{55 \text{ km}}{73 \text{ km}} \qquad \cos y = \frac{48 \text{ km}}{73 \text{ km}} \qquad \tan y = \frac{55 \text{ km}}{48 \text{ km}}$$

$$y = \sin^{-1}\left(\frac{55}{73}\right) \qquad y = \cos^{-1}\left(\frac{48}{73}\right) \qquad y = \tan^{-1}\left(\frac{55}{48}\right)$$

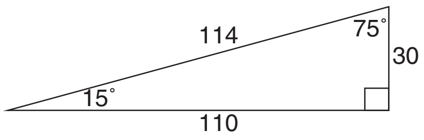
$$y \doteq 48.9^\circ \qquad y \doteq 48.9^\circ \qquad y \doteq 48.9^\circ$$

4. Look at the table in *Lesson 3.2* that shows the tangent, sine, and cosine ratio values for various angles. Notice that $\sin 5^\circ = \cos 85^\circ$, $\sin 10^\circ = \cos 80^\circ$, $\sin 15^\circ = \cos 75^\circ$, etc.

   a. What do the angles in each of the equalities add to?

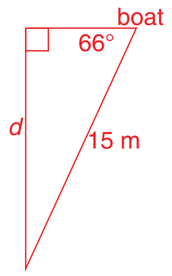
   90°

   b. Use the following triangle to explain the equalities. (Hint: What fraction represents both $\sin 15^\circ$ and $\cos 75^\circ$ in the diagram?)



In the diagram, the side length opposite 15° and the side length adjacent to 75° are both 30. The hypotenuse is 114 for both ratios. This means both sin 15° and cos 75° are equal to $\frac{30}{114}$. In a right triangle, the sine of one acute angle and the cosine of the other acute angle are always equal.

5. A boat is anchored in a river. If there are 15 m of rope between the boat and the bottom of the river and the rope makes an angle of 66° with the surface of the water. How deep is the river to the nearest tenth of a metre?



$$\sin 66^\circ = \frac{\text{length opposite } 66^\circ}{\text{hypotenuse}}$$

$$\sin 66^\circ = \frac{d}{15 \text{ m}}$$

$$15 \text{ m} \cdot \sin 66^\circ = \frac{d}{\cancel{15 \text{ m}}} \cdot \cancel{15 \text{ m}}$$

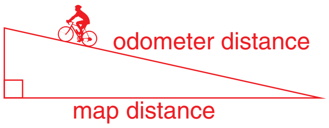
$$13.7 \text{ m} \doteq d$$

The river is approximately 13.7 m deep.

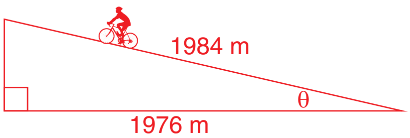
6. Chad cycled up a long straight hill. The odometer on his bike showed that the hill was 1984 m long, while a digital map showed that he was 1976 m from the base of the hill.

a. Assuming Chad rode in a straight line and that both the odometer and the digital map are accurate, why did the two instruments show different distances?

The odometer shows the distance traveled by the bicycle, while the map shows the horizontal distance from start to stop.



b. What is the average angle of elevation of the hill? Express your answer to the nearest degree.



$$\cos \theta = \frac{\text{length adjacent to } \theta}{\text{hypotenuse}}$$

$$\cos \theta = \frac{1976}{1984}$$

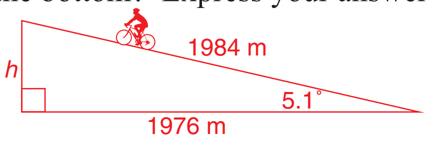
$$\theta = \cos^{-1}\left(\frac{1976}{1984}\right)$$

$$\theta = 5.147...^\circ$$

$$\theta \doteq 5^\circ$$

The average angle of elevation of the hill is approximately 5°.

c. How much higher is Chad when he is at the top of the hill compared to when he is at the bottom? Express your answer to the nearest metre.



$$\sin 5.147...^\circ = \frac{\text{length opposite } 5.147...^\circ}{\text{hypotenuse}}$$

$$\sin 5.147...^\circ = \frac{h}{1984}$$

$$1984 \bullet \sin 5.147...^\circ = \frac{h}{\cancel{1984}} \bullet \cancel{1984}$$

$$178 \text{ m} \doteq h$$

Chad is approximately 178 m higher when he is at the top of the hill.

The Pythagorean theorem could have also been used to solve this problem.

Please complete *Lesson 3.2 Explore Your Understanding Assignment* located in *Workbook 3.2* before proceeding to *Lesson 3.3*.

Lesson 3.3: Solving Problems with Triangles

Practice – IV

1. At the time of this writing, the Burj Khalifa (formerly the Burj Dubai) is the world's tallest building at 829.8 m. If you were to stand 200 m from the centre of the building, what would the angle of elevation be to the top? Express your answer to the nearest degree.

$$\tan \theta = \frac{\text{length opposite } \theta}{\text{length adjacent to } \theta}$$

$$\tan \theta = \frac{829.8 \cancel{\text{m}}}{200 \cancel{\text{m}}}$$

$$\tan \theta = \frac{829.8}{200}$$

$$\theta = \tan^{-1}\left(\frac{829.8}{200}\right)$$

$$\theta \doteq 76^\circ$$

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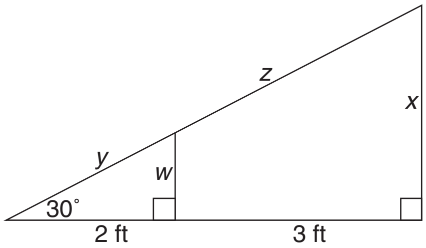
The angle of elevation to the top is approximately 76°.

2. Determine the unknown lengths in the diagram. Express your answers to the nearest tenth of a foot.



$$\tan 30^\circ = \frac{\text{length opposite } 30^\circ}{\text{length adjacent to } 30^\circ}$$
$$\tan 30^\circ = \frac{w}{2\text{ ft}}$$
$$2\text{ ft} \cdot \tan 30^\circ = \frac{w}{\cancel{2\text{ ft}}} \cdot \cancel{2\text{ ft}}$$
$$1.154\ldots\text{ft} = w$$
$$1.2\text{ ft} \doteq w$$

$$\cos 30^\circ = \frac{\text{length adjacent to } 30^\circ}{\text{hypotenuse}}$$
$$\cos 30^\circ = \frac{2\text{ ft}}{y}$$
$$y \cdot \cos 30^\circ = \frac{2\text{ ft}}{\cancel{y}} \cdot \cancel{y}$$
$$\frac{y \cdot \cancel{\cos 30^\circ}}{\cancel{\cos 30^\circ}} = \frac{2\text{ ft}}{\cos 30^\circ}$$
$$y = 2.309\ldots\text{ft}$$
$$y \doteq 2.3\text{ ft}$$

$$\tan 30^\circ = \frac{\text{length opposite } 30^\circ}{\text{length adjacent to } 30^\circ}$$
$$\tan 30^\circ = \frac{x}{5\text{ ft}}$$
$$5\text{ ft} \cdot \tan 30^\circ = \frac{x}{\cancel{5\text{ ft}}} \cdot \cancel{5\text{ ft}}$$
$$2.886\ldots\text{ft} = x$$
$$2.9\text{ ft} \doteq x$$

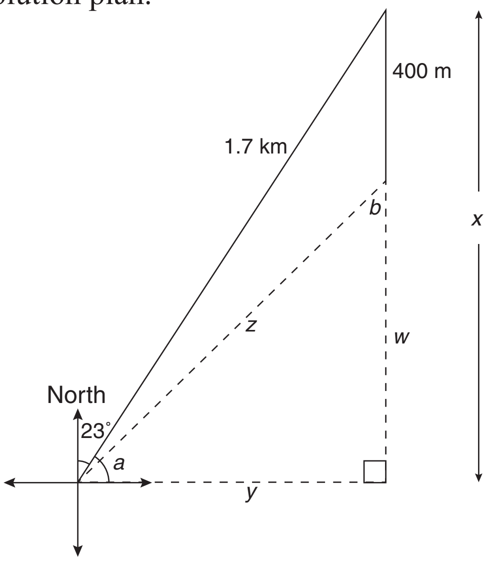
Let v represent the hypotenuse of the larger triangle.

$$a^2 + b^2 = c^2$$
$$(2\text{ ft} + 3\text{ ft})^2 + x^2 = v^2$$
$$(5\text{ ft})^2 + (2.886...\text{ ft})^2 = v^2$$
$$25\text{ ft}^2 + 8.333...\text{ ft}^2 = v^2$$
$$33.333...\text{ ft}^2 = v^2$$
$$\sqrt{33.333...\text{ ft}^2} = \sqrt{v^2}$$
$$5.773...\text{ ft} = v$$

$$z = v - y$$
$$z = 5.773...\text{ ft} - 2.309\ldots\text{ ft}$$
$$z \doteq 3.5\text{ ft}$$

3. While on an orienteering trip, Krastio walks 23º East of North for 1.7 km. He then walks due south for 400 m. To determine his direction and distance back to his original position, Krastio draws the following diagram and solution plan.

   - Use the angle 23º to determine the measure of angle *a*.
   - Use a sine ratio to determine the value of *x*.
   - Use the value of *x* to determine the value of *w*.
   - Use a tangent ratio to determine the value of *y*.
   - Use a tangent ratio to determine the measure of angle *b*.
   - Use the Pythagorean theorem to determine the value of *z*.



   Krastio must return to his original position. Determine the direction and distance that Krastio must walk. Give your answers to the nearest degree and the nearest tenth of a kilometre.

$$23^\circ + a = 90^\circ$$
$$\cancel{23^\circ} + a - \cancel{23^\circ} = 90^\circ - 23^\circ$$
$$a = 67^\circ$$

$$\sin 67^\circ = \frac{\text{length opposite } 67^\circ}{\text{hypotenuse}}$$
$$\sin 67^\circ = \frac{x}{1.7 \text{ km}}$$
$$1.7 \text{ km} \bullet \sin 67^\circ = \frac{x}{\cancel{1.7 \text{ km}}} \bullet \cancel{1.7 \text{ km}}$$
$$1.564\ldots \text{ km} = x$$

$$400 \cancel{\text{m}} \bullet \frac{1 \text{ km}}{1000 \cancel{\text{m}}} = 0.4 \text{ km}$$

$$w = 1.564\ldots \text{ km} - 0.4 \text{ km}$$
$$w = 1.164\ldots \text{ km}$$

$$w^2 + y^2 = z^2$$
$$(1.164\ldots \text{ km})^2 + (0.662\ldots \text{ km})^2 = z^2$$
$$1.356\ldots \text{ km}^2 + 0.438\ldots \text{ km}^2 = z^2$$
$$1.795\ldots \text{ km}^2 = z^2$$
$$\sqrt{1.795\ldots \text{ km}^2} = \sqrt{z^2}$$
$$1.3 \text{ km} \doteq z$$

$$\tan 67^\circ = \frac{\text{length opposite } 67^\circ}{\text{length adjacent to } 67^\circ}$$
$$\tan 67^\circ = \frac{1.56\ldots \text{ km}}{y}$$
$$\tan 67^\circ = \frac{1.56\ldots \text{ km}}{\cancel{y}} \bullet \cancel{y}$$
$$\frac{y \bullet \cancel{\tan 67^\circ}}{\cancel{\tan 67^\circ}} = \frac{1.56 \text{ km}}{\tan 67^\circ}$$
$$y = 0.662\ldots \text{ km}$$

$$\tan b = \frac{\text{length opposite } b}{\text{length adjacent to } b}$$
$$\tan b = \frac{0.662\ldots \cancel{\text{km}}}{1.164\ldots \cancel{\text{km}}}$$
$$\tan b = \frac{0.662\ldots}{1.164\ldots}$$
$$b = \tan^{-1}\left(\frac{0.662\ldots}{1.164\ldots}\right)$$
$$b \doteq 30^\circ$$

Krastio will walk approximately 1.3 km 30° west of south to return to his original position.

Please complete *Lesson 3.3 Explore Your Understanding Assignment*, located in *Workbook 3.3*.