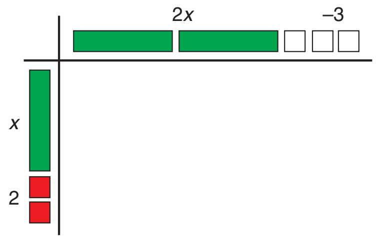
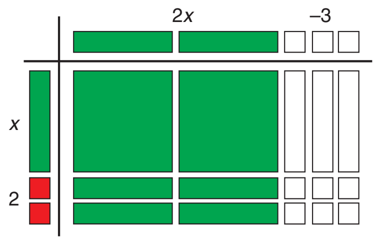
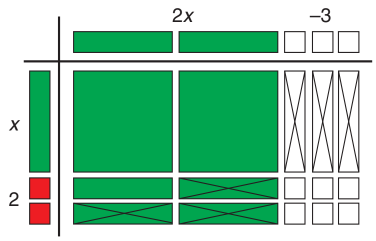
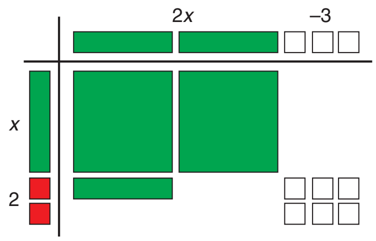
Appendix

Lesson 5.1: Polynomial Multiplication

Practice – I

1. Show the multiplication of $(2x - 3)(x + 2)$ using algebra tiles and symbolically. Show how the steps of the two methods correspond.

Algebra Tiles	Symbolically
	$(2x - 3)(x + 2)$
	$= (2x)(x) + (2x)(2) + (-3)(x) + (-3)(2)$ $= 2x^2 + 4x - 3x - 6$
	$= 2x^2 + (4 - 3)x - 6$
	$= 2x^2 + x - 6$

2. Expand and simplify, if possible.

a. $(1-x)(2-y)$

$$\begin{aligned}(1-x)(2-y) &= (1)(2)+(1)(-y)+(-x)(2)+(-x)(-y)\\ &= 2-y-2x+xy\end{aligned}$$

b. $(n-r)(p+q)$

$$\begin{aligned}(n-r)(p+q) &= (n)(p)+(n)(q)+(-r)(p)+(-r)(q)\\ &= np+nq-rp-rq\end{aligned}$$

c. $(3-x)^2$

$$\begin{aligned}(3-x)(3-x) &= (3)(3)+(3)(-x)+(-x)(3)+(-x)(-x)\\ &= 9-3x-3x+x^2\\ &= 9-6x+x^2\end{aligned}$$

d. $(-z^2-3z+2)(1-z)$

$$\begin{aligned}(-z^2-3z+2)(1-z) &= (-z^2)(1)+(-z^2)(-z)+(-3z)(1)+(-3z)(-z)+(2)(1)+(2)(-z)\\ &= -z^2+z^3-3z+3z^2+2-2z\\ &= z^3+2z^2-5z+2\end{aligned}$$

3. Alex was asked to multiply two binomials: $(6d+2)(7d-5)$

His work is shown below.

$$\begin{aligned}(6d+2)(7d-5) &= (6d)(7d)+(2)(-5)\\ &= 42d^2-10\end{aligned}$$

a. Alex's work is not correct. What error did he make?

Alex did not multiply each term in the first binomial by each term in the second binomial.

b. Write a friendly recommendation to Alex explaining a strategy he could use to improve his solution. In your explanation, suggest how he could numerically verify the product.

Responses will vary. A sample is shown.

Alex, a lot of what you have written is correct, but you are missing some steps.

Using a numerical verification, you can see that your solution has an error. Here I've shown a verification using $d = 2$.

Left Side	Right Side
$(6d+2)(7d-5) = (6(2)+2)(7(2)-5)$ $= (14)(9)$ $= 126$	$42d^2 - 10 = 42(2)^2 - 10$ $= 168 - 10$ $= 158$

If you have the found the correct product, the two sides will be equal for any value of *d*.

To multiply two binomials, you need to multiply each term in the first binomial by each term in the second binomial and then add the products. The four products you should get from multiplying $(6d+2)(7d-5)$ are:

- $(6d)(7d)$
- $(6d)(-5)$
- $(2)(7d)$
- $(2)(-5)$

Then, find the sum of these products and verify your solution.

4. After a book is bound, the three free edges are cut to give the book a clean finish. Suppose a book's pages have uncut dimensions of *l* and *w*, measured in centimetres. Write a binomial multiplication and its product to represent the finished area of a page if 0.75 cm is cut from each free edge.

   $(l-1.5)(w-0.75) = lw - 0.75l - 1.5w + 1.125$

Please complete *Lesson 5.1 Explore Your Understanding Assignment* located in *Workbook 5.1* before proceeding to *Lesson 5.2*.

Lesson 5.2: Common Factors of Polynomials

Practice – II

1. Determine the GCF of $41nr^3$ and $17n^3r$.

   $41nr^3 = 41 \bullet n \bullet r \bullet r \bullet r$

   $17n^3r = 17 \bullet n \bullet n \bullet n \bullet r$

   $n \bullet r = nr$

   The GCF is nr.

2. Explain how to determine the GCF of x^{33}, x^{47}, and x^{25}, by inspection.

   When looking at powers with the same base, the GCF is always equal to the power with the smallest exponent. In this case, the GCF is x^{25}.

3. Write each of $28x^2$ and $42xy^2$ as a product of their GCF and another monomial factor.

   The GCF is $14x$.

   $\frac{28x^2}{14x} = 2x$ $\frac{42xy^2}{14x} = 3y^2$

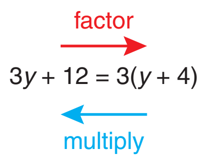
   $28x^2 = (14x)(2x)$ $42xy^2 = (14x)(3y^2)$

4. Write a trinomial with a GCF of $9rs^2$.

   Examples will vary. A sample is shown.

   $(9rs^2)(2r) + (9rs^2)(4s) + (9rs^2)(rs) = 18r^2s^2 + 36rs^3 + 9r^2s^3$

5. This diagram shows that factoring and multiplying are opposite processes. Explain what that means.



   When an expression is factored, the factors can be multiplied to return to the original expression. When two factors are multiplied, the product can be factored to return to the original factors.

6. Factor each of the following polynomials using the greatest common factor.

a. $4x^2 + 10xy - 18y^2$

The GCF is 2.

$\frac{4x^2}{2} = 2x^2 \qquad \frac{10xy}{2} = 5xy \qquad \frac{-18y^2}{2} = -9y^2$

$4x^2 + 10xy - 18y^2 = 2(2x^2 + 5xy - 9y^2)$

b. $-12a^3b^2c^2 - 18a^2b^2c^2 - 36a^2b^3c$

The GCF is $6a^2b^2c$.

$\frac{-12a^3b^2c^2}{6a^2b^2c} = -2ac \qquad \frac{-18a^2b^2c^2}{6a^2b^2c} = -3c \qquad \frac{-36a^2b^3c}{6a^2b^2c} = -6b$

$-12a^3b^2c^2 - 18a^2b^2c^2 - 36a^2b^3c = 6a^2b^2c(-2ac - 3c - 6b)$

Alternatively, $-12a^3b^2c^2 - 18a^2b^2c^2 - 36a^2b^3c = -6a^2b^2c(2ac + 3c + 6b)$.

7. The surface area formulas are shown for three objects.

Right Prism	$SA = 2lw + 2hw + 2lh$
Right Cylinder	$SA = 2\pi r^2 + 2\pi rh$
Right Cone	$SA = \pi r^2 + \pi rs$

Write an alternative surface area formula for each object by factoring using the greatest common factor.

$2lw + 2hw + 2lh = 2(lw + hw + lh)$
$2\pi r^2 + 2\pi rh = 2\pi r(r + h)$
$\pi r^2 + \pi rs = \pi r(r + s)$

8. Chaz factored $4x^2 + 12x - xy - 3y$ as follows.

$$\begin{aligned} 4x^2 + 12x - xy - 3y &= (4x^2 + 12x) + (-xy - 3y) \\ &= (4x)(x + 3) + (-y)(x + 3) \\ &= (x + 3)(4x - y) \end{aligned}$$

Explain Chaz's strategy.

Explanations will vary. A sample is shown.

Chaz began by splitting the polynomial into two separate groups. He then determined the GCF of each group using the distributive property in reverse. A factor of $x + 3$ was common to each group, leaving $4x - y$.

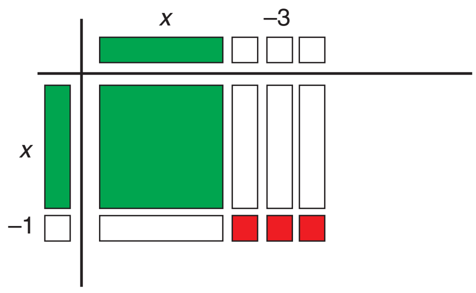
Please complete *Lesson 5.2 Explore Your Understanding Assignment* located in *Workbook 5.2* before proceeding to *Lesson 5.3*.

Lesson 5.3: Factoring Trinomials

Practice – III

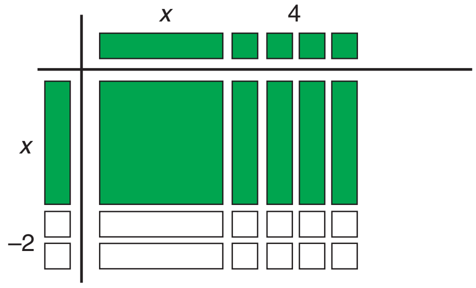
1. Use algebra tiles to factor the following trinomials.

   a. $x^2 - 4x + 3$



   $x^2 - 4x + 3 = (x - 3)(x - 1)$

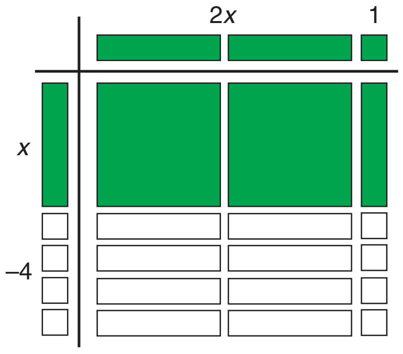
   b. $p^2 + 2p - 8$



   Two pairs of x and $-x$ tiles were added to complete the rectangle.

   $p^2 + 2p - 8 = (p + 4)(p - 2)$

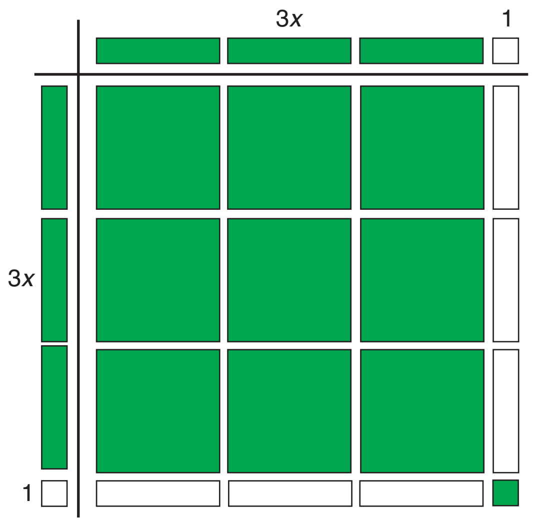
c. $2r^2 - 7r - 4$



A pair of x and $-x$ tiles was added to complete the rectangle.

$2r^2 - 7r - 4 = (2r + 1)(r - 4)$

d. $9x^2 - 6x + 1$



$9x^2 - 6x + 1 = (3x + 1)(3x + 1)$

2. Identify two integers with the given product and sum.

   a. product = 42, sum = 13

      6 and 7

   b. product = 36, sum = –13

      –9 and –4

   c. product = –9, sum = 0

      3 and –3

3. Factor each of the following.

   a. $x^2 + x - 12$

   $b = 1$ and $c = -12$

   The integers 4 and –3 have a sum of 1 and a product of –12.

   $x^2 + x - 12 = (x + 4)(x - 3)$

   b. $i^2 - 10i + 25$

   $b = -10$ and $c = 25$

   The integers –5 and –5 have a sum of –10 and a product of 25.

   $i^2 - 10i + 25 = (i - 5)(i - 5)$

   c. $x^2 - 9$ (Hint: This isn't a trinomial, but it can be factored using the same strategy.)

   $b = 0$ and $c = -9$

   The integers 3 and –3 have a sum of 0 and a product of –9.

   $x^2 - 9 = (x + 3)(x - 3)$

4. Luke factored $x^2 - 9x + 14$ as shown.

   I know –7 and –2 have a sum of –9 and a product of 14, so the factors must be $x - 7$ and $x - 2$.

   Luke showed his work to Destiny, who was working on the same problem. She said that Luke could not be correct because she found different factors for $x^2 - 9x + 14$. Then, she showed Luke her verification.

$$\begin{aligned}(2-x)(7-x) &= (2)(7)+(2)(-x)+(7)(-x)+(-x)(-x)\\ &= 14 - 2x - 7x + x^2\\ &= 14 - 9x + x^2\\ &= x^2 - 9x + 14\end{aligned}$$

   Explain how this discussion could be resolved.

   Both students have shown a correct factorization. It is possible for a trinomial to be factored in different ways, resulting in what might initially appear to be a different set of factors. In this case if Luke's factors are multiplied by $(-1)(-1) = 1$, the result is Destiny's factors.

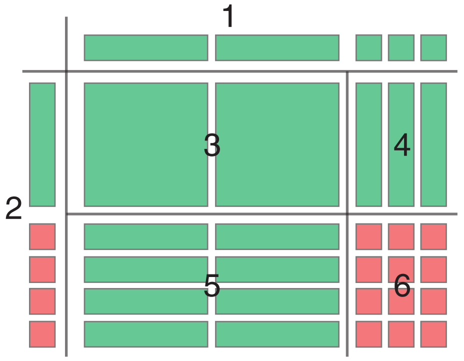
$$\begin{aligned}(x-7)(x-2) &= (1)(x-7)(x-2)\\ &= (-1 \bullet -1)(x-7)(x-2)\\ &= -1(x-7) \bullet -1(x-2)\\ &= (-x+7)(-x+2)\\ &= (7-x)(2-x)\end{aligned}$$

Please return to *Unit 5: Polynomials Lesson 5.3* in the *Module* to continue your exploration.

Practice – IV

1. In *Lesson 5.3*, when we were trying to determine a strategy for factoring trinomials of the form $ax^2 + bx + c, a \neq 1$, the binomial factors $(mx + p)$ and $(nx + q)$ were multiplied to give $mnx^2 + (mq + np)x + pq$. Match each expression to the appropriate section of the algebra tile array shown. Explain your choices.

- mn
- mq
- np
- pq
- $mx + p$
- $nx + q$



The expression mn corresponds to section 3. The product represents the coefficient of x^2 (the a-value) of the trinomial $ax^2 + bx + c$.

The expressions mq and np correspond to sections 4 and 5. The sum of the two products represent the coefficient of x (the b-value) of the trinomial $ax^2 + bx + c$.

The expression pq corresponds to section 6. The product represents the constant term (the c-value) of the trinomial $ax^2 + bx + c$.

The binomials $mx + p$ and $nx + q$ correspond to sections 1 and 2. These binomials represent the factors of the trinomial $ax^2 + bx + c$.

2. Factor each of the following.

a. $15x^2 + 16x + 4$

$ac = 60$ and $b = 16$

The integers 10 and 6 have a product of 60 and a sum of 16.

$$\begin{aligned} 15x^2 + 16x + 4 &= 15x^2 + (10 + 6)x + 4 \\ &= 15x^2 + 10x + 6x + 4 \\ &= (15x^2 + 10x) + (6x + 4) \\ &= 5x(3x + 2) + 2(3x + 2) \\ &= (3x + 2)(5x + 2) \end{aligned}$$

b. $4x^2 - 4x + 1$

$ac = 4$ and $b = -4$

The integers –2 and –2 have a product of 4 and a sum of –4.

$$\begin{aligned} 4x^2 - 4x + 1 &= 4x^2 + (-2 - 2)x + 1 \\ &= 4x^2 - 2x - 2x + 1 \\ &= (4x^2 - 2x) + (-2x + 1) \\ &= 2x(2x - 1) - 1(2x - 1) \\ &= (2x - 1)(2x - 1) \end{aligned}$$

c. $-2a^2 - 7a - 3$

$ac = 6$ and $b = -7$

The integers –6 and –1 have a product of 6 and a sum of –7.

$$\begin{aligned} -2a^2 - 7a - 3 &= -2a^2 + (-6 - 1)a - 3 \\ &= -2a^2 - 6a - a - 3 \\ &= (-2a^2 - 6a) + (-a - 3) \\ &= -2a(a + 3) - 1(a + 3) \\ &= (a + 3)(-2a - 1) \end{aligned}$$

3. Mariah tried to factor $3x^2 + 23x - 36$. Her work is shown.

The value of *ac* is –108 and the value of *b* is 23. Two numbers that add to give 23 and multiply to give –108 are 27 and –4. This means the factors of $3x^2 + 23x - 36$ are $(x + 27)(x - 4)$.

Comment on Mariah's strategy. If she made an error, make the necessary corrections.

The first steps towards factorization are correct. However, when the coefficient of the x^2-term is not 1, decomposition of the *x*-term is necessary. This means that once the integers 27 and –4 are determined, the next step cannot be to simply write the factors as $(x + 27)$ and $(x - 4)$. Instead, she could have finished as shown.

$$\begin{aligned} 3x^2 + 23x - 36 &= 3x^2 + (27 - 4)x - 36 \\ &= 3x^2 + 27x - 4x - 36 \\ &= (3x^2 + 27x) + (-4x - 36) \\ &= 3x(x + 9) - 4(x + 9) \\ &= (x + 9)(3x - 4) \end{aligned}$$

Please return to *Unit 5: Polynomials Lesson 5.3* in the *Module* to continue your exploration.

Practice – V

1. Factor each of the following expressions.

a. $7x^2 + 21x + 14$

$7x^2 + 21x + 14 = 7(x^2 + 3x + 2)$

$b = 3$ and $c = 2$

The integers 2 and 1 have a sum of 3 and a product of 2.

$7(x^2 + 3x + 2) = 7(x + 2)(x + 1)$

b. $6r^2 + 12rs + 6s^2$

$6r^2 + 12rs + 6s^2 = 6(r^2 + 2rs + s^2)$

$b = 2$ and $c = 1$

The integers 1 and 1 have a sum of 2 and a product of 1.

$6(r^2 + 2rs + s^2) = 6(r + s)(r + s)$

c. $4x^2 - 4xy - 8y^2$

$4x^2 - 4xy - 8y^2 = 4(x^2 - xy - 2y^2)$

$b = -1$ and $c = -2$

The integers –2 and 1 have a sum of –1 and a product of –2.

$4(x^2 - xy - 2y^2) = 4(x - 2y)(x + y)$

d. $-6x^2+10xy+4y^2$

$-6x^2+10xy+4y^2=2(-3x^2+5xy+2y^2)$

$ac=-6$ and $b=5$

The integers 6 and –1 have a product of –6 and a sum of 5.

$$\begin{aligned}2(-3x^2+5xy+2y^2)&=2(-3x^2+(6-1)xy+2y^2)\\&=2(-3x^2+6xy-xy+2y^2)\\&=2((-3x^2+6xy)+(-xy+2y^2))\\&=2(3x(-x+2y)+y(-x+2y))\\&=2(-x+2y)(3x+y)\\&\text{OR}\\&=2(2y-x)(3x+y)\end{aligned}$$

2. Using an example, explain why factoring a GCF out of a trinomial can make factoring the trinomial easier.

Examples will vary. A sample is shown.

To factor the trinomial $12x^2-84x+120$ without first removing a GCF, you would need to determine two integers that have a product of $12\times120=1440$ and a sum of -84. These are large numbers, and determining the integers could take a long time.

By factoring out the GCF of 12, you have smaller numbers to work with.

$12x^2-84x+120=12(x^2-7x+10)$

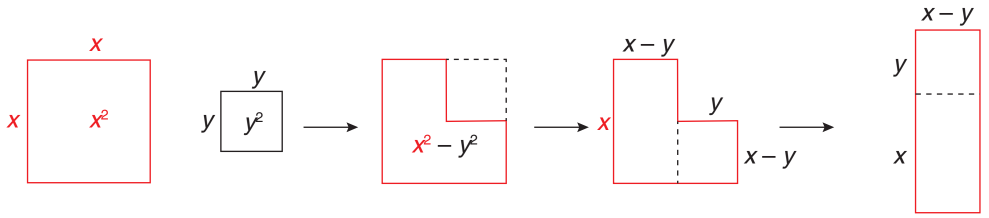
Now, you only need to determine integers that add to –7 and multiply to 10. This is a much easier task.

Please complete *Lesson 5.3 Explore Your Understanding Assignment* located in *Workbook 5.3* before proceeding to *Lesson 5.4*.

Lesson 5.4: Other Factoring Strategies

Practice – VI

1. This diagram was used in *Lesson 5.4* to help explain factoring a difference of squares.



a. Explain the diagram.

Explanations will vary. A sample is shown.

- The first step shows that a square with side length x has an area of x^2 and that a square with a side length y has an area of y^2.
- The second step shows that $x^2 - y^2$ can be represented by removing a square of length y from a square of length x.
- The third section step shows that the remaining area can be split into two rectangles. One has length x and width $x - y$ and the other has length y and width $x - y$.
- The fourth step shows that the two rectangles can be placed side-by-side to form a larger rectangle because both rectangles have a width of $x - y$.

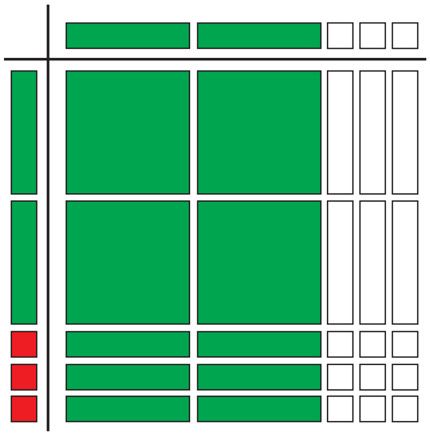
b. How does this diagram show that $x^2 - y^2 = (x + y)(x - y)$?

The area of the shape in the second step can be represented by $x^2 - y^2$. This same area is represented in the final diagram as a rectangle with length $x + y$ and width $x - y$. Because the two areas are equal, $x^2 - y^2$ must equal $(x + y)(x - y)$.

2. Show that $4p^2 - 9$ can be factored using each of the following methods.

   a. algebra tiles

   Six pairs of *x* and –*x* tiles must be introduced to form a rectangle.



   $4p^2 - 9 = (2p - 3)(2p + 3)$

   b. decomposition

   $ac = -36$ and $b = 0$

   The integers –6 and 6 have a product of –36 and a sum of 0.

$$\begin{aligned} 4p^2 - 9 &= 4p^2 - 6p + 6p - 9 \\ &= (4p^2 - 6p) + (6p - 9) \\ &= 2p(2p - 3) + 3(2p - 3) \\ &= (2p - 3)(2p + 3) \end{aligned}$$

   c. a difference of squares

   $\sqrt{4p^2} = 2p$ and $\sqrt{9} = 3$

   $4p^2 - 9 = (2p + 3)(2p - 3)$

3. Factor each of the following expressions.

   a. $121 - p^2$

   $\sqrt{121} = 11$ and $\sqrt{p^2} = p$

   $121 - p^2 = (11 + p)(11 - p)$

b. $a^2 - b^2$

$\sqrt{a^2} = a$ and $\sqrt{b^2} = b$

$a^2 - b^2 = (a+b)(a-b)$

4. Factor each of the following expressions.

a. $n^2 - 4n + 4$

$$\begin{aligned} n^2 - 4n + 4 &= n^2 + 2(n)(-2) + (-2)^2 \\ &= (n-2)^2 \end{aligned}$$

The trinomial $n^2 - 4n + 4$ can also be factored to $(2-n)^2$.

b. $4t^2 + 8t + 4$

$$\begin{aligned} 4t^2 + 8t + 4 &= 4(t^2 + 2t + 1) \\ &= 4(t^2 + 2(t)(1) + 1^2) \\ &= 4(t+1)^2 \end{aligned}$$

5. Arnold said he can multiply some large numbers easily by factoring a difference of squares. He showed the following example.

$$\begin{aligned} (54)(46) &= (50+4)(50-4) \\ &= 50^2 - 4^2 \\ &= 2500 - 16 \\ &= 2484 \end{aligned}$$

Comment on Arnold's procedure.

Responses will vary. A sample is shown.

Arnold's strategy is reasonable, but he never actually factored a difference of squares. A better description may be that he used the difference of squares relationship to multiply large numbers.

Please complete *Lesson 5.4 Explore Your Understanding Assignment*, located in *Workbook 5.4*.