Appendix

Lesson 4.1: Prime Factors, GCF, and LCM

Practice – I

1. A simple cryptosystem uses algorithms with prime factors for encryption and decryption. Given the character code chart below, decrypt the code using prime factorization.

Character Codes

Code	Character	Code	Character	Code	Character
2×5	A	2×13	L	$3 \times 3 \times 5$	W
$2 \times 2 \times 3$	B	$3 \times 3 \times 3$	M	2×23	X
2×7	C	$2 \times 2 \times 7$	N	$2 \times 2 \times 2 \times 2 \times 3$	Y
3×5	D	$2 \times 3 \times 5$	O	7×7	Z
$2 \times 2 \times 2 \times 2$	E	$2 \times 2 \times 2 \times 2 \times 2$	P	$2 \times 5 \times 5$	$
$2 \times 3 \times 3$	F	2×17	Q	$2 \times 2 \times 13$	%
$2 \times 2 \times 5$	G	$2 \times 2 \times 3 \times 3$	R	$2 \times 3 \times 3 \times 3$	*
3×7	H	2×19	S	5×11	+
2×11	I	$2 \times 2 \times 2 \times 5$	T	$2 \times 2 \times 2 \times 7$	–
$2 \times 2 \times 2 \times 3$	J	$2 \times 3 \times 7$	U	2×29	.
5×5	K	$2 \times 2 \times 11$	V	$2 \times 2 \times 3 \times 5$	:

P	R	I	M	E	D		F	O	R
32	36	22	27	16	15		18	30	36

R	A	D	I	C	A	L	S	.
36	10	15	22	14	10	26	38	58

2. Determine the greatest common factor of 54 and 99.

$$\begin{array}{r|l} 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array} \qquad \begin{array}{r|l} 3 & 99 \\ 3 & 33 \\ & 11 \end{array}$$

$$54 = 2 \times 3 \times 3 \times 3$$
$$99 = 3 \times 3 \times 11$$

GCF $= 3 \times 3 = 9$

3. Determine the least common multiple of 5, 48, and 96.

$$\begin{array}{r|l} & 5 \\ & 5 \end{array} \qquad \begin{array}{r|l} 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ & 3 \end{array} \qquad \begin{array}{r|l} 2 & 96 \\ 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ & 3 \end{array}$$

$$5 = 5$$
$$48 = 2 \times 2 \times 2 \times 2 \times 3$$
$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$$

LCM $= 5 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 480$

4. There are 18 male students and 24 female students in a grade 10 math class. The math teacher wants to divide the class into groups that will have equal amounts of girls in each group and equal amounts of boys in each group. What is the greatest number of groups that the teacher can make?

Find the GCF for each gender.

18 – 1, 2, 3, 6, 9, 18

24 – 1, 2, 3, 4, 6, 8, 12, 24

The greatest number of groups the teacher can create is 6 groups, each containing 3 males and 4 females.

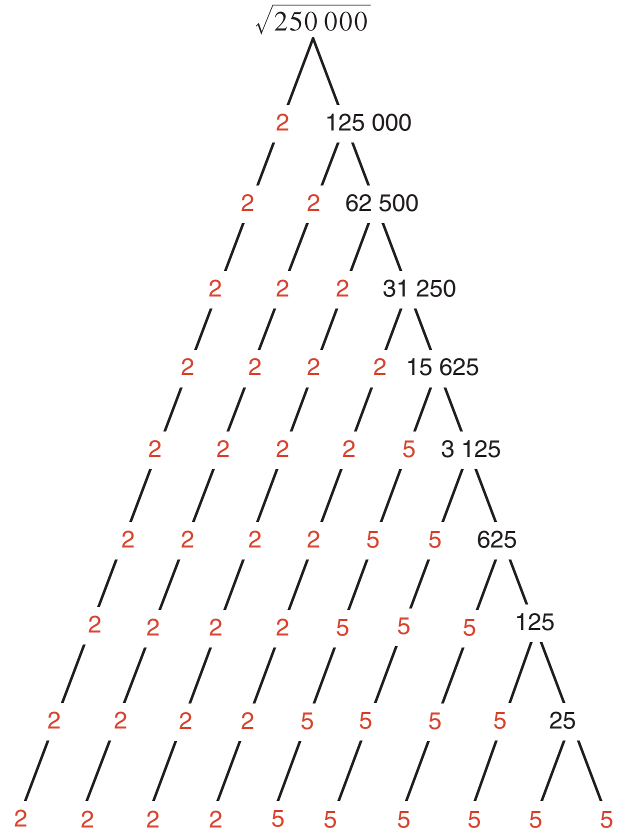
Please complete *Lesson 4.1 Explore Your Understanding Assignment* located in *Workbook 4.1* before proceeding to *Lesson 4.2*.

Lesson 4.2: Mixed and Entire Radicals

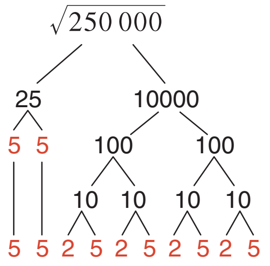
Practice – II

1. Without the use of a calculator, evaluate $\sqrt{250\,000}$.

Prime factorization can be used in a couple of ways.



or



$$\begin{aligned}\sqrt{250\,000} &= \sqrt{2^2\times2^2\times5^2\times5^2\times5^2}\\ &= \sqrt{2^2}\times\sqrt{2^2}\times\sqrt{5^2}\times\sqrt{5^2}\times\sqrt{5^2}\\ &= 2\times2\times5\times5\times5\\ &= (2\times5)\times(2\times5)\times5\\ &= 10\times10\times5\\ &= 500\end{aligned}$$

2. Evaluate the following.

a. $-\sqrt[3]{27}$

$$\begin{aligned}-\sqrt[3]{27} &= -1 \times \sqrt[3]{27} \\ &= -1 \times \sqrt[3]{3^3} \\ &= -1 \times 3 \\ &= -3\end{aligned}$$

b. $\sqrt[3]{\dfrac{125}{512}}$

$$\begin{aligned}\sqrt[3]{\frac{125}{512}} &= \frac{\sqrt[3]{125}}{\sqrt[3]{512}} \\ &= \frac{\sqrt[3]{5^3}}{\sqrt[3]{8^3}} \\ &= \frac{5}{8}\end{aligned}$$

3. Simplify.

a. $\sqrt{72}$

$$\begin{aligned}\sqrt{72} &= \sqrt{36 \times 2} \\ &= \sqrt{36} \times \sqrt{2} \\ &= \sqrt{6^2} \times \sqrt{2} \\ &= 6\sqrt{2}\end{aligned}$$

b. $\sqrt[3]{24}$

$$\begin{aligned}\sqrt[3]{24} &= \sqrt[3]{8 \times 3} \\ &= \sqrt[3]{8} \times \sqrt[3]{3} \\ &= \sqrt[3]{2^3} \times \sqrt[3]{3} \\ &= 2\sqrt[3]{3}\end{aligned}$$

4. Express each of the mixed radicals as an entire radical.

   a. $5\sqrt{2}$

   $$\begin{aligned} 5\sqrt{2} &= \sqrt{5^2 \times 2} \\ &= \sqrt{25 \times 2} \\ &= \sqrt{50} \end{aligned}$$

   b. $2\sqrt[3]{9}$

   $$\begin{aligned} 2\sqrt[3]{9} &= \sqrt[3]{2^3} \times \sqrt[3]{9} \\ &= \sqrt[3]{2^3 \times 9} \\ &= \sqrt[3]{8 \times 9} \\ &= \sqrt[3]{72} \end{aligned}$$

Please complete *Lesson 4.2 Explore Your Understanding Assignment* located in *Workbook 4.2* before proceeding to *Lesson 4.3*.

Lesson 4.3: The Irrational Number System

Practice – III

1. What is the difference between Rational, Irrational, and Real Numbers?
   Rational Numbers can be written as fractions and as repeating or terminating decimals. Irrational Numbers are non-terminating and non-repeating decimals. The Real Number system comprises both Rational and Irrational Numbers.

2. Using benchmarks, what is the approximate value of $\sqrt[3]{2\,185}$?

   $$\sqrt[3]{1\,728} < \sqrt[3]{2\,185} < \sqrt[3]{2\,197}$$
   $$\sqrt[3]{12^3} < \sqrt[3]{2\,185} < \sqrt[3]{13^3}$$
   $$12 < \sqrt[3]{2\,185} < 13$$
   $$\sqrt[3]{2\,185} \doteq 12.9$$

3. Classify each of the following numbers according to the subsets to which they belong.

a. $-\sqrt[3]{-343} = -1 \times \sqrt[3]{(-7)^3} = -1 \times (-7) = 7$
Natural, Whole, Integer, Rational, Real

b. $-\sqrt{81} = -\sqrt{9^2} = -9$ Integer, Rational, Real

c. $-\dfrac{\sqrt[3]{64}}{3} = -\dfrac{\sqrt[3]{64}}{3} = -\dfrac{4}{3}$ Rational, Real

4. Arrange the following numbers from greatest to least.
$-\sqrt[3]{-8}, \sqrt[3]{-8}, \sqrt[3]{-27}, -\sqrt[3]{1}, \sqrt[3]{27}$

$-\sqrt[3]{-8} = -1 \times \sqrt[3]{(-2)^3} = -1 \times -2 = 2$
$\sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2$
$\sqrt[3]{-27} = \sqrt[3]{(-3)^3} = -3$
$-\sqrt[3]{1} = -1 \times \sqrt[3]{1^3} = -1 \times 1 = -1$
$\sqrt[3]{27} = \sqrt[3]{(3)^3} = 3$

The order from greatest to least is: $\sqrt[3]{27}, -\sqrt[3]{-8}, -\sqrt[3]{1}, \sqrt[3]{-8}, \sqrt[3]{-27}$.

Please complete *Lesson 4.3 Explore Your Understanding Assignment* located in *Workbook 4.3* before proceeding to *Lesson 4.4*.

Lesson 4.4: Exponent Laws

Practice – IV

1. Apply the exponent laws to simplify the following expressions.

a. $\dfrac{(2x^{12}y^2)(7x^{-4}y^7)}{(28x^2y)(xy^2)}$

$$\frac{(2x^{12}y^2)(7x^{-4}y^7)}{(28x^2y)(xy^2)} = \frac{14x^8y^9}{28x^3y^3}$$
$$= \frac{x^5y^6}{2}$$

b. $\left(\frac{5a^5b^{-6}}{6a^{-2}b^2}\right)^{-2}$

$$\begin{aligned}\left(\frac{5a^5b^{-6}}{6a^{-2}b^2}\right)^{-2} &= \left(\frac{6a^{-2}b^2}{5a^5b^{-6}}\right)^2\\ &= \frac{6^2a^{-4}b^4}{5^2a^{10}b^{-12}}\\ &= \frac{36a^{(-4-10)}b^{(4-(-12))}}{25}\\ &= \frac{36a^{-14}b^{16}}{25}\\ &= \frac{36b^{16}}{25a^{14}}\end{aligned}$$

c. $(64a^{24}b^8)^{\frac{1}{2}}$

$$\begin{aligned}(64a^{24}b^8)^{\frac{1}{2}} &= 64^{\frac{1}{2}} \bullet a^{24\cdot\frac{1}{2}}b^{8\cdot\frac{1}{2}}\\ &= \sqrt{64} \bullet a^{\frac{24}{2}}b^{\frac{8}{2}}\\ &= 8a^{12}b^4\end{aligned}$$

d. $\left(\frac{343}{216}\right)^{-\frac{2}{3}}$

$$\begin{aligned}\left(\frac{343}{216}\right)^{-\frac{2}{3}} &= \left(\frac{216}{343}\right)^{\frac{2}{3}}\\ &= \left(\frac{\sqrt[3]{216}}{\sqrt[3]{343}}\right)^2\\ &= \left(\frac{6}{7}\right)^2\\ &= \frac{36}{49}\end{aligned}$$

e. $\left(\frac{1}{32}\right)^{-\frac{1}{5}}$

$$\begin{aligned}\left(\frac{1}{32}\right)^{-\frac{1}{5}} &= \left(\frac{32}{1}\right)^{\frac{1}{5}}\\ &= \left(\frac{\sqrt[5]{32}}{\sqrt[5]{1}}\right)^1\\ &= \frac{\sqrt[5]{2^5}}{\sqrt[5]{1^5}}\\ &= 2\end{aligned}$$

Please complete *Lesson 4.4 Explore Your Understanding Assignment*, located in *Workbook 4.4*.