Unit 5 Assignment

Work slowly and carefully. If you are having difficulty, go back and review the appropriate *Lesson*.

As your final exam does not allow a calculator, it is best to attempt all questions in this *Assignment* without a calculator.

Be sure to proofread your assignment carefully.

For full marks, show all calculations, steps, and/or explain your answers.

Total: 76 marks.

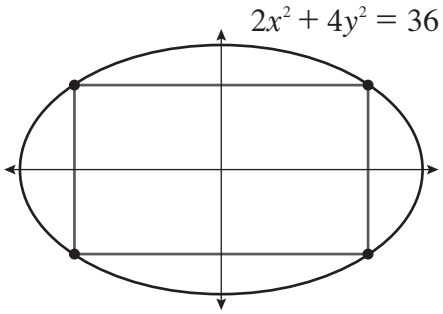
(5) 1. The sum of a number and the square of another number is 27. Find the numbers so that their product is a maximum.

(6) 2. A rectangular box with an open top has a square base. The volume of the box is $108\ \text{cm}^3$. If the box is made of the least amount of material, what must be its dimensions?

(6) 3. A farmer wants to build two adjacent and identical rectangular cattle pens on $60\ 000\ \text{m}^2$ of land. What is the least amount of fencing material required?

(5) 4. A children's picture book is being designed so that each page contains $320\ \text{cm}^2$ of print and pictures, surrounded completely by a white border. The border must be $2\ \text{cm}$ wide at the bottom and on each side and $3\ \text{cm}$ wide at the top. Calculate the dimensions of the page of smallest possible area.
Note: Verification that the dimensions yield a minimum is not needed for this question.

(6) 5. What is the area of the largest rectangle that can be inscribed in the ellipse $2x^2 + 4y^2 = 36$?
Note: Verification that the area is a maximum is not needed for this question.



(7) 6. Find the dimension of a right-circular cylinder of maximum volume that can be inscribed in a sphere of radius $30\ \text{cm}$.

(6) 7. At one end of a $4\ \text{km}$ long, straight beach, there is a boat anchored at point A, $3\ \text{km}$ offshore. At the other end of the beach, there is another boat anchored at point B, $5\ \text{km}$ offshore. A sailor from the first boat must bring a passenger to the beach, and then proceed to the second boat to pick up another passenger. At what point, C, along the beach should the first passenger be dropped in order to minimize the distance travelled by the sailor?
Note: Verification that the distance is a minimum is not needed for this question.

(3) 8. Sarah is in a kayak $250\ \text{m}$ offshore from point A on a straight beach. A storm is brewing and Sarah wishes to go to a shelter located $900\ \text{m}$ down the beach from point A. If Sarah can paddle her kayak at $2\ \text{km/h}$ and walk at $3\ \text{km/h}$, how far down the shore should she beach her kayak to reach the shelter in the shortest amount of time?
Note: Verification that the time is a minimum is not needed for this question.

(7) 9. A property management company manages an apartment block containing 150 units. All 150 units are rented at a monthly rate of $\$460$ per unit and each unit costs the property management company $\$72.50$/month for utilities and repairs. For every $\$25$ rent increase, four fewer apartments are occupied. What rent should be charged in order to realize the most profit?

10. A spherical balloon is being inflated.

(2) a. Find the rate of change of the volume when the radius is $\frac{1}{2}\ \text{cm}$ and changing at a rate of $2\ \text{cm/min}$.

(3) b. If the volume of the balloon is increasing at a rate of $10\ \text{cm}^3/\text{min}$, how fast is the diameter increasing when the radius is $3\ \text{cm}$?

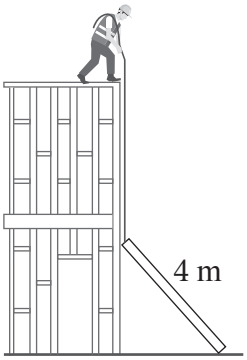
(3) 11. A block of ice has a square top and bottom and rectangular sides. At a certain point in time, the square top and bottom each have a length of $30\ \text{cm}$, which are decreasing at a rate of $2\ \text{cm/h}$. At the same time, the height of the ice block is $20\ \text{cm}$ and decreasing at $3\ \text{cm/h}$. How fast is the ice melting?

(3) 12. The angle of elevation of the sun is decreasing at $\frac{1}{3}$ rad/h. How fast is the shadow cast by a tree 10 m tall lengthening when the angle of elevation of the sun is $\frac{\pi}{6}$ rad?

(5) 13. Two sides of a triangle measure 12 cm and 15 cm in length, and the angle between them is increasing at a rate of 2 rad/min. Determine the rate at which the third side is increasing when the angle between the sides of fixed length is $\frac{\pi}{3}$.

(3) 14. A student leaves school on his skateboard heading east. He travels at a rate of $20\ \text{km/h}$. At the same time, another student leaves the school heading directly south at a rate of $22\ \text{km/h}$. At what rate is the distance between the two students increasing when the skateboarder is $5\ \text{km}$ from the school and the biker is $12\ \text{km}$ from the school?

(3) 15. A construction worker pulls a 4 m plank up the side of a building by means of a rope tied to the end of the plank. The opposite end of the plank is being dragged along the ground. If the worker is pulling at a rate of 20 cm/s, how fast is the end of the plank sliding along the ground when it is 2 m from the wall of the building? Express the solution as an exact value.



(3) 16. A water tank is built in the shape of a circular cone with a height of $5\ \text{m}$ and a diameter of $6\ \text{m}$ at the top. Water is pumped into the tank at a rate of $\frac{8}{5}\ \text{m}^3/\text{min}$. Find the rate at which the water level is rising when the water is $2\ \text{m}$ deep.