C. Solving Problems with Linear Relations

Many relationships are linear and can be modelled using a linear equation or a linear graph. Use what you have learned so far in this lesson to explore the relationship between the Celsius and Fahrenheit temperature scales.

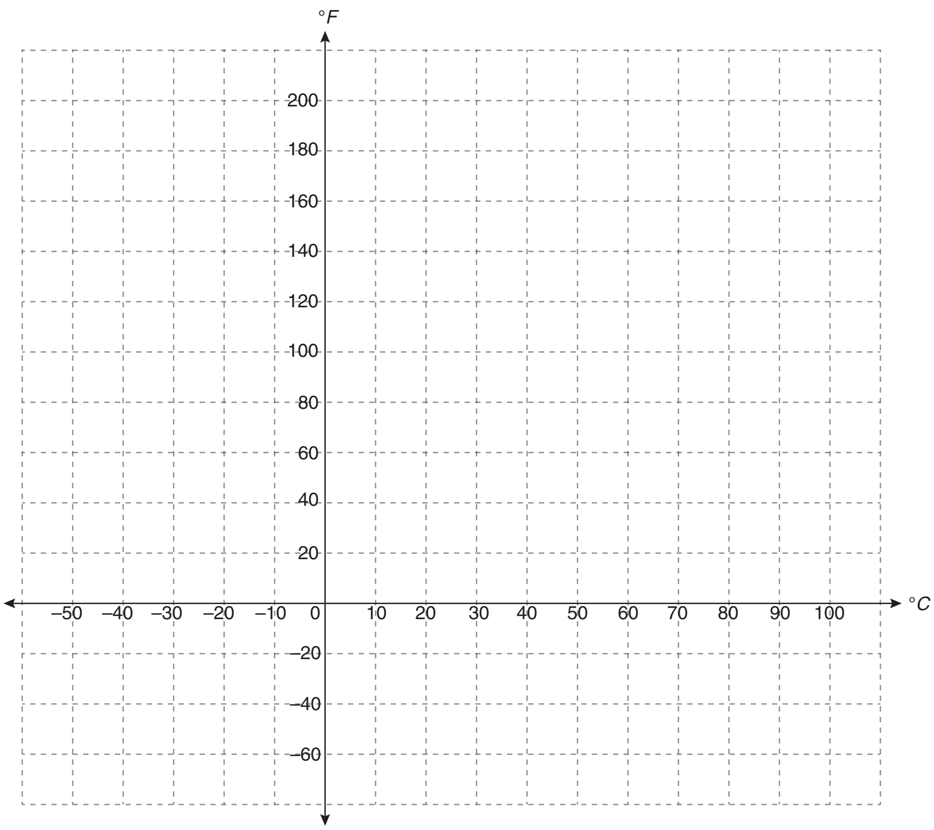
Check Up

There is a linear relationship between the Celsius and Fahrenheit temperature scales. The following table shows some equivalent temperatures.

°C	°F
–40	–40
10	50
35	95
60	140

1. Use the table to sketch a graph of the relation relating the Celsius and Fahrenheit scales.

Temperature Conversions



2. a. Estimate the slope and y-intercept of your graph.

   b. Determine an equation for the graph, in slope-intercept form.

   c. How did your estimated slope and y-intercept values compare to the calculated values?

3. a. What does the slope represent in this scenario?

   b. What does the y-intercept represent in this scenario?

4. a. What is 140 ºC in Fahrenheit?

   b. What is –80 ºF in Celsius?

5. Theoretically, the coldest possible temperature occurs at approximately −273.15 °C and is sometimes referred to as “absolute zero”. Use this information to state the domain and range of the temperature conversion relation.

Compare your answers.

There is a linear relationship between the Celsius and Fahrenheit temperature scales. The following table shows some equivalent temperatures.

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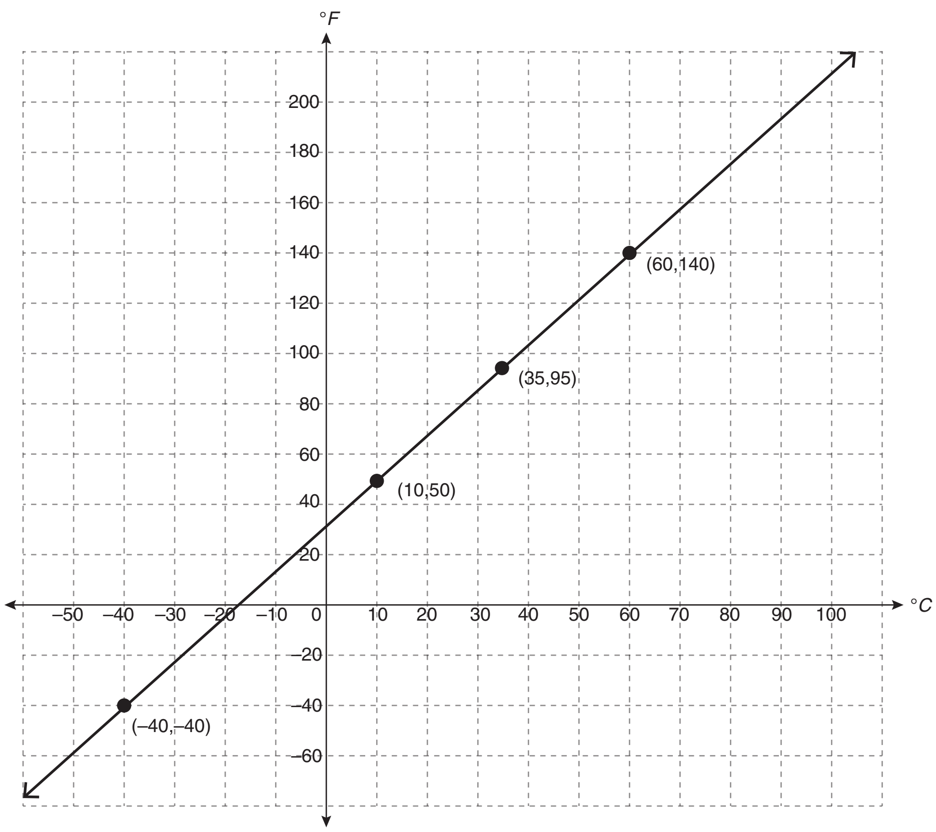

2. a. Estimate the slope and *y*-intercept of your graph.

Estimates will vary.

b. Determine an equation for the graph, in slope-intercept form.

Use the slope formula and two points on the graph to determine the slope.

$$\begin{aligned} m &= \frac{F_2 - F_1}{C_2 - C_1} \\ &= \frac{95 - 50}{35 - 10} \\ &= \frac{45}{25} \\ &= \frac{9}{5} \end{aligned}$$

Substitute a point into $F = \frac{9}{5}C + b$ to determine the b-value.

$$\begin{aligned} F &= \frac{9}{5}C + b \\ 50 &= \frac{9}{5}(10) + b \\ 50 &= 18 + b \\ 50 - 18 &= \cancel{18} + b \cancel{- 18} \\ 32 &= b \end{aligned}$$

An equation relating the Celsius and Fahrenheit scales is $F = \frac{9}{5}C + 32$.

c. How did your estimated slope and y-intercept values compare to the calculated values?

Responses will vary.

3. a. What does the slope represent in this scenario?

The slope represents how much the Fahrenheit scale changes for each change of 1 °C. For every increase of 1 °C, the Fahrenheit change is $\frac{9}{5}$ = 1.8 °F.

b. What does the y-intercept represent in this scenario?

The y-intercept represents the Fahrenheit temperature at 0 °C.
This means 0 °C = 32 °F.

4. a. What is 140 °C in Fahrenheit?

$$F = \frac{9}{5}C + 32$$
$$F = \frac{9}{5}(140) + 32$$
$$F = 252 + 32$$
$$F = 284$$

140 °C = 284 °F

b. What is –80 ºF in Celsius?

$$\begin{aligned} F &= \frac{9}{5}C + 32 \\ -80 &= \frac{9}{5}C + 32 \\ -80 - 32 &= \frac{9}{5}C \cancel{+32} \cancel{-32} \\ -112 &= \frac{9}{5}C \\ -112 \cdot \frac{5}{9} &= \frac{\cancel{9}}{\cancel{5}}C \cdot \frac{\cancel{5}}{\cancel{9}} \\ -\frac{560}{9} &= C \\ -62 &\doteq C \end{aligned}$$

–80 °F ≐ –62 °C

5. Theoretically, the coldest possible temperature occurs at approximately -273.15 °C and is sometimes referred to as "absolute zero". Use this information to state the domain and range of the temperature conversion relation.

The minimum Celsius temperature corresponds to the minimum Fahrenheit temperature.

$$F = \frac{9}{5}C + 32$$
$$F = \frac{9}{5}(-273.15) + 32$$
$$F = -491.67 + 32$$
$$F = -459.67$$

Domain: $\{C \mid C \geq -273.15, C \in \mathrm{R}\}$ or $[-273.15, \infty)$

Range: $\{F \mid F \geq -459.67, F \in \mathrm{R}\}$ or $[-459.67, \infty)$