▤ Unit 6 Assignment

Work slowly and carefully. If you are having difficulty, go back and review the appropriate *Lesson*.

As your final exam does not allow calculators, it is best to attempt all questions in this *Assignment* without a calculator.

Be sure to proofread your assignment carefully.

For full marks, show all calculations, steps, and/or explain your answers.

Total: 75 marks.

1. Find the derivative of the following functions. Simplify as much as possible.

(1)   a. $f(x) = e^{5x}$

(1)   b. $y = e^x + e^{-x}$

(1)   c. $y = \sin(e^x)$

(2)   d. $f(x) = e^{\cos 2x}$

(2)   e. $y = \sqrt{x}\, e^x$

(2)   f. $f(x) = \dfrac{e^{4x}}{x^2 + 1}$

(3)   2.   Find the derivative of $\dfrac{x}{y} + e^{y} = e$. State the solution in the form $\dfrac{a}{b}$, where a and b contain only positive exponents. Simplify as much as possible.

(3)   3.   Find the equation of the line tangent to the curve $f(x) = e^{2x}$ at $x = 2$ in general form.

④   4.   If $f(x) = e^{-x}x^4$, determine the intervals of increase and decrease.

5.   Algebraically determine the value of x.

①   a. $\log_{27}x = -\dfrac{4}{3}$

①   b. $\log_{\frac{1}{8}}16 = x$

(1)   c. $\log_x 64 = 6$

(1)   d. $2^{\log_3 27} = x$

(2)   e. $\dfrac{4^{\log_2 8\sqrt{5}}}{4^{\log_2 \sqrt{5}}} = x$

6.   If $\log_3 4 = x$, write the following expressions in terms of x.

(2)   a. $\log_3 64$

(2)   b. $\log_3 144$

7. Evaluate the following expressions.

(3)   a. $\log_{\sqrt{2}} 8 + 2\log_9 3$

(2)   b. $\log_2 6 - \log_2 3 + 2\log_2 \sqrt{8}$

8.   Write each expression as a single logarithm.

(1)   a. $\log_a x - 2\log_a p + 3\log_a r - \dfrac{3}{4}\log_a z$

(1)   b. $-1 + 3\log y + \dfrac{5}{4}\log x - \log a$

(2)   9.   Determine the inverse of $y = 3\ln(x+5)$. Write the answer in the form $y = f^{-1}(x)$.

10.   Solve the following equations.

(1)   a. $\ln 5x = 4$

(1)   b. $e^{\frac{1}{2}x} = 3$

(1)   c. $\ln \sqrt[4]{e} = x$

(1)   d. $\ln(3x - 7) = 0$

(1)   e. $\ln(\ln x) = 3$

11.   Rewrite each expression as a single logarithm.

(1)   a. $\dfrac{\ln a}{2} - \ln b - 3 \ln c$

(1)   b. $5 \ln x + \dfrac{3}{2} \ln(x^3 + 1) - 4 \ln(x^4 - 5)$

12.   Find the derivative of each of the following functions. Simply as much as possible.

(1)   a. $f(x) = \ln(x - 2)$

(1)   b. $y = \ln x^2 + \ln 5$

(3)   c. $f(x) = \ln\left(\dfrac{1 + x^2}{1 - x^3}\right)$

(1)   d. $y = 5^{2x+1}$

(2)   e. $y = x \cdot 3^{\sqrt{x}}$

(3)   f. $y = 3\ln(x \cdot 5^{2x})$

(2)   g. $y = \ln^2(\tan x)$

(2)   h. $y = \sin(\ln(2x^5))$

(3)   i. $x^y = y^x$

(3)   13.   Find the equation of the line tangent to the curve $f(x) = \dfrac{\ln x}{x}$ at $(1, 0)$ in general form.

14. Use logarithmic differentiation to find $\dfrac{dy}{dx}$.

(3)   a. $y = (4x)^{x+9}$

(3)   b. $y = (\ln x)^x$

③ 15. A student wanted to sketch the graph of the function $y = \ln(x^2 - 9)$. She wrote down the following steps:

$$y = \ln(x^2 - 9)$$
$$= \ln(x + 3)(x - 3)$$
$$= \ln(x + 3) + \ln(x - 3)$$

She went on to sketch the graph of $y = \ln(x + 3) + \ln(x - 3)$. Her teacher told her that her graph was only half complete. What did she do wrong? Explain.