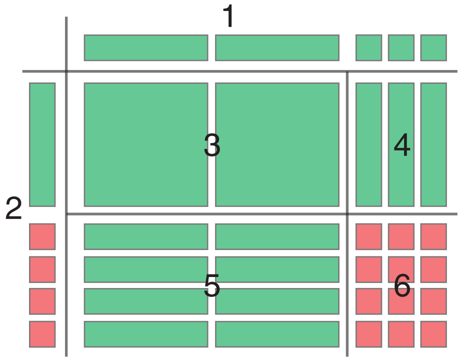
Practice – IV

1. In *Lesson 5.3*, when we were trying to determine a strategy for factoring trinomials of the form $ax^2 + bx + c, a \neq 1$, the binomial factors $(mx + p)$ and $(nx + q)$ were multiplied to give $mnx^2 + (mq + np)x + pq$. Match each expression to the appropriate section of the algebra tile array shown. Explain your choices.

- mn
- mq
- np
- pq
- $mx + p$
- $nx + q$



The expression mn corresponds to section 3. The product represents the coefficient of x^2 (the a-value) of the trinomial $ax^2 + bx + c$.

The expressions mq and np correspond to sections 4 and 5. The sum of the two products represent the coefficient of x (the b-value) of the trinomial $ax^2 + bx + c$.

The expression pq corresponds to section 6. The product represents the constant term (the c-value) of the trinomial $ax^2 + bx + c$.

The binomials $mx + p$ and $nx + q$ correspond to sections 1 and 2. These binomials represent the factors of the trinomial $ax^2 + bx + c$.

2. Factor each of the following.

a. $15x^2 + 16x + 4$

$ac = 60$ and $b = 16$

The integers 10 and 6 have a product of 60 and a sum of 16.

$$\begin{aligned}15x^2 + 16x + 4 &= 15x^2 + (10 + 6)x + 4\\ &= 15x^2 + 10x + 6x + 4\\ &= (15x^2 + 10x) + (6x + 4)\\ &= 5x(3x + 2) + 2(3x + 2)\\ &= (3x + 2)(5x + 2)\end{aligned}$$

b. $4x^2 - 4x + 1$

$ac = 4$ and $b = -4$

The integers –2 and –2 have a product of 4 and a sum of –4.

$$\begin{aligned} 4x^2 - 4x + 1 &= 4x^2 + (-2-2)x + 1 \\ &= 4x^2 - 2x - 2x + 1 \\ &= (4x^2 - 2x) + (-2x + 1) \\ &= 2x(2x-1) - 1(2x-1) \\ &= (2x-1)(2x-1) \end{aligned}$$

c. $-2a^2 - 7a - 3$

$ac = 6$ and $b = -7$

The integers –6 and –1 have a product of 6 and a sum of –7.

$$\begin{aligned} -2a^2 - 7a - 3 &= -2a^2 + (-6-1)a - 3 \\ &= -2a^2 - 6a - a - 3 \\ &= (-2a^2 - 6a) + (-a - 3) \\ &= -2a(a+3) - 1(a+3) \\ &= (a+3)(-2a-1) \end{aligned}$$

3. Mariah tried to factor $3x^2 + 23x - 36$. Her work is shown.

The value of ac is –108 and the value of b is 23. Two numbers that add to give 23 and multiply to give –108 are 27 and –4. This means the factors of $3x^2 + 23x - 36$ are $(x+27)(x-4)$.

Comment on Mariah's strategy. If she made an error, make the necessary corrections.

The first steps towards factorization are correct. However, when the coefficient of the x^2-term is not 1, decomposition of the x-term is necessary. This means that once the integers 27 and –4 are determined, the next step cannot be to simply write the factors as $(x+27)$ and $(x-4)$. Instead, she could have finished as shown.

$$\begin{aligned} 3x^2 + 23x - 36 &= 3x^2 + (27-4)x - 36 \\ &= 3x^2 + 27x - 4x - 36 \\ &= (3x^2 + 27x) + (-4x - 36) \\ &= 3x(x+9) - 4(x+9) \\ &= (x+9)(3x-4) \end{aligned}$$

Please return to *Unit 5: Polynomials Lesson 5.3* in the *Module* to continue your exploration.