# Lesson 8.2: Solving Systems of Linear Equations by Substitution

## Practice – III

1. Solve and verify each of the following systems of linear equations.

   a. $y = 2x$ and $3x + y = -5$

$$\begin{aligned} 3x + y &= -5 \\ 3x + (2x) &= -5 \\ 5x &= -5 \\ x &= -1 \end{aligned}$$

$$\begin{aligned} y &= 2x \\ y &= 2(-1) \\ y &= -2 \end{aligned}$$

The solution is (–1,–2).

Verify the solution.

$y = 2x$

| Left Side | Right Side |
|---|---|
| $y$<br>$-2$ | $2x$<br>$2(-1)$<br>$-2$ |
| LS = RS | |

$3x + y = -5$

| Left Side | Right Side |
|---|---|
| $3x + y$<br>$3(-1) + (-2)$<br>$-5$ | $-5$ |
| LS = RS | |

   b. $x + 2y = 2$ and $x - 2y = 6$

$$\begin{aligned} x + 2y &= 2 \\ x &= 2 - 2y \end{aligned}$$

$$\begin{aligned} x - 2y &= 6 \\ (2 - 2y) - 2y &= 6 \\ 2 - 4y &= 6 \\ -4y &= 4 \\ y &= -1 \end{aligned}$$

$x = 2 - 2y$
$x = 2 - 2(-1)$
$x = 4$

The solution is (4,–1).

Verify the solution.

$x + 2y = 2$

| Left Side | Right Side |
|---|---|
| $x + 2y$<br>$4 + 2(-1)$<br>$2$ | $2$ |
| LS = RS | |

$x - 2y = 6$

| Left Side | Right Side |
|---|---|
| $x - 2y$<br>$4 - 2(-1)$<br>$6$ | $6$ |
| LS = RS | |

c. $y = \frac{1}{2}x + 1$ and $y = -\frac{3}{2}x - 1$

$\frac{1}{2}x + 1 = -\frac{3}{2}x - 1$
$2x = -2$
$x = -1$

$y = \frac{1}{2}x + 1$
$y = \frac{1}{2}(-1) + 1$
$y = \frac{1}{2}$

The solution is $\left(-1, \frac{1}{2}\right)$.

Verify the solution.

$y = \frac{1}{2}x + 1$

| Left Side | Right Side |
|---|---|
| $y$<br>$\frac{1}{2}$ | $\frac{1}{2}x + 1$<br>$\frac{1}{2}(-1) + 1$<br>$\frac{1}{2}$ |
| LS = RS | |

$y = -\frac{3}{2}x - 1$

| Left Side | Right Side |
|---|---|
| $y$<br>$\frac{1}{2}$ | $-\frac{3}{2}x - 1$<br>$-\frac{3}{2}(-1) - 1$<br>$\frac{1}{2}$ |
| LS = RS | |

d. $6a - 2b - 8 = 0$ and $7b - 14a + 26 = 0$

$$\begin{aligned} 6a - 2b - 8 &= 0 \\ 6a - 8 &= 2b \\ 3a - 4 &= b \end{aligned}$$

$$\begin{aligned} 7b - 14a + 26 &= 0 \\ 7(3a - 4) - 14a + 26 &= 0 \\ 21a - 28 - 14a + 26 &= 0 \\ 7a - 2 &= 0 \\ 7a &= 2 \\ a &= \frac{2}{7} \end{aligned}$$

$$\begin{aligned} 3a - 4 &= b \\ 3\left(\frac{2}{7}\right) - 4 &= b \\ \frac{6}{7} - \frac{28}{7} &= b \\ -\frac{22}{7} &= b \end{aligned}$$

The solution is $a = \frac{2}{7}$ and $b = -\frac{22}{7}$.

Verify the solution.

$6a - 2b - 8 = 0$

| Left Side | Right Side |
|---|---|
| $6a - 2b - 8$<br>$6\left(\frac{2}{7}\right) - 2\left(-\frac{22}{7}\right) - 8$<br>$0$ | $0$ |
| LS = RS | |

$7b - 14a + 26 = 0$

| Left Side | Right Side |
|---|---|
| $7b - 14a + 26$<br>$7\left(-\frac{22}{7}\right) - 14\left(\frac{2}{7}\right) + 26$<br>$0$ | $0$ |
| LS = RS | |

2. Jakub attempted to solve the system of equations $5x + y + 20 = 0$ and $2x + 2y + 9 = 0$. His work is shown.

$$5x + y + 20 = 0$$
$$y = -5x - 20$$

$$5x + (-5x - 20) + 20 = 0$$
$$0 = 0$$

Uneasy about the result, Jakub graphed the relations and found the two lines intersect at (–3,–6).

a. What error did Jakub make?

Jakub isolated $y$ in the first equation and then substituted an equivalent expression for $y$ back into that same equation. Jakub should have substituted the equivalent expression for $y$ into the second equation.

b. Suggest to Jakub a general strategy that he can always use to solve linear systems of equations by substitution.

Strategies will vary. A sample is shown.

- Isolate one variable in one equation.
- Substitute the isolated variable's equivalent expression into the other equation.
- Solve the new equation to determine the value of one variable.
- Substitute the known value into one of the original equations to determine the value of the other variable.
- Verify the solution.

3. a. Explain how a solution to a system of linear equations can be verified algebraically.

Substitute the solution values into each of the original equations. If the left side equals the right side for both equations, the solution is verified.

b. Explain how a solution to a system of linear equations can be verified graphically.

Graph the two relations and determine the point of intersection. If the coordinates of the intersection point match the solution, the solution is verified.

c. Describe an advantage to verifying a solution using each method.

The algebraic method doesn't require graph paper or technology. This method is often faster for 'nice' numbers because it can be completed mentally.

The graphing method is more visual and often can be completed quickly using a graphing calculator or graphing program.

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4. Glen and Warren competed in a cycling race where participants began the race at different locations based on their previous cycling performances. Glen's position $t$ hours after the race began is represented by $d = 32t + 14$ and Warren's position is represented by $d = 29t + 12$.

   a. If the race was 100 km long, what are the domain and range of Glen's relation?

   Glen's minimum time was 0 hours.

   Glen's maximum time occurred at 100 km.

   $$\begin{aligned} d &= 32t + 14 \\ 100 &= 32t + 14 \\ 86 &= 32t \\ \frac{43}{16} &= t \end{aligned}$$

   The maximum time was $\frac{43}{16}$ hours.

   Domain: $\left\{t \mid 0 \le t \le \frac{43}{16}, t \in \mathrm{R}\right\}$

   Glen's initial position occurred at time 0.

   $$\begin{aligned} d &= 32t + 14 \\ d &= 32(0) + 14 \\ d &= 14 \end{aligned}$$

   In other words, Glen had a 14 km advantage (he began the race at the 14 km point) due to his past performance.

   The initial position was 14 km and the final position was 100 km.

   Range: $\{d \mid 14 \le d \le 100, d \in \mathrm{R}\}$

   b. Solve the system of equations.

   $$\begin{aligned} d &= 32t + 14 \\ 29t + 12 &= 32t + 14 \\ -3t &= 2 \\ t &= -\frac{2}{3} \end{aligned}$$

   $$\begin{aligned} d &= 29t + 12 \\ d &= 29\left(-\frac{2}{3}\right) + 12 \\ d &= -\frac{22}{3} \end{aligned}$$

c. Explain the meaning of the solution.

The solution should represent the time and location where Warren caught up to Glen.

However, the solution occurs outside the domain and range of the functions. This means there is no solution to the scenario and Warren did not catch up to Glen during the race.

Please complete *Lesson 8.2 Explore Your Understanding Assignment* located in *Workbook 8.2* before proceeding to *Lesson 8.3*.