Lesson 8.4: Solving Problems using Linear Systems

Practice – V

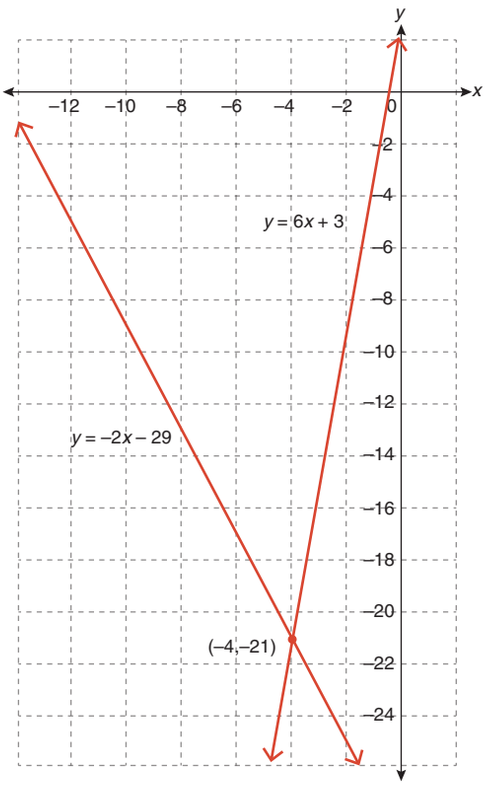
1. Consider the following system of equations.

$y = 6x + 3$

$y = -2x - 29$

a. Solve the system by graphing, by substitution, and by elimination.

Graphing



The solution is (–4, –21).

Substitution

$$y = 6x + 3$$
$$-2x - 29 = 6x + 3$$
$$-32 = 8x$$
$$-4 = x$$

$y = 6x + 3$
$y = 6(-4) + 3$
$y = -21$

The solution is (–4,–21).

Elimination

$$\begin{array}{rrcl} & y & = & 6x + 3 \\ -\ (& y & = & -2x - 29\ ) \\ \hline & 0 & = & 8x + 32 \\ & -32 & = & 8x \\ & -4 & = & x \end{array}$$

$y = 6x + 3$
$y = 6(-4) + 3$
$y = -21$

The solution is (–4,–21).

b. Which method do you prefer for this system? Why?

Choices and explanations will vary.

2. Matt claims that if a linear system includes a vertical or a horizontal line, the best method for solving the system is usually by substitution. Do you agree with Matt? Why or why not?

Responses will vary. Equations of horizontal and vertical lines either have a variable that is already isolated or a variable that is easy to isolate, so substitution is usually a good method.

3. From the table below, select the system of equations that best represents each of the following problems.

$A + B = 12$ $A - B = 2$	$C = 190M$ $C = 120M + 3400$	$C = 795$ $C = 66D$	$C = 450M + 2000$ $C = 350M + 5000$

a. Tyler wants to know how many days he will need to visit the ski hill for a season's pass to cost less than buying a pass for each visit.

$C = 795$
$C = 66D$

b. Elizabeth is comparing the cost of continuing to run an old furnace with the cost of buying and running a new, high-efficiency furnace.

$C = 190M$
$C = 120M + 3400$

c. The sum and difference of two numbers is known. What are the numbers?

$A + B = 12$
$A - B = 2$

d. A car dealership charges a down payment and a monthly payment for someone to finance a vehicle. After how many months will a car with a large down payment and a small monthly payment cost the same as a car with a small down payment and a large monthly payment?

$C = 450M + 2\,000$
$C = 350M + 5\,000$

4. Arial has been offered two jobs. One job pays \$10/h plus a commission of 2% of all sales. The other job pays \$12/h plus a commission of 0.5% of all sales. How much merchandise would Arial need to sell in each 8 hour shift for the two jobs to pay the same?

a. Write a system of equations that can be used to represent this scenario.

Let E be Arial's earnings and let S be the amount sold.

	Hourly Rate (\$/h)	Daily Wage (\$)	Commission Rate	Amount Sold (\$)	Commission Earnings (\$)	Total Earnings
Job 1	10	$10 \times 8 = 80$	0.02	S	$0.02S$	$80 + 0.02S$
Job 2	12	$12 \times 8 = 96$	0.005	S	$0.005S$	$96 + 0.005S$

Arial will earn $0.02S$ commission and a wage of 80 dollars from the first job. Her total earnings per day from Job 1 will be $E = 0.02S + 80$.

Arial will earn $0.005S$ commission and a wage of 96 dollars from the second job. Her total earnings per day from Job 2 will be $E = 0.005S + 96$.

b. Solve the system of equations.

$$\begin{aligned} E &= 0.02S + 80 \\ -\ (E &= 0.005S + 96\ ) \\ \hline 0 &= 0.015S - 16 \\ 16 &= 0.015S \\ 1\,066.\overline{6} &= S \end{aligned}$$

$E = 0.02S + 80$
$E = 0.02(1\,066.\overline{6}) + 80$
$E = 101.\overline{3}$

c. How much merchandise would Arial need to sell in each 8 hour shift for the two jobs to pay the same?

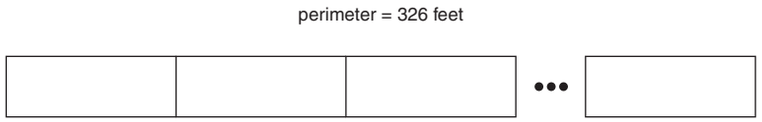
Arial would need to sell approximately $1 066.67 worth of merchandise each shift for the two jobs to pay the same.

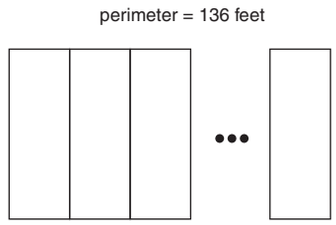
d. Does the conclusion agree with the original information?

Yes. If Arial sells approximately $1 066.67 worth of merchandise at either job, she will earn approximately $101.33.

5. When twenty identical rectangular tables are placed end-to-end, their perimeter is 326 feet. When they are placed side-by-side, their perimeter is 136 feet.

a. Sketch a diagram to represent this problem. (You don't need to show all 20 tables, just a pattern.)





b. Write a linear system to model the situation.

Let L be the length of a table and let W be the width of a table.

$2W + 40L = 326$

$40W + 2L = 136$

c. What are the dimensions of each table?

$$\begin{aligned} 2W + 40L &= 326 \\ 20(2W + 40L) &= 20(326) \\ 40W + 800L &= 6\,520 \end{aligned}$$

$$\begin{array}{rrcr} 40W & + \ 800L & = & 6\,520 \\ -\ (\ 40W & + \ \ 2L & = & \ \ 136\ ) \\ \hline & 798L & = & 6\,384 \\ & L & = & \ \ 8 \end{array}$$

$$2W + 40L = 326$$
$$2W + 40(8) = 326$$
$$2W + 320 = 326$$
$$2W = 6$$
$$W = 3$$

The width of each table is three feet and the length is eight feet.

6. Medicine Hat and Lethbridge are 169 km apart. If Kiran leaves Medicine Hat at noon and travels towards Lethbridge at 110 km/h, and Leela leaves Lethbridge at noon and travels towards Medicine Hat at 100 km/h, when and where will the two meet?

Let t be time after noon and let d be the distance from Medicine Hat.

	Kiran	Leela
Distance, d, from Medicine Hat at Start	0	169
Time Driving (h)	t	t
Speed (km/h)	110	100
Distance Driven (km)	$110t$	$100t$
Distance from Medicine Hat at time t (km)	$0 + 110t$	$169 - 100t$

Kiran's distance from Medicine Hat is represented by $d = 110t$.

Leela's distance from Medicine Hat is represented by $d = 169 - 100t$.

$$d = 110t$$
$$169 - 100t = 110t$$
$$169 = 210t$$
$$0.804... = t$$

$$d = 110t$$
$$d = 110(0.804)$$
$$d = 88.523...$$

The two will meet after driving for approximately 0.8 hours, at a distance of approximately 89 km from Medicine Hat.

Please complete *Lesson 8.4 Explore Your Understanding Assignment*, located in *Workbook 8.4*.