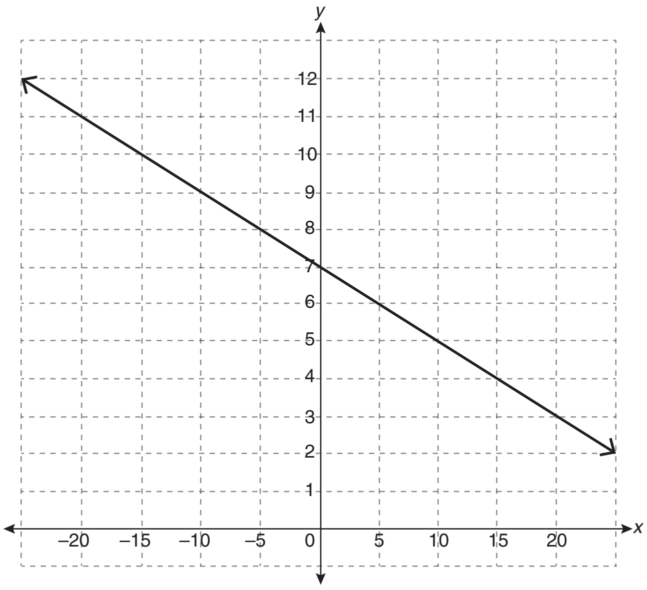
Appendix

Lesson 7.1: Slope-Intercept Form of a Linear Equation

Practice – I

1. State the slope and y-intercept of the following graph. Explain how you determined each.



The slope is $-\frac{1}{5}$ and the y-intercept is 7.

The slope can be determined using two points and the slope formula. The points (15, 4) and (0, 7) are used below.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{7-4}{0-15} \\ &= -\frac{3}{15} \\ &= -\frac{1}{5} \end{aligned}$$

The y-intercept can be determined by looking at where the graph of the relation crosses the y-axis. This happens at (0, 7), so the y-intercept is 7.

2. Write each of the following equations in slope-intercept form.

   a. $y + 6 = 3x$

$$y + 6 = 3x$$
$$y + \cancel{6} - \cancel{6} = 3x - 6$$
$$y = 3x - 6$$

   b. $x = 3y - 18$

$$x = 3y - 18$$
$$x + 18 = 3y - \cancel{18} + \cancel{18}$$
$$\frac{x + 18}{3} = \frac{\cancel{3}y}{\cancel{3}}$$
$$\frac{1}{3}x + 6 = y$$

   c. $3x + 12y + 22 = 0$

$$3x + 12y + 22 = 0$$
$$3x + \cancel{12y} + 22 - \cancel{12y} = 0 - 12y$$
$$3x + 22 = -12y$$
$$\frac{3x + 22}{-12} = \frac{\cancel{-12}y}{\cancel{-12}}$$
$$-\frac{3}{12}x - \frac{22}{12} = y$$
$$-\frac{1}{4}x - \frac{11}{6} = y$$

3. Consider the slope-intercept form of a linear equation, $y = mx + b$.

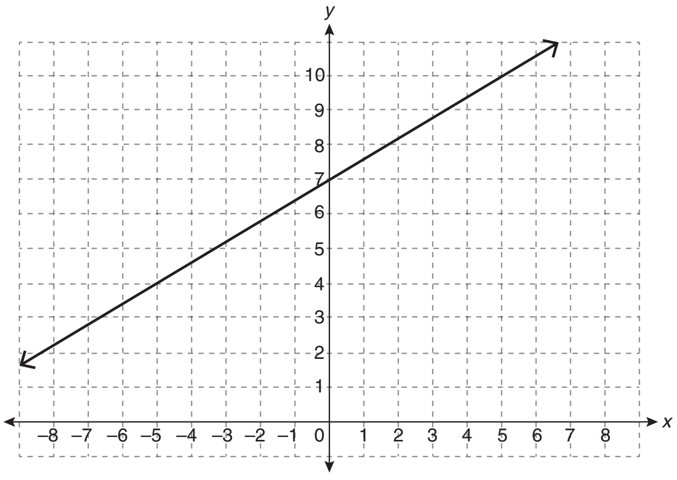
   a. Explain how this form can be used to graph a relation by hand.

   Graphing strategies will vary. Strategies include

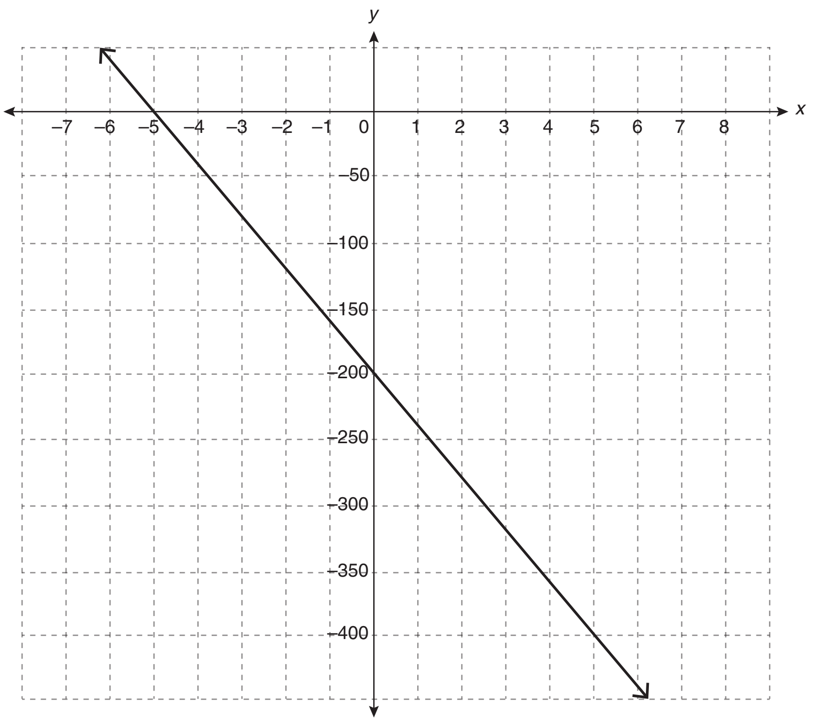
   - Make a table of x and y-values using the equation of the relation. Plot these points and draw a line through them.
   - Use the b-value to plot the y-intercept. Use the slope to determine at least one other point on the graph of the relation. Draw a line through the points.

b. Graph each of the following.

i. $y = \frac{3}{5}x + 7$



ii. $y = -200 - 40x$



4. Explain how technology could be used to check your graphs from question 3.

The graphs in question 3 can be checked by graphing the same relations using a graphing calculator or graphing program and verifying that the two graphs look the same when the same scales are used.

5. The graph of a linear relation with a slope of 5.8 passes through the point (–2, –5). Determine an equation for the relation, in slope-intercept form.

$$y = mx + b$$
$$-5 = 5.8(-2) + b$$
$$-5 = -11.6 + b$$
$$-5 + 11.6 = -\cancel{11.6} + b + \cancel{11.6}$$
$$6.6 = b$$

The equation is $y = 5.8x + 6.6$.

6. a. Explain how you can use the equation $y = 4.592x - 8.387$ to determine points on the corresponding graph.

Explanations will vary. A sample is shown.

Select an x-value and substitute it into the equation to determine the corresponding value of y. This ordered pair represents a point on the graph. Repeat this procedure using different x-values to determine other points on the graph.

b. State three points that could be used to graph the relation $y = 4.592x - 8.387$.

Points will vary. Some possible points are listed.

x	y
–5	–31.347
–4	–26.755
–3	–22.163
–2	–17.571
–1	–12.979
0	–8.387
1	–3.795
2	0.797
3	5.389
4	9.981
5	14.573

7. A plumbing company installs tankless hot water heaters and charges for both installation time and materials used.

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a. The heater and supplies cost $1 800 and the shop charges $110/h for a plumber and an apprentice. Write an equation to represent the total cost to the customer. Be sure to state what each variable represents.

Let C be the total cost and let T be the installation time in hours. The equation is $C = 110T + 1\,800$.

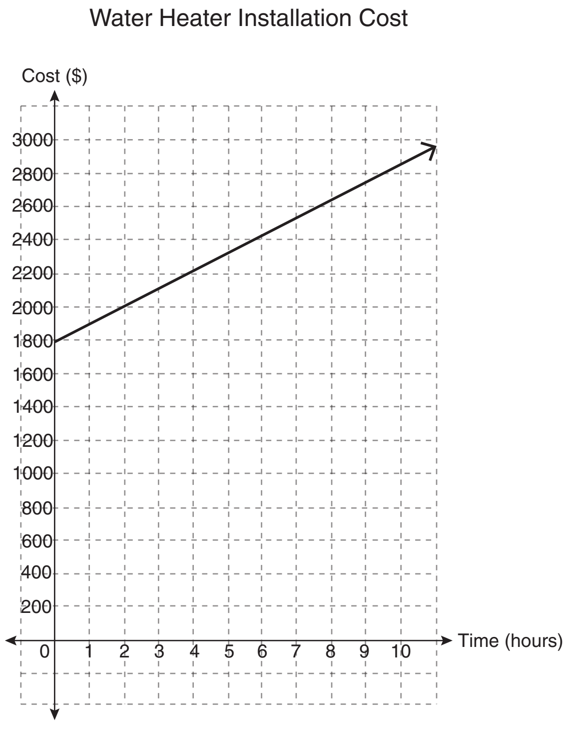
b. i. What is the slope of the relation?

110

ii. What is the vertical-axis intercept of the relation?

1 800

c. Sketch the graph of the relation.



d. If the installation takes 2 hours, how much will the customer be charged?

$C = 110t + 1\,800$

$C = 110(2) + 1\,800$

$C = 2\,020$

The installation will cost $2 020.

e. If a customer was charged $2 185, how long did the installation take?

$$
\begin{aligned}
C &= 110t + 1\,800 \\
2185 &= 110t + 1\,800 \\
2\,185 - 1\,800 &= 110t + \cancel{1\,800} - \cancel{1\,800} \\
385 &= 110t \\
\frac{385}{110} &= \frac{\cancel{110}t}{\cancel{110}} \\
3.5 &= t
\end{aligned}
$$

The installation took 3.5 hours.

Please complete *Lesson 7.1 Explore Your Understanding Assignment* located in *Workbook 7.1* before proceeding to *Lesson 7.2*.

Lesson 7.2: General Form of a Linear Equation

Practice – II

1. Rewrite each of the following equations in general form, $Ax + By + C = 0$.

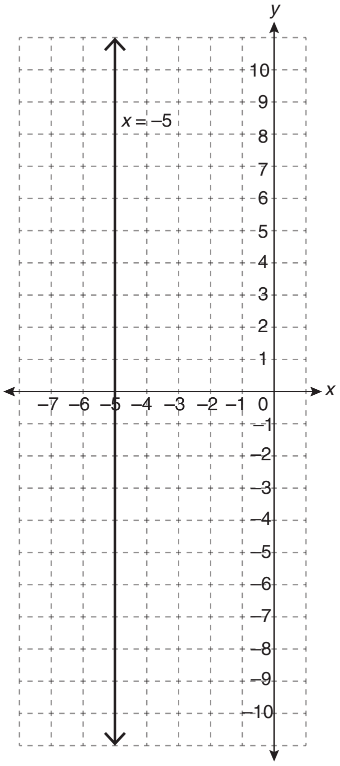
a. $y = -3x - 6$

$$
\begin{aligned}
y &= -3x - 6 \\
y + 3x + 6 &= -\cancel{3x} - \cancel{6} + \cancel{3x} + \cancel{6} \\
3x + y + 6 &= 0
\end{aligned}
$$

b. $y = \frac{2}{3}x - 7$

$$
\begin{aligned}
y &= \frac{2}{3}x - 7 \\
\cancel{y} - \cancel{y} &= \frac{2}{3}x - 7 - y \\
0 &= \frac{2}{3}x - y - 7 \\
3(0) &= 3\left(\frac{2}{3}x - y - 7\right) \\
0 &= 2x - 3y - 21
\end{aligned}
$$

2. Sketch the graph of $x = -5$.



3. The y-axis has can be represented by the equation $x = 0$.

   a. What is the x-intercept of the y-axis?

      0

   b. The line $x = 0$ has an infinite number of y-intercepts. Explain what this means.

      The line $x = 0$ and the y-axis are the same line, so every point on the line is a y-intercept. There are an infinite number of points on each line, so there are an infinite number of y-intercepts.

4. State the equation of a vertical line that passes through the point $(5, -7)$.

   $x = 5$

5. Pravin is planning a garden of tomatoes and pumpkins. His garden has a total area of 300 ft^2. Pravin writes the following equation to represent the number of plants he can include.

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$$4t + 25p = 300$$

a. Explain what you expect each term of Pravin's equation to represent.

The $4t$ represents the total area the tomatoes will occupy. If t represents the number of tomato plants, each plant will occupy 4 ft^2. The $25p$ represents the total area the pumpkins will occupy. If p represents the number of pumpkin plants, each plant will occupy 25 ft^2. The 300 represents the total area of the garden, in square feet.

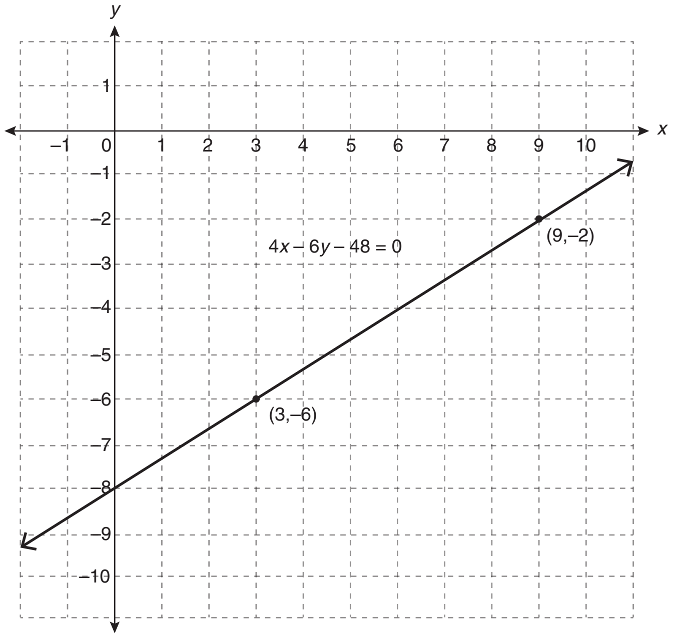
b. If Pravin plans to plant 30 tomato plants, how many pumpkin plants can he use?

$$4t + 25p = 300$$
$$4(30) + 25p = 300$$
$$120 + 25p = 300$$
$$120 + 25p - 120 = 300 - 120$$
$$25p = 180$$
$$\frac{25p}{25} = \frac{180}{25}$$
$$p = 7.2$$

Pravin will be able to plant 7 pumpkin plants in his garden.

6. Ryan says that he can graph a linear relation that is in general form without using x- and y-axis intercepts. Below is his work showing how to graph $4x - 6y - 48 = 0$ using this strategy.

$$\begin{aligned} 4x - 6y - 48 &= 0 \\ 4(3) - 6y - 48 &= 0 \\ 12 - 6y - 48 &= 0 \\ -36 - 6y &= 0 \\ -6y &= 36 \\ y &= -6 \end{aligned} \qquad \begin{aligned} 4x - 6y - 48 &= 0 \\ 4(9) - 6y - 48 &= 0 \\ 36 - 6y - 48 &= 0 \\ -12 - 6y &= 0 \\ -6y &= 12 \\ y &= -2 \end{aligned}$$



a. Explain Ryan's strategy.

Any ordered pair that satisfies the equation of a linear relation will correspond to a point on its graph. By inputting x-values, and solving for y, Ryan determined two such ordered pairs. Once two points on a line are known, the line can be drawn.

b. Give a reason people might prefer to use the intercepts instead of Ryan's method.

Determining the intercepts is a special case of Ryan's strategy. Intercepts tend to be easier to use because substituting 0 for a variable eliminates that term, resulting in a simpler equation.

7. Galaxy High School students want to raise \$1 200 to support their student government activities. They sell sweatshirts for a profit of \$5.75 and t-shirts for a profit of \$3.50.

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a. Write a linear equation that represents the number of each type of shirt needing to be sold to reach their goal.

Let *s* represent the number of sweatshirts sold and let *t* represent the number of t-shirts sold.

This means 5.75*s* represents the income from the sweatshirts and 3.5*t* represents the income from the t-shirts. These two expressions can be added to give the total desired income.

$5.75s + 3.5t = 1\,200$

b. State the domain and range of the graph of the relation if the students plan to stop selling once they have raised \$1 200.

The minimum number of t-shirts is 0. The maximum number of t-shirts occurs when no sweatshirts are sold.

$$\begin{aligned} 5.75s + 3.5t &= 1\,200 \\ 5.75(0) + 3.5t &= 1\,200 \\ 3.5t &= 1\,200 \\ \frac{\cancel{3.5}t}{\cancel{3.5}} &= \frac{1\,200}{3.5} \\ t &\doteq 342.9 \end{aligned}$$

The maximum number of t-shirts sold can be 343.

The minimum number of sweatshirts is 0. The maximum number of sweatshirts occurs when no t-shirts are sold.

$$\begin{aligned} 5.75s + 3.5t &= 1\,200 \\ 5.75s + 3.5(0) &= 1\,200 \\ 5.75s &= 1\,200 \\ \frac{\cancel{5.75}s}{\cancel{5.75}} &= \frac{1\,200}{5.75} \\ s &\doteq 208.7 \end{aligned}$$

The maximum number of sweatshirts sold can be 209.

Domain: $\{s \mid 0 \le s \le 209, s \in W\}$ or $[0, 209]$

Range: $\{t \mid 0 \le t \le 343, t \in W\}$ or $[0, 343]$

There is no clear independent or dependent variable, so the domain and range can be switched.

Please complete *Lesson 7.2 Explore Your Understanding Assignment* located in *Workbook 7.2* before proceeding to *Lesson 7.3*.

Lesson 7.3: Slope-Point Form of a Linear Equation

Practice – III

1. Convert the equation $y-5=-\frac{3}{8}(x-12)$ into

   a. slope-intercept form

$$y-5=-\frac{3}{8}(x-12)$$
$$y-5=-\frac{3}{8}x+\frac{9}{2}$$
$$y=-\frac{3}{8}x+\frac{19}{2}$$

   b. general form

$$y-5=-\frac{3}{8}(x-12)$$
$$y-5=-\frac{3}{8}x+\frac{9}{2}$$
$$\frac{3}{8}x+y-\frac{19}{2}=0$$
$$8\left(\frac{3}{8}x+y-\frac{19}{2}\right)=8(0)$$
$$3x+8y-76=0$$

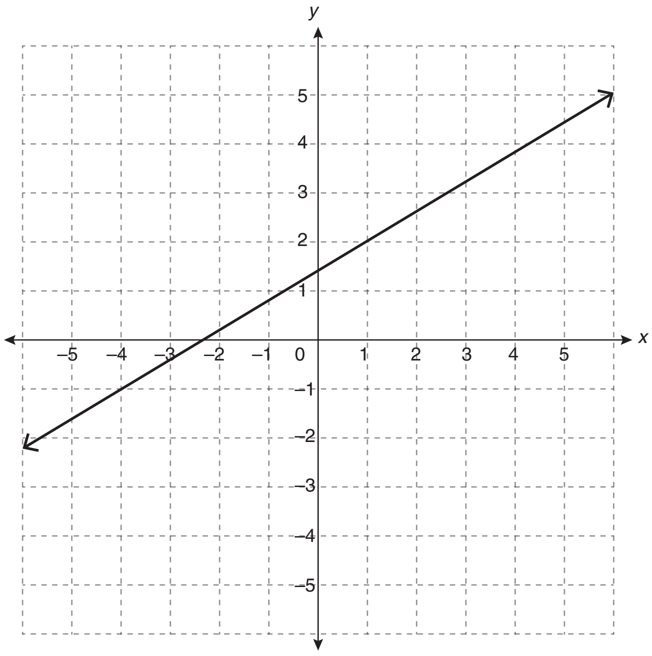
2. A line passes through the points $(-5,-5)$ and $(19,-3)$. Determine the equation of this line, in slope-point form.

$$\begin{aligned} m &= \frac{y_2-y_1}{x_2-x_1} \\ &= \frac{-3-(-5)}{19-(-5)} \\ &= \frac{2}{24} \\ &= \frac{1}{12} \end{aligned}$$

Use one of the points to write the equation in slope-point form.

$$y-y_1=m(x-x_1)$$
$$y-(-5)=\frac{1}{12}(x-(-5))$$
$$y+5=\frac{1}{12}(x+5)$$

3. State two different equations in slope-point form that represent the graph of the relation shown.



The slope of the line is $\frac{3}{5}$, and the points (–4, –1) and (1, 2) are both on the line.

$$y - y_1 = m(x - x_1) \qquad y - y_1 = m(x - x_1)$$

$$y - (-1) = \frac{3}{5}(x - (-4)) \qquad y - 2 = \frac{3}{5}(x - 1)$$

$$y + 1 = \frac{3}{5}(x + 4)$$

Two possible equations are $y + 1 = \frac{3}{5}(x + 4)$ and $y - 2 = \frac{3}{5}(x - 1)$.

4. The graph of a linear relation has a slope of 16.5 and an x-intercept of 121. Determine the y-intercept.

The x-intercept corresponds to the point $(121,0)$.

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 16.5(x - 121)$$
$$y = 16.5x - 1\,996.5$$

Despite starting with the slope-point form, the equation simplified to slope-intercept form, making the y-intercept easy to identify.

The y-intercept is –1 996.5.

5. While planning a trip to Europe, Brian and Donna exchanged some Canadian dollars for euros. Brian bought €300 for $430 and Donna bought €450 for $640 from a merchant who uses a linear relation to calculate the rate.

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a. Let *x* represent the euros purchased and let *y* represent the cost, in Canadian dollars. Determine the slope of the graph of the relation.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{640 - 430}{450 - 300} \\ &= \frac{210}{150} \\ &= 1.4 \end{aligned}$$

b. What does the slope represent in this scenario?

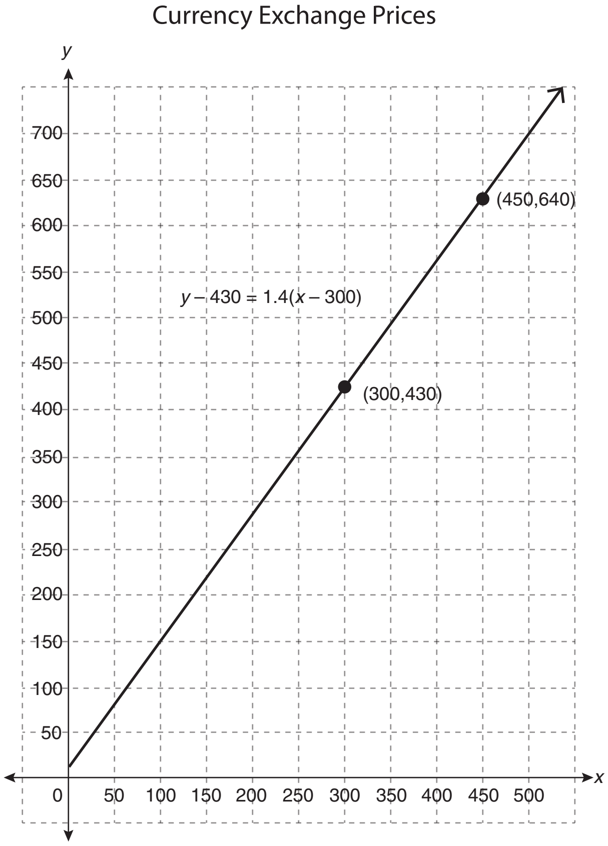
The slope represents the number of Canadian dollars required to buy one euro. Including units, the slope can be written as 1.4 CAD/EUR.

c. Write a currency exchange equation in slope-point form.

Equations will vary depending on the point used. The equation shown uses the point (300,430).

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 430 &= 1.4(x - 300) \end{aligned}$$

d. Graph the relation represented by the currency exchange equation.



e. The merchant charges a service fee for each exchange. What characteristic of the graph represents the service fee? What is the service fee?

The y-intercept represents the service fee, in Canadian dollars.

$y - 430 = 1.4(x - 300)$

$y - 430 = 1.4(0 - 300)$

$y - 430 = -420$

$y = 10$

The service fee is \$10.

Please complete *Lesson 7.3 Explore Your Understanding Assignment* located in *Workbook 7.3* before proceeding to *Lesson 7.4*.

Lesson 7.4: Parallel and Perpendicular Lines

Practice – IV

1. Decide if each pair of lines is parallel, perpendicular, or neither. Explain your choice.

   a. $y = 9x + 4$ and $18x - 2y + 13 = 0$

   Find the slopes to determine if the lines are parallel, perpendicular, or neither.

   The slope of $y = 9x + 4$ is 9.

   $$\begin{aligned} 18x - 2y + 13 &= 0 \\ 18x + 13 &= 2y \\ \frac{18x + 13}{2} &= y \\ 9x + \frac{13}{2} &= y \end{aligned}$$

   The slope of $18x - 2y + 13 = 0$ is also 9, so the two lines are parallel.

   b. $y - 7 = \frac{3}{2}(x + 5)$ and $y = \frac{2}{3}x$

   Find the slopes to determine if the lines are parallel, perpendicular, or neither.

   The slope of $y - 7 = \frac{3}{2}(x + 5)$ is $\frac{3}{2}$ and the slope of $y = \frac{2}{3}x$ is $\frac{2}{3}$. Although the two slopes are reciprocals, they are not negative reciprocals. The two lines are neither parallel nor perpendicular.

   c. $y = 2.5x + 1$ and $y = -0.4x - 1$

   The slope of $y = 2.5x + 1$ is 2.5 and the slope of $y = -0.4x - 1$ is –0.4. Multiply the two slopes to determine whether the product is –1.

   $2.5 \times (-0.4) = -1$

   The product is –1, so the slopes are negative reciprocals and the two lines are perpendicular.

2. Line A passes through the points $(-1,-1)$ and $(5,3)$. Line B passes through the points $(7,-5)$ and $(1,r)$. Determine a value of r such that the two lines are

   a. parallel

$$\begin{aligned} m_A &= \frac{y_2 - y_1}{x_2 - x_1} \\ m_A &= \frac{3-(-1)}{5-(-1)} \\ &= \frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

The slope of line A is $\frac{2}{3}$. For lines A and B to be parallel, line B must also have a slope of $\frac{2}{3}$. The slope formula can be used to determine the value of r.

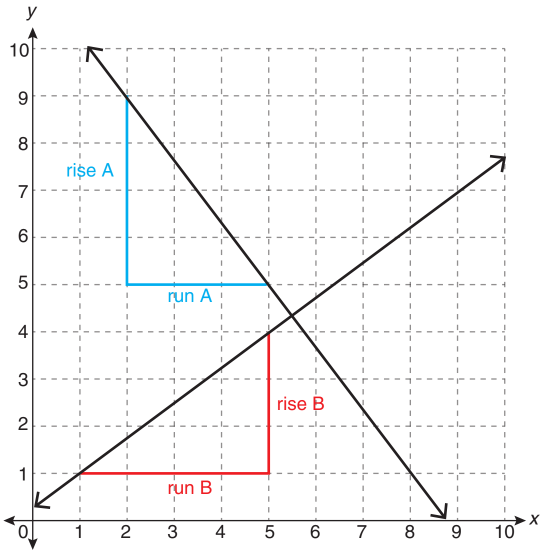
$$\begin{aligned} m_B &= \frac{y_2 - y_1}{x_2 - x_1} \\ \frac{2}{3} &= \frac{r-(-5)}{1-7} \\ \frac{2}{3} &= \frac{r+5}{-6} \\ \frac{2}{3}(-6) &= \frac{r+5}{\cancel{-6}}(\cancel{-6}) \\ -4 &= r+5 \\ -4-5 &= r\cancel{+5}\cancel{-5} \\ -9 &= r \end{aligned}$$

   b. perpendicular

The slope of line A is $\frac{2}{3}$. The slope of line B must be the negative reciprocal of $\frac{2}{3}$ for the lines to be perpendicular, so the slope of line B must be $-\frac{3}{2}$. Again, use the slope formula to determine the value of r.

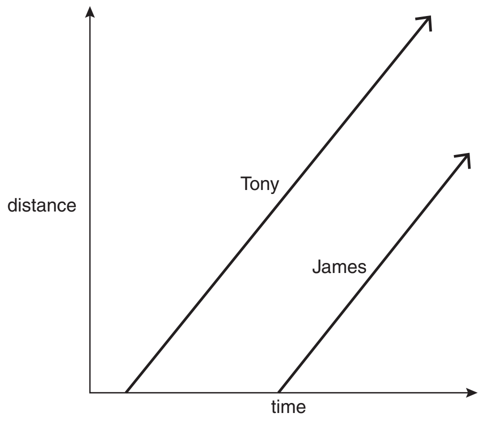
$$\begin{aligned} m_B &= \frac{y_2 - y_1}{x_2 - x_1} \\ -\frac{3}{2} &= \frac{r-(-5)}{1-7} \\ -\frac{3}{2} &= \frac{r+5}{-6} \\ -\frac{3}{2}(-6) &= \frac{r+5}{\cancel{-6}}(\cancel{-6}) \\ 9 &= r+5 \\ 9-5 &= r\cancel{+5}\cancel{-5} \\ 4 &= r \end{aligned}$$

3. The grid shows two perpendicular lines. Use the information provided on the grid to show that the slopes of the lines have a product of –1.



$$\frac{\text{rise A}}{\text{run A}} \times \frac{\text{rise B}}{\text{run B}} = \frac{-4}{3} \times \frac{3}{4}$$
$$= -\frac{12}{12}$$
$$= -1$$

4. Tony and James both walked home from school, as shown in the graph provided.



a. Describe a scenario that would lead to this graph.

Situations may vary. A sample is shown.

Tony started to walk home from school before James did.

b. The two lines in the graph are parallel. Explain what this means in the given context.

The parallel lines have the same slope, or rate of change. In this case, the rate of change is a speed. The two boys walked home from school at the same speed.

c. Suppose the two lines were not parallel. Would this guarantee that James and Tony will meet? Explain.

No. If the both lines extended in both directions, this would be true because non-parallel lines must intersect in exactly one place. However, they both start walking from a distance of 0. In this case, the two lines will only intersect if James walks faster than Tony (the slope of James' line would have to be steeper) and reaches Tony while they are both still walking.

Please complete *Lesson 7.4 Explore Your Understanding Assignment*, located in *Workbook 7.4*.