Appendix

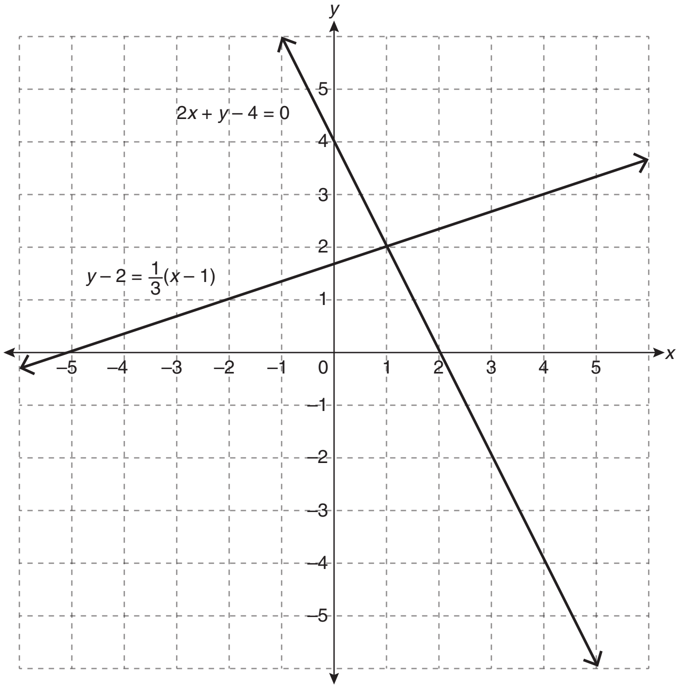
Lesson 8.1: Systems of Linear Equations and Graphs

Practice – I

1. Graph the following system of equations.

   $2x + y - 4 = 0$

   $y - 2 = \frac{1}{3}(x - 1)$

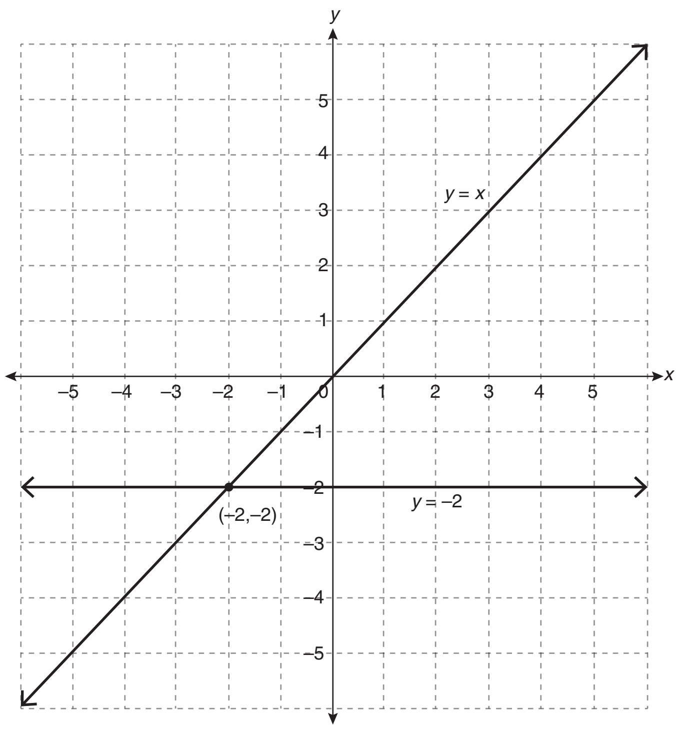


2. Explain what the intersection point of lines represents.

   The intersection point represents the solution to the corresponding system of equations.

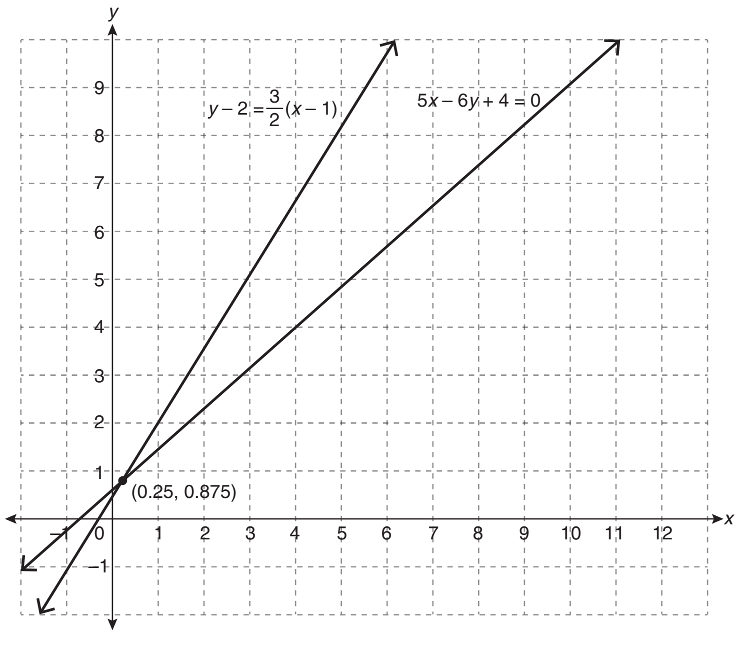
3. Graphically solve each of the following linear systems.

   a. $y = x$ and $y = -2$



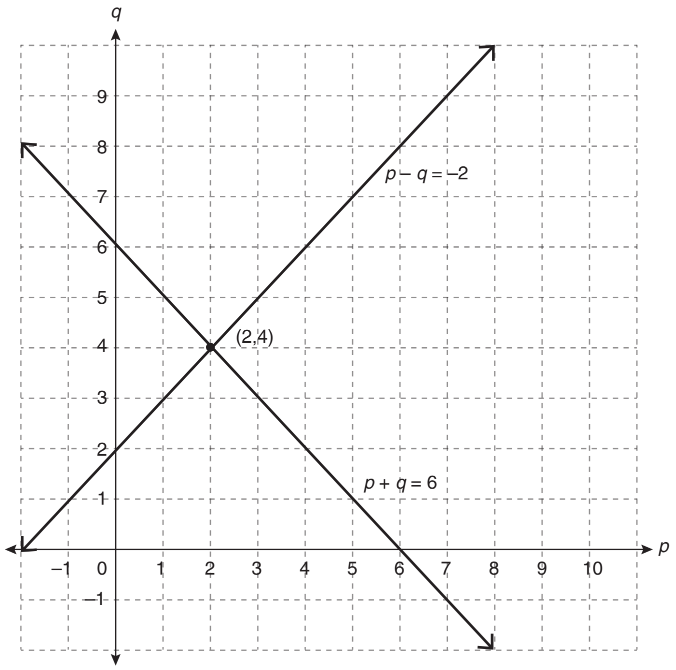
The solution is (–2,–2).

   b. $5x - 6y + 4 = 0$ and $y - 2 = \frac{3}{2}(x - 1)$



The solution is (0.25, 0.875).

c. $p+q=6$ and $p-q=-2$



The solution is (2,4).

4. Megan was trying to determine whether the point (–3,0) was a solution to the following system of equations.

$y=6x-20$

$y=-\frac{1}{3}x-1$

She wrote the following and concluded (–3,0) was not a solution.

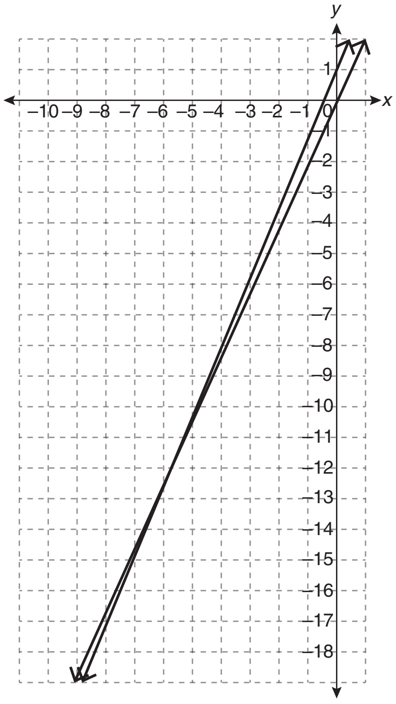
Left Side	Right Side
y	$6x-20$
0	$6(-3)-20$
	-38
LS ≠ RS	

Megan then showed her work to Faith, who said it was incomplete because Megan didn't check both equations.

Explain how this discussion could be resolved.

Once it is shown that a solution does not satisfy one of the equations, you know it is not a solution to the system of equations. Megan's work is sufficient.

5. Toby drew the following graph while trying to determine the solution to a linear system of equations.



a. Explain why it is difficult to use Toby's graph to determine a solution to the system of equations.

The lines appear to have a significant overlap, so determining the exact point of intersection is difficult.

b. Suggest an improvement to the graph that will make determining a solution easier.

Suggestions will vary. Toby may be able to see the solution more clearly by adjusting one or both scales of the graph. Toby may also use technology and an intersection command to determine the intersection point.

Please return to *Unit 8: Systems of Linear Equations Lesson 8.1* in the *Module* to continue your exploration.

Lesson 8.1: Systems of Linear Equations and Graphs

Practice – II

1. Describe a strategy that can be used to determine the number of solutions to a system of linear equations.

   Strategies will vary. One strategy is to use the slope and y-intercept to determine if the lines are parallel, coincident, or neither.

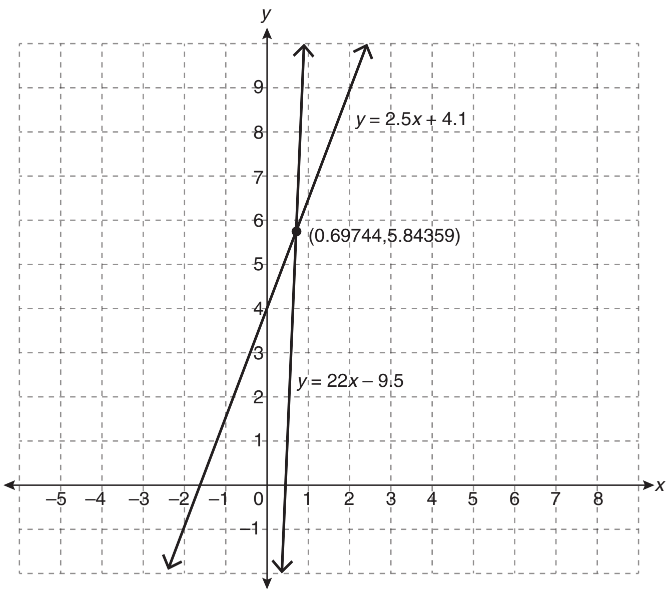
Relationship	Parallel	Coincident	Neither Parallel Nor Coincident
Slope	same	same	different
y-Intercept	different	same	any
Number of Solutions	0	infinite	1

2. Austin has found that both (13,17) and (24,61) are solutions to a system of linear equations. How are the two lines related? Explain.

   The system has more than one solution, so the lines must be coincident.

3. Use technology to determine an approximate solution to each of the following systems of linear equations.

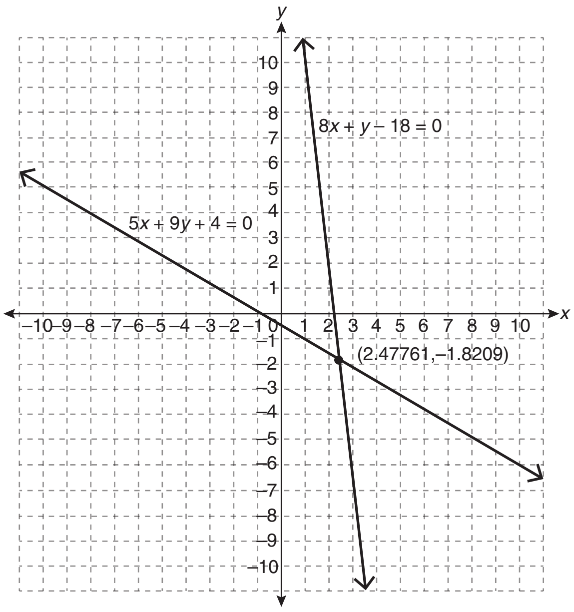
   a. $y = 22x - 9.5$ and $y = 2.5x + 4.1$.



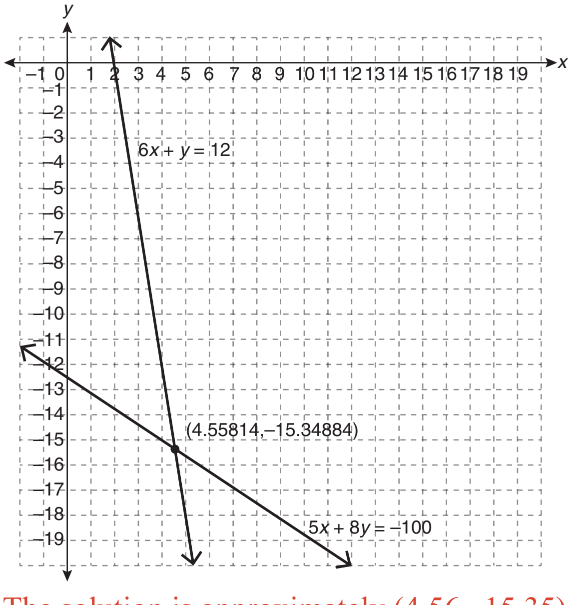
The solution is approximately (0.70,5.84).

b. $8x + y - 18 = 0$ and $5x + 9y + 4 = 0$

The solution is approximately (2.48,–1.82).



c. $6x + y = 12$ and $5x + 8y = -100$



The solution is approximately (4.56,–15.35).

Please complete *Lesson 8.1 Explore Your Understanding Assignment* located in *Workbook 8.1* before proceeding to *Lesson 8.2*.

Lesson 8.2: Solving Systems of Linear Equations by Substitution

Practice – III

1. Solve and verify each of the following systems of linear equations.

   a. $y = 2x$ and $3x + y = -5$

$$\begin{aligned} 3x + y &= -5 \\ 3x + (2x) &= -5 \\ 5x &= -5 \\ x &= -1 \end{aligned}$$

$$\begin{aligned} y &= 2x \\ y &= 2(-1) \\ y &= -2 \end{aligned}$$

The solution is (–1,–2).

Verify the solution.

$y = 2x$

Left Side	Right Side
y	$2x$
-2	$2(-1)$
	-2
LS = RS	

$3x + y = -5$

Left Side	Right Side
$3x + y$	-5
$3(-1) + (-2)$	
-5	
LS = RS	

   b. $x + 2y = 2$ and $x - 2y = 6$

$$\begin{aligned} x + 2y &= 2 \\ x &= 2 - 2y \end{aligned}$$

$$\begin{aligned} x - 2y &= 6 \\ (2 - 2y) - 2y &= 6 \\ 2 - 4y &= 6 \\ -4y &= 4 \\ y &= -1 \end{aligned}$$

$x = 2 - 2y$
$x = 2 - 2(-1)$
$x = 4$

The solution is (4,–1).

Verify the solution.

$x + 2y = 2$

Left Side	Right Side
$x + 2y$ $4 + 2(-1)$ 2	2
LS = RS	

$x - 2y = 6$

Left Side	Right Side
$x - 2y$ $4 - 2(-1)$ 6	6
LS = RS	

c. $y = \frac{1}{2}x + 1$ and $y = -\frac{3}{2}x - 1$

$\frac{1}{2}x + 1 = -\frac{3}{2}x - 1$
$2x = -2$
$x = -1$

$y = \frac{1}{2}x + 1$
$y = \frac{1}{2}(-1) + 1$
$y = \frac{1}{2}$

The solution is $\left(-1, \frac{1}{2}\right)$.

Verify the solution.

$y = \frac{1}{2}x + 1$

Left Side	Right Side
y $\frac{1}{2}$	$\frac{1}{2}x + 1$ $\frac{1}{2}(-1) + 1$ $\frac{1}{2}$
LS = RS	

$y = -\frac{3}{2}x - 1$

Left Side	Right Side
y $\frac{1}{2}$	$-\frac{3}{2}x - 1$ $-\frac{3}{2}(-1) - 1$ $\frac{1}{2}$
LS = RS	

d. $6a - 2b - 8 = 0$ and $7b - 14a + 26 = 0$

$$\begin{aligned} 6a - 2b - 8 &= 0 \\ 6a - 8 &= 2b \\ 3a - 4 &= b \end{aligned}$$

$$\begin{aligned} 7b - 14a + 26 &= 0 \\ 7(3a - 4) - 14a + 26 &= 0 \\ 21a - 28 - 14a + 26 &= 0 \\ 7a - 2 &= 0 \\ 7a &= 2 \\ a &= \frac{2}{7} \end{aligned}$$

$$\begin{aligned} 3a - 4 &= b \\ 3\left(\frac{2}{7}\right) - 4 &= b \\ \frac{6}{7} - \frac{28}{7} &= b \\ -\frac{22}{7} &= b \end{aligned}$$

The solution is $a = \frac{2}{7}$ and $b = -\frac{22}{7}$.

Verify the solution.

$6a - 2b - 8 = 0$

Left Side	Right Side
$6a - 2b - 8$ $6\left(\frac{2}{7}\right) - 2\left(-\frac{22}{7}\right) - 8$ 0	0
LS = RS	

$7b - 14a + 26 = 0$

Left Side	Right Side
$7b - 14a + 26$ $7\left(-\frac{22}{7}\right) - 14\left(\frac{2}{7}\right) + 26$ 0	0
LS = RS	

2. Jakub attempted to solve the system of equations $5x + y + 20 = 0$ and $2x + 2y + 9 = 0$. His work is shown.

$$5x + y + 20 = 0$$
$$y = -5x - 20$$

$$5x + (-5x - 20) + 20 = 0$$
$$0 = 0$$

Uneasy about the result, Jakub graphed the relations and found the two lines intersect at (–3,–6).

a. What error did Jakub make?

Jakub isolated y in the first equation and then substituted an equivalent expression for y back into that same equation. Jakub should have substituted the equivalent expression for y into the second equation.

b. Suggest to Jakub a general strategy that he can always use to solve linear systems of equations by substitution.

Strategies will vary. A sample is shown.

- Isolate one variable in one equation.
- Substitute the isolated variable's equivalent expression into the other equation.
- Solve the new equation to determine the value of one variable.
- Substitute the known value into one of the original equations to determine the value of the other variable.
- Verify the solution.

3. a. Explain how a solution to a system of linear equations can be verified algebraically.

Substitute the solution values into each of the original equations. If the left side equals the right side for both equations, the solution is verified.

b. Explain how a solution to a system of linear equations can be verified graphically.

Graph the two relations and determine the point of intersection. If the coordinates of the intersection point match the solution, the solution is verified.

c. Describe an advantage to verifying a solution using each method.

The algebraic method doesn't require graph paper or technology. This method is often faster for 'nice' numbers because it can be completed mentally.

The graphing method is more visual and often can be completed quickly using a graphing calculator or graphing program.

4. Glen and Warren competed in a cycling race where participants began the race at different locations based on their previous cycling performances. Glen's position t hours after the race began is represented by $d = 32t + 14$ and Warren's position is represented by $d = 29t + 12$.

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a. If the race was 100 km long, what are the domain and range of Glen's relation?

Glen's minimum time was 0 hours.

Glen's maximum time occurred at 100 km.

$$d = 32t + 14$$
$$100 = 32t + 14$$
$$86 = 32t$$
$$\frac{43}{16} = t$$

The maximum time was $\frac{43}{16}$ hours.

Domain: $\left\{t \mid 0 \le t \le \frac{43}{16}, t \in \mathrm{R}\right\}$

Glen's initial position occurred at time 0.

$$d = 32t + 14$$
$$d = 32(0) + 14$$
$$d = 14$$

In other words, Glen had a 14 km advantage (he began the race at the 14 km point) due to his past performance.

The initial position was 14 km and the final position was 100 km.

Range: $\{d \mid 14 \le d \le 100, d \in \mathrm{R}\}$

b. Solve the system of equations.

$$d = 32t + 14$$
$$29t + 12 = 32t + 14$$
$$-3t = 2$$
$$t = -\frac{2}{3}$$

$$d = 29t + 12$$
$$d = 29\left(-\frac{2}{3}\right) + 12$$
$$d = -\frac{22}{3}$$

c. Explain the meaning of the solution.

The solution should represent the time and location where Warren caught up to Glen.

However, the solution occurs outside the domain and range of the functions. This means there is no solution to the scenario and Warren did not catch up to Glen during the race.

Please complete *Lesson 8.2 Explore Your Understanding Assignment* located in *Workbook 8.2* before proceeding to *Lesson 8.3*.

Lesson 8.3: Solving Systems of Linear Equations by Elimination

Practice – IV

1. Use the following example to explain why the order of subtraction is not important when solving systems of equations by elimination.

$$\begin{array}{rrrrl} & 5x & + & 9y = & 7 \\ -(& 6x & + & 9y = & 25 \;) \\ \hline & -x & + & 0y = & -18 \end{array} \qquad \begin{array}{rrrrl} & 6x & + & 9y = & 25 \\ -(& 5x & + & 9y = & 7 \;) \\ \hline & x & + & 0y = & 18 \end{array}$$

The order of subtraction is not important because the resulting equations are equivalent. If both sides of the first equation are multiplied by –1, the second equation is produced. Both equations also simplify to $x = 18$.

2. The subtraction of two equations is shown.

$$\begin{array}{rrrrrrl} & 5x & + & 3y & - & 1 = & 0 \\ -(& 2x & - & y & + & 4 = & 0 \;) \\ \hline & 3x & + & 4y & - & 5 = & 0 \end{array}$$

Explain why this subtraction is not useful for solving the linear system $5x + 3y - 1 = 0$ and $2x - y + 4 = 0$.

When using the elimination method, the purpose of subtracting two equations is to produce a new equation with fewer variables. In the subtraction shown, there is no reduction in variables.

3. Solve the following systems of equations by elimination. Verify the solutions.

a. $52 - a = 4b$

$70 - a = 6b$

$$\begin{array}{rrrrl} & 52 & - & a = & 4b \\ -(& 70 & - & a = & 6b \;) \\ \hline & -18 & + & 0a = & -2b \\ & 18 & & = & 2b \\ & 9 & & = & b \end{array}$$

$$52 - a = 4b$$
$$52 - a = 4(9)$$
$$52 - a = 36$$
$$-a = -16$$
$$a = 16$$

The solution is $a = 16$ and $b = 9$.

Verify the solution.

$52 - a = 4b$

Left Side	Right Side
$52 - a$ $52 - 16$ 36	$4b$ $4(9)$ 36
LS = RS	

$70 - a = 6b$

Left Side	Right Side
$70 - a$ $70 - 16$ 54	$6b$ $6(9)$ 54
LS = RS	

b. $3x + 5y = -2$

$x - y = -6$

$$x - y = -6$$
$$3(x - y) = 3(-6)$$
$$3x - 3y = -18$$

$$\begin{array}{rrrrrrr} & 3x & + & 5y & = & -2 & \\ -\ (& 3x & - & 3y & = & -18 &) \\ \hline & 0x & + & 8y & = & 16 & \\ & & & 8y & = & 16 & \\ & & & y & = & 2 & \end{array}$$

$$x - y = -6$$
$$x - 2 = -6$$
$$x = -4$$

The solution is (–4,2).

Verify the solution.

$3x+5y=-2$

Left Side	Right Side
$3x+5y$ $3(-4)+5(2)$ -2	-2
LS = RS	

$x-y=-6$

Left Side	Right Side
$x-y$ $-4-2$ -6	-6
LS = RS	

c. $7x=11+5y$

$8y=-6x-9$

$$\begin{aligned}7x&=11+5y\\7x-5y&=11\\8(7x-5y)&=8(11)\\56x-40y&=88\end{aligned}$$

$$\begin{aligned}8y&=-6x-9\\6x+8y&=-9\\5(6x+8y)&=5(-9)\\30x+40y&=-45\end{aligned}$$

$$\begin{array}{rrrrrrl} & 56x & + & 40y & = & 88 & \\ +\ (& 30x & - & 40y & = & -45 &) \\ \hline & 86x & + & 0y & = & 43 & \\ & 86x & & & = & 43 & \\ & x & & & = & \frac{1}{2} & \end{array}$$

$$\begin{aligned}7x&=11+5y\\7\left(\frac{1}{2}\right)&=11+5y\\\frac{7}{2}&=11+5y\\-\frac{15}{2}&=5y\\-\frac{3}{2}&=y\end{aligned}$$

The solution is $\left(\frac{1}{2},-\frac{3}{2}\right)$.

Verify the solution.

$7x = 11 + 5y$

Left Side	Right Side
$7x$ $7\left(\frac{1}{2}\right)$ $\frac{7}{2}$	$11 + 5y$ $11 + 5\left(-\frac{3}{2}\right)$ $\frac{7}{2}$
LS = RS	

$8y = -6x - 9$

Left Side	Right Side
$8y$ $8\left(-\frac{3}{2}\right)$ -12	$-6x - 9$ $-6\left(\frac{1}{2}\right) - 9$ -12
LS = RS	

d. $A - 2B = -4$

$2A + 3B = 10$

$$A - 2B = -4$$
$$2(A - 2B) = 2(-4)$$
$$2A - 4B = -8$$

$$\begin{array}{rrrrr} & 2A & - & 4B & = & -8 \\ -(& 2A & + & 3B & = & 10\ ) \\ \hline & 0A & - & 7B & = & -18 \\ & & & -7B & = & -18 \\ & & & B & = & \frac{18}{7} \end{array}$$

$$A - 2B = -4$$
$$A - 2\left(\frac{18}{7}\right) = -4$$
$$A = \frac{8}{7}$$

The solution is $A = \frac{8}{7}$ and $B = \frac{18}{7}$.

Verify the solution.

$A - 2B = -4$

Left Side	Right Side
$A - 2B$ $\frac{8}{7} - 2\left(\frac{18}{7}\right)$ -4	-4
LS = RS	

$2A + 3B = 10$

Left Side	Right Side
$2A + 3B$ $2\left(\frac{8}{7}\right) + 3\left(\frac{18}{7}\right)$ 10	10
LS = RS	

4. Attempt to solve the following systems of equations. How is each pair of lines related?

a. $x + 3y = 11$

$4x + 12y = 44$

$$x + 3y = 11$$
$$4(x + 3y) = 4(11)$$
$$4x + 12y = 44$$

$$\begin{array}{rrrrrr} & 4x & + & 12y & = & 44 \\ -\ (& 4x & + & 12y & = & 44\ ) \\ \hline & 0x & + & 0y & = & 0 \\ & & & 0 & = & 0 \end{array}$$

Attempting to solve the system produced a true statement, so there are an infinite number of solutions and the two lines are coincident.

b. $2x - 6y = 9$

$3x - 9y = 12$

$$3(2x - 6y) = 3(9)$$
$$6x - 18y = 27$$

$$2(3x - 9y) = 2(12)$$
$$6x - 18y = 24$$

$$\begin{array}{rrrrr} & 6x & - & 18y & = & 27 \\ - & 6x & - & 18y & = & 24 \\ \hline & & & 0 & = & 3 \end{array}$$

Attempting to solve the system produced a false statement, so there is no solution and the two lines are parallel.

Please complete *Lesson 8.3 Explore Your Understanding Assignment* located in *Workbook 8.3* before proceeding to *Lesson 8.4*.

Lesson 8.4: Solving Problems using Linear Systems

Practice – V

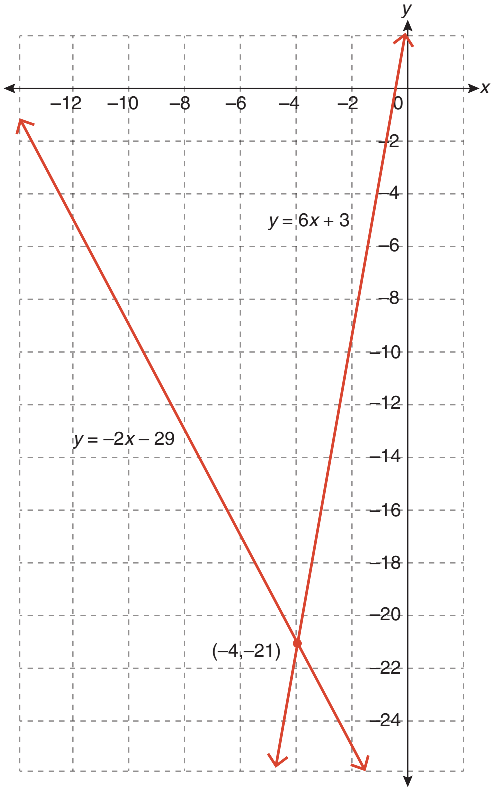
1. Consider the following system of equations.

$y = 6x + 3$

$y = -2x - 29$

a. Solve the system by graphing, by substitution, and by elimination.

Graphing



The solution is (–4, –21).

Substitution

$$\begin{aligned} y &= 6x + 3 \\ -2x - 29 &= 6x + 3 \\ -32 &= 8x \\ -4 &= x \end{aligned}$$

$y = 6x + 3$
$y = 6(-4) + 3$
$y = -21$

The solution is (–4,–21).

Elimination

$$\begin{array}{rrcrcrl} & y & = & 6x & + & 3 & \\ -\ (& y & = & -2x & - & 29 &) \\ \hline & 0 & = & 8x & + & 32 & \\ & -32 & = & 8x & & & \\ & -4 & = & x & & & \end{array}$$

$y = 6x + 3$
$y = 6(-4) + 3$
$y = -21$

The solution is (–4,–21).

b. Which method do you prefer for this system? Why?

Choices and explanations will vary.

2. Matt claims that if a linear system includes a vertical or a horizontal line, the best method for solving the system is usually by substitution. Do you agree with Matt? Why or why not?

Responses will vary. Equations of horizontal and vertical lines either have a variable that is already isolated or a variable that is easy to isolate, so substitution is usually a good method.

3. From the table below, select the system of equations that best represents each of the following problems.

$A + B = 12$ $A - B = 2$	$C = 190M$ $C = 120M + 3400$	$C = 795$ $C = 66D$	$C = 450M + 2000$ $C = 350M + 5000$

a. Tyler wants to know how many days he will need to visit the ski hill for a season's pass to cost less than buying a pass for each visit.

$C = 795$
$C = 66D$

b. Elizabeth is comparing the cost of continuing to run an old furnace with the cost of buying and running a new, high-efficiency furnace.

$C = 190M$
$C = 120M + 3\,400$

c. The sum and difference of two numbers is known. What are the numbers?

$A + B = 12$
$A - B = 2$

d. A car dealership charges a down payment and a monthly payment for someone to finance a vehicle. After how many months will a car with a large down payment and a small monthly payment cost the same as a car with a small down payment and a large monthly payment?

$C = 450M + 2\,000$
$C = 350M + 5\,000$

4. Arial has been offered two jobs. One job pays \$10/h plus a commission of 2% of all sales. The other job pays \$12/h plus a commission of 0.5% of all sales. How much merchandise would Arial need to sell in each 8 hour shift for the two jobs to pay the same?

a. Write a system of equations that can be used to represent this scenario.

Let E be Arial's earnings and let S be the amount sold.

	Hourly Rate (\$/h)	Daily Wage (\$)	Commission Rate	Amount Sold (\$)	Commission Earnings (\$)	Total Earnings
Job 1	10	$10 \times 8 = 80$	0.02	S	$0.02S$	$80 + 0.02S$
Job 2	12	$12 \times 8 = 96$	0.005	S	$0.005S$	$96 + 0.005S$

Arial will earn $0.02S$ commission and a wage of 80 dollars from the first job. Her total earnings per day from Job 1 will be $E = 0.02S + 80$.

Arial will earn $0.005S$ commission and a wage of 96 dollars from the second job. Her total earnings per day from Job 2 will be $E = 0.005S + 96$.

b. Solve the system of equations.

$$\begin{array}{rrcrcl} & E & = & 0.02S & + & 80 \\ -\ (& E & = & 0.005S & + & 96\ ) \\ \hline & 0 & = & 0.015S & - & 16 \\ & 16 & = & 0.015S & & \\ & 1\,066.\overline{6} & = & S & & \end{array}$$

$E = 0.02S + 80$
$E = 0.02(1\,066.\overline{6}) + 80$
$E = 101.\overline{3}$

c. How much merchandise would Arial need to sell in each 8 hour shift for the two jobs to pay the same?

Arial would need to sell approximately \$1 066.67 worth of merchandise each shift for the two jobs to pay the same.

d. Does the conclusion agree with the original information?

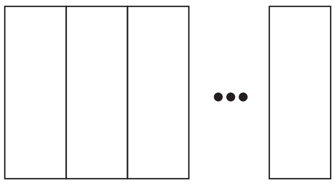
Yes. If Arial sells approximately \$1 066.67 worth of merchandise at either job, she will earn approximately \$101.33.

5. When twenty identical rectangular tables are placed end-to-end, their perimeter is 326 feet. When they are placed side-by-side, their perimeter is 136 feet.

a. Sketch a diagram to represent this problem. (You don't need to show all 20 tables, just a pattern.)

perimeter = 326 feet

perimeter = 136 feet



b. Write a linear system to model the situation.

Let L be the length of a table and let W be the width of a table.

$2W + 40L = 326$

$40W + 2L = 136$

c. What are the dimensions of each table?

$$\begin{aligned} 2W + 40L &= 326 \\ 20(2W + 40L) &= 20(326) \\ 40W + 800L &= 6\,520 \end{aligned}$$

$$\begin{array}{rrcrcr} & 40W & + & 800L & = & 6\,520 \\ -\ (& 40W & + & 2L & = & 136\ ) \\ \hline & & & 798L & = & 6\,384 \\ & & & L & = & 8 \end{array}$$

$$\begin{aligned} 2W + 40L &= 326 \\ 2W + 40(8) &= 326 \\ 2W + 320 &= 326 \\ 2W &= 6 \\ W &= 3 \end{aligned}$$

The width of each table is three feet and the length is eight feet.

6. Medicine Hat and Lethbridge are 169 km apart. If Kiran leaves Medicine Hat at noon and travels towards Lethbridge at 110 km/h, and Leela leaves Lethbridge at noon and travels towards Medicine Hat at 100 km/h, when and where will the two meet?

Let t be time after noon and let d be the distance from Medicine Hat.

	Kiran	Leela
Distance, d, from Medicine Hat at Start	0	169
Time Driving (h)	t	t
Speed (km/h)	110	100
Distance Driven (km)	$110t$	$100t$
Distance from Medicine Hat at time t (km)	$0 + 110t$	$169 - 100t$

Kiran's distance from Medicine Hat is represented by $d = 110t$.

Leela's distance from Medicine Hat is represented by $d = 169 - 100t$.

$$\begin{aligned} d &= 110t \\ 169 - 100t &= 110t \\ 169 &= 210t \\ 0.804... &= t \end{aligned}$$

$$\begin{aligned} d &= 110t \\ d &= 110(0.804) \\ d &= 88.523... \end{aligned}$$

The two will meet after driving for approximately 0.8 hours, at a distance of approximately 89 km from Medicine Hat.

Please complete *Lesson 8.4 Explore Your Understanding Assignment*, located in *Workbook 8.4*.