Unit 1B Assignment

Work slowly and carefully. If you are having difficulty, go back and review the appropriate *Lesson*.

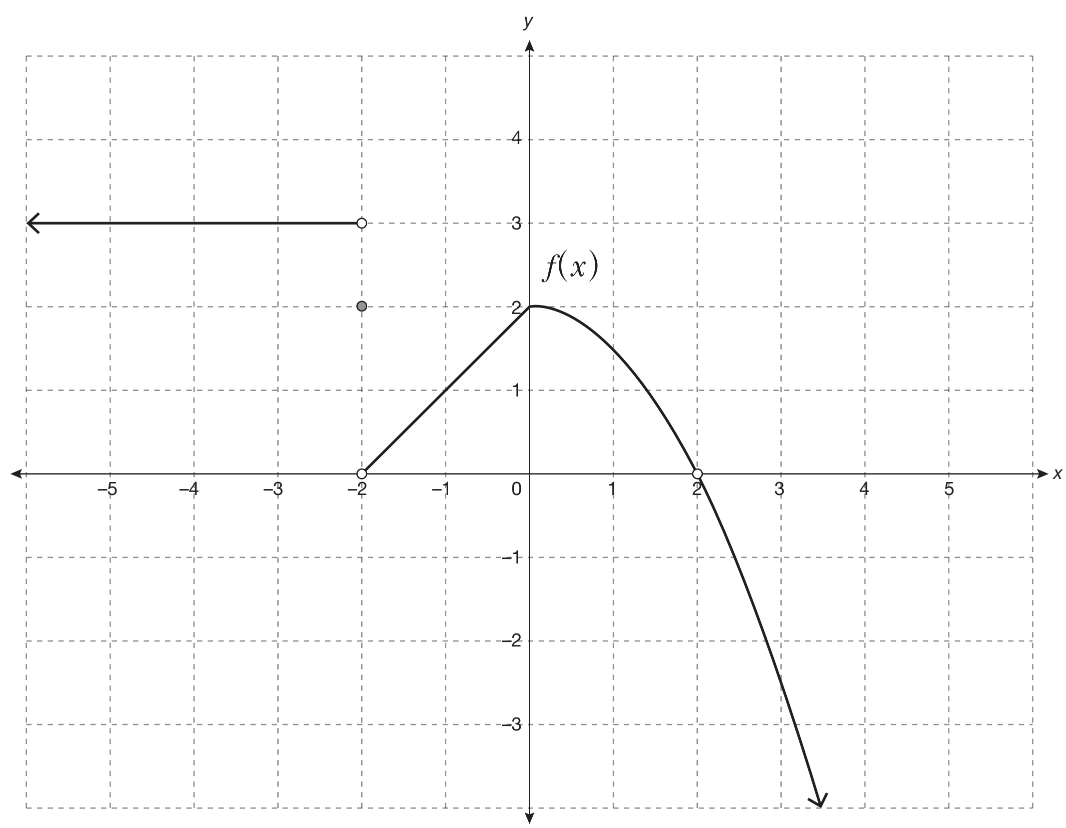
As your midterm and final exams do not allow calculators, it is best to attempt all questions in this *Assignment* without a calculator.

Be sure to proofread your assignment carefully.

For full marks, show all calculations, steps, and/or explain your answers.

Total: 76 marks.

1. Use the graph of $y = f(x)$, shown below, to answer the following questions.



6 a. Complete the chart.

	Limit	Answer
i.	$\lim_{x \to -2^-} f(x)$	
ii.	$\lim_{x \to -2^+} f(x)$	
iii.	$\lim_{x \to -2} f(x)$	
iv.	$f(-2)$	
v.	$\lim_{x \to 0^-} f(x)$	
vi.	$\lim_{x \to 0^+} f(x)$	
vii.	$\lim_{x \to 0} f(x)$	
viii.	$\lim_{x \to 2^-} f(x)$	
ix.	$\lim_{x \to 2^+} f(x)$	
x.	$\lim_{x \to 2} f(x)$	
xi.	$f(2)$	
xii.	$f(-1)$	

2 b. Is this a continuous or discontinuous function? Explain.

2. A piecewise function is defined as follows.

$$f(x) = \begin{cases} (x+2)^2 - 1 & -3 \leq x < 0 \\ 1 & x = 0 \\ x - 2 & 0 < x < 4 \\ 2 & x > 4 \end{cases}$$

4 a. Sketch the graph of $y = f(x)$.

3 b. Determine the following limits, if they exist.

i. $\lim\limits_{x \to 0^-} f(x)$ ____________________

ii. $\lim\limits_{x \to 0^+} f(x)$ ____________________

iii. $\lim\limits_{x \to 0} f(x)$ ____________________

iv. $\lim\limits_{x \to 4^-} f(x)$ ____________________

v. $\lim\limits_{x \to 4^+} f(x)$ ____________________

vi. $\lim\limits_{x \to 4} f(x)$ ____________________

2 c. State any points of discontinuity on the graph of $y = f(x)$. Explain.

3 3. Evaluate the following limits for the piecewise function defined below.

$$f(x) = \begin{cases} x^2 + 2x - 3, & x < -4 \\ x, & x = -4 \\ -\frac{3}{4}x, & x > -4 \end{cases}$$

a. $\lim\limits_{x \to -4^+} f(x)$

b. $\lim\limits_{x \to -4^-} f(x)$

c. $\lim\limits_{x \to -4} f(x)$

4. Find each limit algebraically.

1 a. $\lim\limits_{x \to -1} \dfrac{2x+1}{x}$

1 b. $\lim\limits_{x \to 8} (-3)$

2 c. $\lim\limits_{h \to 2} \dfrac{h^2+2h-8}{h-2}$

2 d. $\lim\limits_{x \to 2} \dfrac{x^2-4x+4}{x^2+x-6}$

2 e. $\lim_{x \to 0} \frac{x^2 + x}{x}$

2 f. $\lim_{x \to 2} \frac{x^2 - 9}{2x^2 - x - 15}$

2 g. $\lim_{x \to 0} \frac{x}{\sqrt{x+4} - 2}$

2 h. $\lim_{x \to 1} \frac{\frac{1}{x^2} - x}{x - 1}$

2 i. $\lim_{x \to 1} \frac{\frac{1}{\sqrt{x}} - 1}{x - 1}$

2 j. $\lim_{x \to 0} \frac{(1-x)^3 - 1}{x}$

4 5. Show $\lim_{x \to 2}(x^3 - 2x) = (\lim_{x \to 2} x)^3 - \lim_{x \to 2} 2x$

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6. Using limit theorems, evaluate the following limits, where $\lim_{x \to a} f(x) = 1$, $\lim_{x \to a} g(x) = 4$, and $\lim_{x \to a} h(x) = -2$. Show all work.

2 a. $\lim_{x \to a}\left(\frac{f(x)g(x)}{h(x)}\right)$

4 b. $\lim_{x \to a}\left(\frac{\sqrt{h(x)+6}}{[g(x)]^2}f(x)\right)$

7. Evaluate the following limits, if they exist. Where applicable, show all work.

1 a. $\lim_{x \to \infty}\frac{x^2}{99}$

1 b. $\lim_{x \to \infty}\left(\frac{29}{14}\right)^x$

1 c. $\lim_{x \to \infty}\left(\frac{7}{9}\right)^x$

1 d. $\lim_{x \to -\infty} 3^{-x}$

2 e. $\lim_{x \to \infty} \frac{2-x}{x+4}$

2 f. $\lim_{x \to -\infty} \frac{6x^2+5x-7}{3x^4+10}$

2 g. $\lim_{x \to -\infty} \frac{4x^6}{x^3 - 8}$

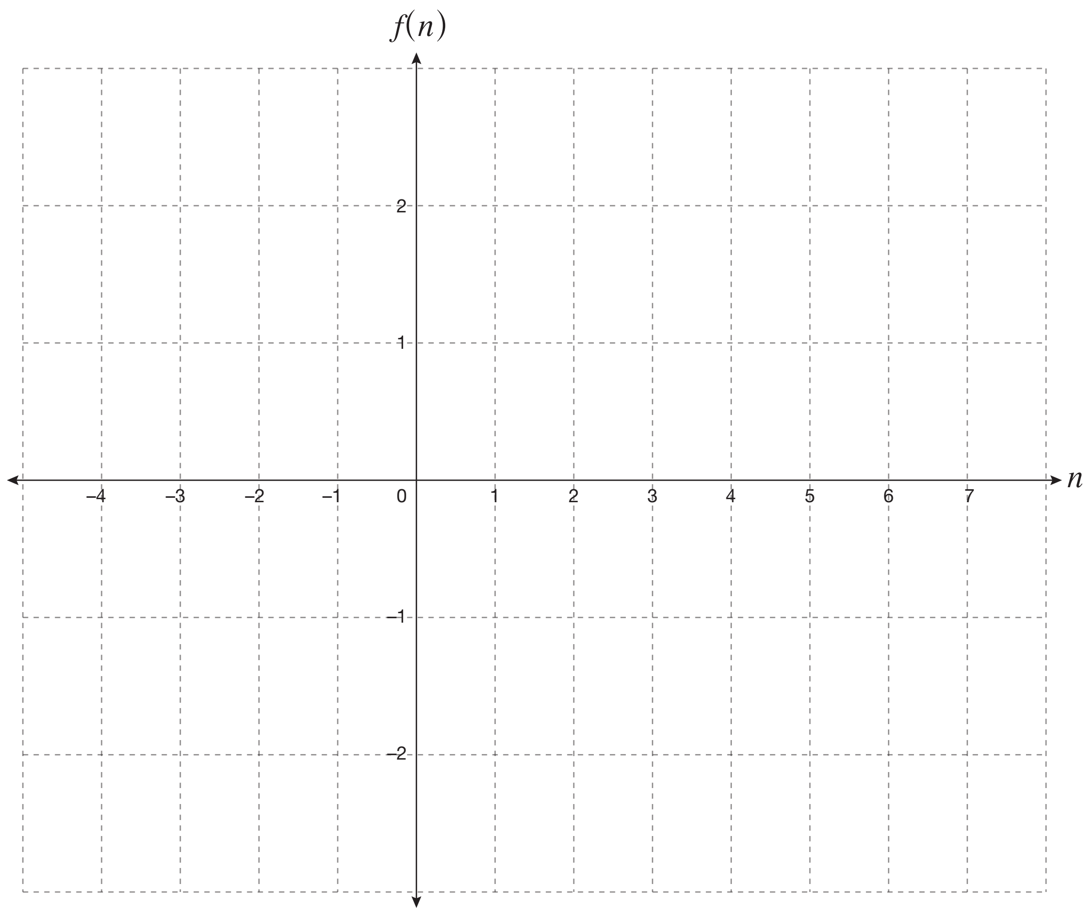
4 h. $\lim_{x \to \pm\infty} \frac{\sqrt{2x^2 - 1}}{x + 3}$

(2) 8. Sketch a possible graph for a function with the following properties.

$$\lim_{x \to \infty} f(x) = -3$$
$$\lim_{x \to 1^-} f(x) = \infty$$
$$\lim_{x \to -\infty} f(x) = -3$$
$$\lim_{x \to 1^+} f(x) = -\infty$$

9. The general term of a sequence is defined by $f(n) = (-1)^{n+1}\left(\frac{2n-1}{n}\right)$.

2 a. Find the first five terms of the sequence, and graph the results.



2 b. Determine if the sequence is convergent or divergent. If the sequence is convergent, determine its limit. If the sequence is divergent, explain why.

2 10. A pendulum on its first swing traces out a path 50 cm in length. On each successive swing, it traces out a path that is 90% as long. The total distance swept out by the swings of the pendulum forms an infinite geometric series. Determine the total distance the pendulum travels before coming to rest.

2 11. Find the sum of the infinite geometric $1 + (x + 1) + (x + 1)^2 + (x + 1)^3 + \ldots$ if $|x + 1| < 1$.

2 12. Find the values of x for which the infinite geometric series $(x-2)^1+(x-2)^2+(x-2)^3+\ldots$ is convergent.

2 13. The second term in an infinite geometric series is $-\frac{1}{2}$ and the third term is $\frac{3}{2}$. Is it possible to determine the sum of this series? Explain.