Lesson 8.3: Solving Systems of Linear Equations by Elimination

Practice – IV

1. Use the following example to explain why the order of subtraction is not important when solving systems of equations by elimination.

$$\begin{array}{rrrrr} & 5x & + 9y & = & 7 \\ -\ (& 6x & + 9y & = & 25\ ) \\ \hline & -x & + 0y & = & -18 \end{array} \qquad \begin{array}{rrrrr} & 6x & + 9y & = & 25 \\ -\ (& 5x & + 9y & = & 7\ ) \\ \hline & x & + 0y & = & 18 \end{array}$$

The order of subtraction is not important because the resulting equations are equivalent. If both sides of the first equation are multiplied by –1, the second equation is produced. Both equations also simplify to $x = 18$.

2. The subtraction of two equations is shown.

$$\begin{array}{rrrrrr} & 5x & + 3y & - 1 & = & 0 \\ -\ (& 2x & - y & + 4 & = & 0\ ) \\ \hline & 3x & + 4y & - 5 & = & 0 \end{array}$$

Explain why this subtraction is not useful for solving the linear system $5x + 3y - 1 = 0$ and $2x - y + 4 = 0$.

When using the elimination method, the purpose of subtracting two equations is to produce a new equation with fewer variables. In the subtraction shown, there is no reduction in variables.

3. Solve the following systems of equations by elimination. Verify the solutions.

a. $52 - a = 4b$

$70 - a = 6b$

$$\begin{array}{rrrrr} & 52 & - a & = & 4b \\ -\ (& 70 & - a & = & 6b\ ) \\ \hline & -18 & + 0a & = & -2b \\ & 18 & & = & 2b \\ & 9 & & = & b \end{array}$$

$52 - a = 4b$
$52 - a = 4(9)$
$52 - a = 36$
$-a = -16$
$a = 16$

The solution is $a = 16$ and $b = 9$.

Verify the solution.

$52 - a = 4b$

Left Side	Right Side
$52 - a$	$4b$
$52 - 16$	$4(9)$
36	36
LS = RS	

$70 - a = 6b$

Left Side	Right Side
$70 - a$	$6b$
$70 - 16$	$6(9)$
54	54
LS = RS	

b. $3x + 5y = -2$

$x - y = -6$

$x - y = -6$
$3(x - y) = 3(-6)$
$3x - 3y = -18$

$$\begin{array}{rrrrr} & 3x & + 5y & = & -2 \\ -\ (& 3x & - 3y & = & -18\ ) \\ \hline & 0x & + 8y & = & 16 \\ & & 8y & = & 16 \\ & & y & = & 2 \end{array}$$

$x - y = -6$
$x - 2 = -6$
$x = -4$

The solution is (–4,2).

Verify the solution.

$3x + 5y = -2$

Left Side	Right Side
$3x + 5y$ $3(-4) + 5(2)$ -2	-2
LS = RS	

$x - y = -6$

Left Side	Right Side
$x - y$ $-4 - 2$ -6	-6
LS = RS	

c. $7x = 11 + 5y$

$8y = -6x - 9$

$$7x = 11 + 5y$$
$$7x - 5y = 11$$
$$8(7x - 5y) = 8(11)$$
$$56x - 40y = 88$$

$$8y = -6x - 9$$
$$6x + 8y = -9$$
$$5(6x + 8y) = 5(-9)$$
$$30x + 40y = -45$$

$$\begin{array}{rrcr} 56x & + \ 40y & = & 88 \\ + \ (\ 30x & - \ 40y & = & -45\ ) \\ \hline 86x & + \ 0y & = & 43 \\ 86x & & = & 43 \\ x & & = & \frac{1}{2} \end{array}$$

$$7x = 11 + 5y$$
$$7\left(\frac{1}{2}\right) = 11 + 5y$$
$$\frac{7}{2} = 11 + 5y$$
$$-\frac{15}{2} = 5y$$
$$-\frac{3}{2} = y$$

The solution is $\left(\frac{1}{2}, -\frac{3}{2}\right)$.

Verify the solution.

$7x = 11 + 5y$

Left Side	Right Side
$7x$	$11 + 5y$
$7\left(\frac{1}{2}\right)$	$11 + 5\left(-\frac{3}{2}\right)$
$\frac{7}{2}$	$\frac{7}{2}$
LS = RS	

$8y = -6x - 9$

Left Side	Right Side
$8y$	$-6x - 9$
$8\left(-\frac{3}{2}\right)$	$-6\left(\frac{1}{2}\right) - 9$
-12	-12
LS = RS	

d. $A - 2B = -4$

$2A + 3B = 10$

$$A - 2B = -4$$
$$2(A - 2B) = 2(-4)$$
$$2A - 4B = -8$$

$$\begin{array}{rrcl} 2A & - \ 4B & = & -8 \\ -\ (\ 2A & + \ 3B & = & 10\ ) \\ \hline 0A & - \ 7B & = & -18 \\ & -7B & = & -18 \\ & B & = & \frac{18}{7} \end{array}$$

$$A - 2B = -4$$
$$A - 2\left(\frac{18}{7}\right) = -4$$
$$A = \frac{8}{7}$$

The solution is $A = \frac{8}{7}$ and $B = \frac{18}{7}$.

Verify the solution.

$A - 2B = -4$

Left Side	Right Side
$A - 2B$	-4
$\frac{8}{7} - 2\left(\frac{18}{7}\right)$	
-4	
LS = RS	

$2A + 3B = 10$

Left Side	Right Side
$2A + 3B$	10
$2\left(\frac{8}{7}\right) + 3\left(\frac{18}{7}\right)$	
10	
LS = RS	

4. Attempt to solve the following systems of equations. How is each pair of lines related?

   a. $x + 3y = 11$

   $4x + 12y = 44$

   $$x + 3y = 11$$
   $$4(x + 3y) = 4(11)$$
   $$4x + 12y = 44$$

   $$\begin{array}{rrcl} & 4x + 12y & = & 44 \\ -\ (& 4x + 12y & = & 44\ ) \\ \hline & 0x + 0y & = & 0 \\ & 0 & = & 0 \end{array}$$

   Attempting to solve the system produced a true statement, so there are an infinite number of solutions and the two lines are coincident.

   b. $2x - 6y = 9$

   $3x - 9y = 12$

   $$3(2x - 6y) = 3(9)$$
   $$6x - 18y = 27$$

   $$2(3x - 9y) = 2(12)$$
   $$6x - 18y = 24$$

   $$\begin{array}{rrcl} & 6x - 18y & = & 27 \\ -\ & 6x - 18y & = & 24 \\ \hline & 0 & = & 3 \end{array}$$

   Attempting to solve the system produced a false statement, so there is no solution and the two lines are parallel.

Please complete *Lesson 8.3 Explore Your Understanding Assignment* located in *Workbook 8.3* before proceeding to *Lesson 8.4*.