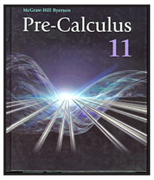
Enhance Your Understanding

Lesson 1.1: Arithmetic Sequences



Refer to Page 16 in *Pre-Calculus 11* for more practice.

- Page 16, #1, 2ac, 3, 4ac, 5bd, 7, 8, 10, 11, 16, 19, and 21

Question 1, page 16

a. $d = t_n - t_{n-1}$

$d = 32 - 16 = 16$
$d = 48 - 32 = 16$
$d = 64 - 48 = 16$
$d = 80 - 64 = 16$

The differences between pairs of consecutive terms are constant; therefore, the sequence is arithmetic. The common difference is 16. The first term, t_1, is 16.

The next three terms are: $t_6 = 80 + 16 = 96$
$t_7 = 96 + 16 = 112$
$t_8 = 112 + 16 = 128$

b. $d = t_n - t_{n-1}$

$d = 4 - 2 = 2$
$d = 8 - 4 = 4$
$d = 16 - 8 = 8$
$d = 32 - 16 = 16$

The sequence is not arithmetic because the differences between pairs of consecutive terms are not constant.

c. $d = t_n - t_{n-1}$

$d = -7 - (-4) = -3$
$d = -10 - (-7) = -3$
$d = -13 - (-10) = -3$
$d = -16 - (-13) = -3$

The differences between pairs of consecutive terms are constant; therefore, the sequence is arithmetic. The common difference is –3. The first term, t_1, is –4.

The next three terms are: $t_6 = -16 + (-3) = -19$
$t_7 = -19 + (-3) = -22$
$t_8 = -22 + (-3) = -25$

d. $d = t_n - t_{n-1}$

$d = 0 - 3 = -3$
$d = -3 - 0 = -3$
$d = -6 - (-3) = -3$
$d = -9 - (-6) = -3$

The differences between pairs of consecutive terms are constant; therefore, the sequence is arithmetic. The common difference is –3. The first term, t_1, is 3.

The next three terms are: $t_6 = -9 + (-3) = -12$
$t_7 = -12 + (-3) = -15$
$t_8 = -15 + (-3) = -18$

Question 2, page 16

a. $t_n = t_{n-1} + d$

$t_1 = 5$
$t_2 = 5 + 3 = 8$
$t_3 = 8 + 3 = 11$
$t_4 = 11 + 3 = 14$

c. $t_n = t_{n-1} + d$

$t_1 = 4$
$t_2 = 4 + \frac{1}{5} = 4\frac{1}{5}$
$t_3 = 4\frac{1}{5} + \frac{1}{5} = 4\frac{2}{5}$
$t_4 = 4\frac{2}{5} + \frac{1}{5} = 4\frac{3}{5}$

Question 3, page 16

a. $t_1 = 3(1) + 8$

$t_1 = 11$

b. $t_7 = 3(7) + 8$

$t_7 = 29$

c. $t_{14} = 3(14) + 8$

$t_{14} = 50$

Question 4, page 16

a. $t_4 = 19, t_5 = 23$

$$\begin{aligned}
d &= t_5 - t_4 \\
d &= 23 - 19 = 4 \\
t_n &= t_1 + (n-1)d \\
t_4 &= t_1 + (4-1)4 \\
19 &= t_1 + 12 \\
7 &= t_1
\end{aligned}$$

$$\begin{aligned}
t_1 &= 7 \\
t_2 &= 7 + 4 = 11 \\
t_3 &= 11 + 4 = 15
\end{aligned}$$

The missing terms are 7, 11, and 15.

c. $t_2 = 4, t_5 = 10$

Substitute into $t_n = t_1 + (n-1)d$

Equation I	Equation II
$t_2 = t_1 + (2-1)d$	$t_5 = t_1 + (5-1)d$
$4 = t_1 + d$	$10 = t_1 + 4d$

Subtract Equation I from Equation II

$$\begin{array}{rrcl}
 & 10 = & \not{t_1} + & 4d \\
-\,(& 4 = & \not{t_1} + & d\ ) \\
\hline
 & 6 = & & 3d \\
 & 2 = & & d
\end{array}$$

$$\begin{aligned}
4 &= t_1 + d \\
4 &= t_1 + 2 \\
2 &= t_1
\end{aligned}$$

$$\begin{aligned}
t_1 &= 2 \\
t_3 &= 4 + 2 = 6 \\
t_4 &= 6 + 2 = 8
\end{aligned}$$

The missing terms are 2, 6, and 8.

Question 5, page 16

b. $t_1 = 2\frac{1}{5}$

$d = 2 - 2\frac{1}{5} = -\frac{1}{5}$

$$t_n = t_1 + (n-1)d$$
$$-14 = 2\frac{1}{5} + (n-1)\left(-\frac{1}{5}\right)$$
$$-14 = 2\frac{1}{5} - \frac{1}{5}n + \frac{1}{5}$$
$$\frac{1}{5}n = 14 + 2\frac{2}{5}$$
$$\frac{1}{5}n = 16\frac{2}{5}$$
$$\cancel{5}\left(\frac{1}{\cancel{5}}n\right) = 5\left(16\frac{2}{5}\right)$$
$$n = 82$$

So, –14 is the 82nd term of the sequence.

d. $t_1 = 14$

$d = 12.5 - 14 = -1.5$

$$t_n = t_1 + (n-1)d$$
$$-10 = 14 + (n-1)(-1.5)$$
$$-10 = 14 - 1.5n + 1.5$$
$$1.5n = 15.5 + 10$$
$$1.5n = 25.5$$
$$\frac{\cancel{1.5}n}{\cancel{1.5}} = \frac{25.5}{1.5}$$
$$n = 17$$

So, –10 is the 17th term of the sequence.

Question 7, page 17

a. The *y*-values of each point are the values of the terms. It appears that the first five terms are: 5, 8, 11, 14, and 17.

b. $t_1 = 5$

$d = 8 - 5 = 3$
$t_n = t_1 + (n-1)d$
$t_n = 5 + (n-1)(3)$
$t_n = 5 + 3n - 3$
$t_n = 2 + 3n$

c. $t_n = 2 + 3n$

$t_{50} = 2 + 3(50)$ $t_{200} = 2 + 3(200)$
$t_{50} = 152$ $t_{200} = 602$

d. Take any two points on the graph to calculate slope.

$m = \frac{y_2 - y_1}{x_2 - x_1}$
$m = \frac{8 - 5}{2 - 1}$
$m = 3$

The slope is equal to the coefficient of n, which corresponds to the common difference of the sequence.

e. If a line were drawn through the points, the y-intercept would be 2. This is the same as the constant value in the general term formula, $t_n = 2 + 3n$.

Question 8, page 17

Consider A:

$t_n = 6 + (n-1)4$
$34 = 6 + 4n - 4$
$34 = 2 + 4n$
$32 = 4n$
$n = 8$

The sequence defined by $t_n = 6 + (n-1)4$ has 34 as its eighth term.

Consider B:

$$t_n = 3n - 1$$
$$34 = 3n - 1$$
$$35 = 3n$$
$$n = 11.6666...$$

Because n is not a natural number, 34 is not a term in this sequence.

Consider C:

$t_1 = 12, d = 5.5$

$$t_n = t_1 + (n-1)d$$
$$t_n = 12 + (n-1)5.5$$
$$34 = 12 + 5.5n - 5.5$$
$$34 = 6.5 + 5.5n$$
$$34 - 6.5 = 5.5n$$
$$27.5 = 5.5n$$
$$\frac{27.5}{5.5} = n$$
$$n = 5$$

The sequence for which $t_1 = 12$ and $d = 5.5$ has 34 as its fifth term.

Consider D:

$t_1 = 3, d = 4$

$$t_n = 3 + (n-1)4$$
$$34 = 3 + 4n - 4$$
$$34 = -1 + 4n$$
$$34 + 1 = 4n$$
$$35 = 4n$$
$$n = 8.75$$

Because n is not a natural number; 34 is not a term in this sequence.

Question 10, page 17

$t_1 = 5y, d = -3y$
$t_n = t_1 + (n-1)d$
$t_n = 5y + (n-1)(-3y)$
$t_n = 5y - 3ny + 3y$
$t_n = 8y - 3ny$

For t_{15}, substitute $n = 15$.

$t_{15} = 8y - 3(15)y$
$t_{15} = 8y - 45y$
$t_{15} = -37y$

Question 11, page 17

The differences between pairs of consecutive terms is the same for terms of an arithmetic sequence.

$$\begin{aligned} t_2 - t_1 &= t_3 - t_2 \\ (7x-4)-(5x+2) &= (10x+6)-(7x-4) \\ 7x - 4 - 5x - 2 &= 10x + 6 - 7x + 4 \\ 2x - 6 &= 3x + 10 \\ -x &= 16 \\ x &= -16 \end{aligned}$$

Substitute $x = -16$ to find the three terms.

$$\begin{aligned} 5x + 2 &= 5(-16) + 2 \\ &= -78 \\ 7x - 4 &= 7(-16) - 4 \\ &= -116 \\ 10x + 6 &= 10(-16) + 6 \\ &= -154 \end{aligned}$$

The three terms are –78, –116, and –154.

Question 16, page 18

a. $t_6 = 11, t_{15} = 29$

Substitute into $t_n = t_1 + (n-1)d$.

Equation I	Equation II
$t_6 = t_1 + (6-1)d$	$t_{15} = t_1 + (15-1)d$
$11 = t_1 + 5d$	$29 = t_1 + 14d$

Subtract Equation I from Equation II

$$\begin{array}{rcl} 29 &=& \cancel{t_1} + 14d \\ -\,(\;11 &=& \cancel{t_1} + 5d\;) \\ \hline 18 &=& 9d \\ 2 &=& d \end{array}$$

Then, calculate t_1.

$11 = t_1 + 5d$
$11 = t_1 + 5(2)$
$11 = t_1 + 10$
$1 = t_1$
$t_n = t_1 + (n-1)d$
$t_n = 1 + (n-1)(2)$
$t_n = 1 + 2n - 2$
$t_n = 2n - 1$

The general term that relates the number of sit-ups to the number of days is $t_n = 2n - 1$.

b. $t_n = 2n - 1$

$100 = 2n - 1$
$101 = 2n$
$50.5 = n$

Susan will be able to do 100 sit-ups on the 51st day of her program.

c. Answers may vary. The assumption is that Susan is physically able to continue increasing the number of sit-ups by two each day.

Question 19, page 19

a. $t_1 = 14.7$

$d = 14.7$	$t_1 = 14.7$
$t_n = t_1 + (n-1)d$	$t_2 = t_1 + d = 14.7 + 14.7 = 29.4$
$t_n = 14.7 + (n-1)(14.7)$	$t_3 = t_2 + d = 29.4 + 14.7 = 44.1$
$t_n = 14.7 + 14.7n - 14.7$	$t_4 = t_3 + d = 44.1 + 14.7 = 58.8$
$t_n = 14.7n$	

The first four terms of the sequence of water pressure with depth are 14.7, 29.4, 44.1, and 58.8. The general term of the sequence is $t_n = 14.7n$, where n is the number of 30 ft descents.

b. Determine n for 1 000 ft.

$$\frac{1\,000}{30} = 33\frac{1}{3}$$

$$\begin{aligned}\text{Water Pressure} &= 14.7n\\ &= 14.7\left(33\frac{1}{3}\right)\\ &= 490\end{aligned}$$

The pressure at a depth of 1 000 ft is 490 psi.

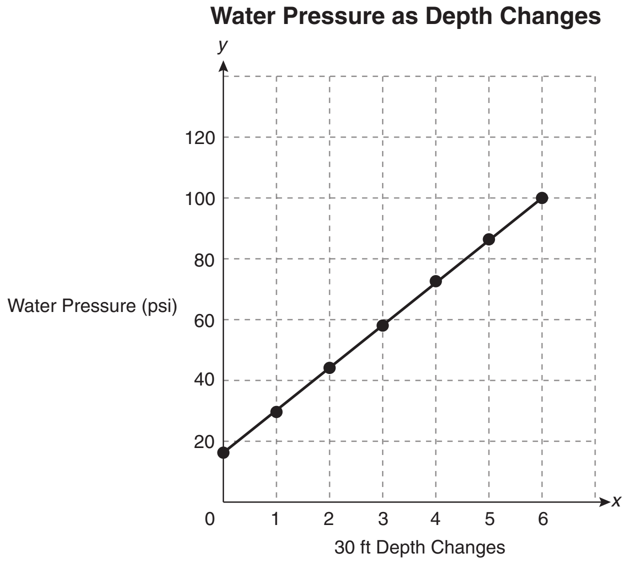
Determine n for 2 000 ft.

$$\frac{2\,000}{30} = 66\frac{2}{3}$$

$$\begin{aligned}\text{Water Pressure} &= 14.7n\\ &= 14.7\left(66\frac{2}{3}\right)\\ &= 980\end{aligned}$$

The pressure at a depth of 2 000 ft is 980 psi.

c.



d. The y-intercept is 14.7.

e. The slope is 14.7.

f. The y-intercept is t_1 and the slope is the common difference, d.

Question 21, page 19

a.

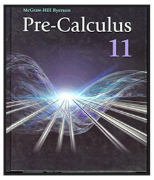
Term Number	Number of Minutes	Number of Degrees
1	4	1
2	8	2
3	12	3
4	16	4
5	20	5

b. $t_1 = 4$

$d = 8 - 4 = 4$
$t_n = t_1 + (n-1)d$
$t_n = 4 + (n-1)(4)$
$t_n = 4 + 4n - 4$
$t_n = 4n$
The number of minutes, t_n, is defined by $t_n = 4n$, where n is the number of degrees of rotation.

c. The time for a rotation of 80° is $4(80) = 320$ minutes.

Lesson 1.2: Arithmetic Series



Refer to Page 27 in *Pre-Calculus 11* for more practice.

- Page 27, #1ad, 2bd, 3bd, 4bc, 5b, 7a, 9, 10, 16, 19, 21, and 23

Question 1, page 27

a. $t_1 = 5, t_n = 53, d = 8 - 5 = 3$

$t_n = t_1 + (n-1)d$
$53 = 5 + (n-1)3$
$53 = 5 + 3n - 3$
$53 = 2 + 3n$
$51 = 3n$
$17 = n$

$S_n = \frac{n}{2}[t_1 + t_n]$
$S_{17} = \frac{17}{2}[5 + 53]$
$S_{17} = 8.5(58)$
$S_{17} = 493$

d. $t_1 = \frac{2}{3}, t_n = \frac{41}{3}, d = \frac{5}{3} - \frac{2}{3} = \frac{3}{3} = 1$

$$\begin{aligned} t_n &= t_1 + (n-1)d \\ \frac{41}{3} &= \frac{2}{3} + (n-1)1 \\ \frac{41}{3} &= \frac{2}{3} + n - 1 \\ \frac{41}{3} &= -\frac{1}{3} + n \\ 14 &= n \end{aligned}$$

$$\begin{aligned} S_n &= \frac{n}{2}[t_1 + t_n] \\ S_{14} &= \frac{14}{2}\left[\frac{2}{3} + \frac{41}{3}\right] \\ S_{14} &= 7\left[\frac{43}{3}\right] \\ S_{14} &= \frac{301}{3} \text{ or } 100\frac{1}{3} \end{aligned}$$

Question 2, page 27

b. $t_1 = 40, n = 11, d = 35 - 40 = -5$

$$\begin{aligned} S_n &= \frac{n}{2}[2t_1 + (n-1)d] \\ S_{11} &= \frac{11}{2}[2(40) + (11-1)(-5)] \\ S_{11} &= \frac{11}{2}[80 - 50] \\ S_{11} &= 165 \end{aligned}$$

d. $t_1 = -3.5, n = 6, d = (-1.25) - (-3.5) = 2.25$

$$\begin{aligned} S_n &= \frac{n}{2}[2t_1 + (n-1)d] \\ S_6 &= \frac{6}{2}[2(-3.5) + (6-1)(2.25)] \\ S_6 &= 3[-7 + 11.25] \\ S_6 &= 12.75 \end{aligned}$$

Question 3, page 27

b. $t_1 = 58, t_n = -7, n = 26$

$$\begin{aligned} S_n &= \frac{n}{2}(t_1 + t_n) \\ S_{26} &= \frac{26}{2}(58 + (-7)) \\ S_{26} &= 13(51) \\ S_{26} &= 663 \end{aligned}$$

d. $t_1 = 12, d = 8, n = 9$

$$S_n = \frac{n}{2}[2t_1 + (n-1)d]$$
$$S_9 = \frac{9}{2}[2(12) + (9-1)(8)]$$
$$S_9 = \frac{9}{2}[24 + 64]$$
$$S_9 = 396$$

Question 4, page 27

b. $d = -6, S_n = 32, n = 13$

$$S_n = \frac{n}{2}[2t_1 + (n-1)d]$$
$$32 = \frac{13}{2}[2t_1 + (13-1)(-6)]$$
$$32 = \frac{13}{2}[2t_1 - 72]$$
$$32 = 13t_1 - 468$$
$$500 = 13t_1$$
$$\frac{500}{13} = t_1$$

c. $d = 0.5, S_n = 218.5, n = 23$

$$S_n = \frac{n}{2}[2t_1 + (n-1)d]$$
$$218.5 = \frac{23}{2}[2t_1 + (23-1)(0.5)]$$
$$218.5 = \frac{23}{2}[2t_1 + 11]$$
$$218.5 = 23t_1 + 126.5$$
$$92 = 23t_1$$
$$4 = t_1$$

Question 5, page 27

b. $t_1 = -6, t_n = 21, S_n = 75$

$$S_n = \frac{n}{2}(t_1 + t_n)$$
$$75 = \frac{n}{2}(-6 + 21)$$
$$75 = \frac{n}{2}(15)$$
$$75 = 7.5n$$
$$10 = n$$

Question 7, page 27

a. $t_1 = 4, t_n = 996, d = 4$

$$\begin{aligned} t_n &= t_1 + (n-1)d \\ 996 &= 4 + (n-1)4 \\ 996 &= 4 + 4n - 4 \\ 996 &= 4n \\ 249 &= n \end{aligned}$$

$$\begin{aligned} S_n &= \frac{n}{2}[t_1 + t_n] \\ S_{249} &= \frac{249}{2}[4 + 996] \\ S_{249} &= 124\,500 \end{aligned}$$

Question 9, page 28

a. $t_5 = 14, d = 3, n = 5$

$$\begin{aligned} t_n &= t_1 + (n-1)d \\ 14 &= t_1 + (5-1)3 \\ 14 &= t_1 + (4)3 \\ 14 &= t_1 + 12 \\ 2 &= t_1 \end{aligned}$$

The pilot flew 2 circuits on the first day.

b. $t_1 = 2, t_5 = 14, d = 3, n = 5$

$$\begin{aligned} S_n &= \frac{n}{2}(t_1 + t_n) \\ S_5 &= \frac{5}{2}(2 + 14) \\ S_5 &= 40 \end{aligned}$$

The pilot flew 40 circuits by the end of the fifth day.

c. $t_1 = 2, d = 3$

$$\begin{aligned} S_n &= \frac{n}{2}[2t_1 + (n-1)d] \\ S_n &= \frac{n}{2}[2(2) + (n-1)3] \\ S_n &= \frac{n}{2}[4 + 3n - 3] \\ S_n &= \frac{n}{2}[1 + 3n] \end{aligned}$$

The pilot flew $\frac{n}{2}[1 + 3n]$ circuits by the end of the n^{th} day.

Question 10, page 28

$t_2 = 40, t_5 = 121$

Substitute into $t_n = t_1 + (n-1)d$.

Equation I	Equation II
$t_2 = t_1 + (2-1)d$	$t_5 = t_1 + (5-1)d$
$40 = t_1 + d$	$121 = t_1 + 4d$

Subtract Equation I from Equation II.

$$\begin{array}{rrcl} & 121 & = & \cancel{t_1} + 4d \\ -\,(& 40 & = & \cancel{t_1} + d\ ) \\ \hline & 81 & = & 3d \\ & 27 & = & d \end{array}$$

Then, calculate t_1.

$t_1 = t_2 - d$

$t_1 = 40 - 27$

$t_1 = 13$

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$

$S_{25} = \frac{25}{2}[2(13) + (25-1)27]$

$S_{25} = \frac{25}{2}[26 + (24)(27)]$

$S_{25} = \frac{25}{2}[26 + 648]$

$S_{25} = 8\,425$

The sum of the first 25 terms of the series is 8 425.

Question 16, page 29

There are 18 rings of diameter 20 cm through to diameter 3 cm. There are 17 overlaps of 2 cm each. So, find the sum of the 18 diameters, and subtract the overlap.

$t_1 = 20, t_n = 3, n = 18$

$S_n = \frac{n}{2}(t_1 + t_n)$

$S_{18} = \frac{18}{2}(20 + 3)$

$S_{18} = 9(23)$

$S_{18} = 207$

The distance from the top of the top ring to the bottom of the lowest ring is $207 - 2(17) = 173\text{ cm}$.

Question 19, page 30

a. Amount by the end of the seventh hour $= 240 + 250 + 260 + 270 + 280 + 290 + 300$.

b. $t_1 = 240, d = 10$

$$S_n = \frac{n}{2}[2t_1 + (n-1)d]$$
$$S_n = \frac{n}{2}[2(240) + (n-1)10]$$
$$S_n = \frac{n}{2}[480 + 10n - 10]$$
$$S_n = \frac{n}{2}[470 + 10n]$$
$$S_n = 235n + 5n^2$$

c. $n = 7$

$$S_7 = 235(7) + 5(7)^2$$
$$S_7 = 1\,645 + 245$$
$$S_7 = 1\,890$$

Question 21, page 30

The formula that Pierre used is, in effect, the same as the one that Jeannette used. In the first formula, substitute $t_n = t_1 + (n-1)d$, and simplify.

$$S_n = \frac{n}{2}(t_1 + t_n)$$

$$S_n = \frac{n}{2}[t_1 + t_1 + (n-1)d]$$
$$S_n = \frac{n}{2}[2t_1 + (n-1)d]$$

Question 23, page 31

a. The tenth triangular number is given by $1 + 2 + 3 + ... + 9 + 10$.

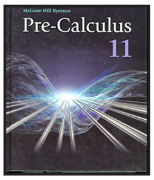
$t_1 = 1, t_n = 10, n = 10$

$$S_n = \frac{n}{2}(t_1 + t_n)$$
$$S_{10} = \frac{10}{2}(1 + 10)$$
$$S_{10} = 5(11)$$
$$S_{10} = 55$$

b. $t_1 = 1, d = 1$

$$S_n = \frac{n}{2}[2t_1 + (n-1)d]$$
$$S_n = \frac{n}{2}[2(1) + (n-1)1]$$
$$S_n = \frac{n}{2}[2 + n - 1]$$
$$S_n = \frac{n}{2}(n+1)$$

Lesson 1.3: Geometric Sequences



Refer to Page 39 in *Pre-Calculus 11* for more practice.

- Page 39, #1, 2c, 5, 9, 10, 14, 19, and 20

Question 1, page 39

a. $t_1 = 1$

$$r = \frac{t_n}{t_{n-1}}$$
$$r = \frac{2}{1} = 2$$
$$r = \frac{4}{2} = 2$$
$$r = \frac{8}{4} = 2$$

Because $r = 2$ and is constant, this is a geometric sequence. The general term is $t_n = 1(2)^{n-1}$ or $t_n = 2^{n-1}$.

b. $r = \frac{t_n}{t_{n-1}}$

$$r = \frac{4}{2} = 2$$
$$r = \frac{6}{4} = 1.5$$
$$r = \frac{8}{6} = 1.3333...$$

Because r is not constant, this is not a geometric sequence, and a general term cannot be determined.

c. $t_1 = 3$

$$r = \frac{t_n}{t_{n-1}}$$
$$r = \frac{-9}{3} = -3$$
$$r = \frac{27}{-9} = -3$$
$$r = \frac{-81}{27} = -3$$

Because $r = -3$ and is constant, this is a geometric sequence. The general term is $t_n = 3(-3)^{n-1}$.

d. $$r = \frac{t_n}{t_{n-1}}$$
$$r = \frac{1}{1} = 1$$
$$r = \frac{2}{1} = 2$$
$$r = \frac{4}{2} = 2$$
$$r = \frac{8}{4} = 2$$

Because r is not constant, this is not a geometric sequence, and a general term cannot be determined.

e. $t_1 = 10$

$$r = \frac{t_n}{t_{n-1}}$$
$$r = \frac{15}{10} = 1.5$$
$$r = \frac{22.5}{15} = 1.5$$
$$r = \frac{33.75}{22.5} = 1.5$$

Because $r = 1.5$ and is constant, this is a geometric sequence. The general term is $t_n = 10(1.5)^{n-1}$.

f. $t_1 = -1$

$$r = \frac{t_n}{t_{n-1}}$$
$$r = \frac{-5}{-1} = 5$$
$$r = \frac{-25}{-5} = 5$$
$$r = \frac{-125}{-25} = 5$$

Because $r = 5$ and is constant, this is a geometric sequence. The general term is $t_n = -1(5)^{n-1}$.

Question 2, page 39

c.

Geometric Sequence	Common Ratio	6th Term	10th Term
$\frac{1}{5}, \frac{3}{5}, \frac{9}{5}$	$r = \frac{t_n}{t_{n-1}}$ $r = \frac{\frac{3}{5}}{\frac{1}{5}} = 3$	$t_n = t_1(r)^{n-1}$ $t_6 = \frac{1}{5}(3)^{6-1}$ $t_6 = \frac{1}{5}(3)^5$ $t_6 = \frac{243}{5}$ or 48.6	$t_n = t_1(r)^{n-1}$ $t_{10} = \frac{1}{5}(3)^{10-1}$ $t_{10} = \frac{1}{5}(3)^9$ $t_{10} = \frac{19\,683}{5}$ or 3 936.6

Question 5, page 39

a. $r = 2, t_1 = 3$

$$t_n = t_1(r)^{n-1}$$
$$t_n = 3(2)^{n-1}$$

b. $t_1 = 192$

$$r = \frac{t_n}{t_{n-1}}$$
$$r = \frac{-48}{192} = -\frac{1}{4}$$
$$t_n = t_1(r)^{n-1}$$
$$t_n = 192\left(-\frac{1}{4}\right)^{n-1}$$

c. $t_3 = 5, t_6 = 135$

$$\begin{aligned} t_n &= t_1(r)^{n-1} \\ t_3 &= t_1(r)^{3-1} \\ 5 &= t_1 r^2 \\ \frac{5}{r^2} &= t_1 \end{aligned} \qquad \begin{aligned} t_n &= t_1(r)^{n-1} \\ t_6 &= t_1(r)^{6-1} \\ 135 &= \frac{5}{r^2}(r)^5 \\ 135 &= 5r^3 \\ 27 &= r^3 \\ \sqrt[3]{27} &= r \\ 3 &= r \end{aligned}$$

Then, $t_1 = \frac{5}{r^2}$,

$$\begin{aligned} &= \frac{5}{3^2} \\ &= \frac{5}{9} \end{aligned}$$

and $t_n = \frac{5}{9}(3)^{n-1}$.

d. $t_1 = 4, t_{13} = 16\,384$

$$\begin{aligned} t_n &= t_1(r)^{n-1} \\ t_{13} &= t_1(r)^{13-1} \\ 16\,384 &= 4(r)^{12} \\ 4\,096 &= r^{12} \\ \sqrt[12]{4\,096} &= r \\ 2 &= r \end{aligned}$$

So, $t_n = 4(2)^{n-1}$.

Question 9, page 40

a. $t_1 = 3.0, r = 0.75$

b. $t_n = t_1(r)^{n-1}$

$t_n = 3.0(0.75)^{n-1}$

c. After the sixth bounce, the ball will reach its seventh height.

$t_7 = 3.0(0.75)^{7-1}$
$t_7 = 0.5339...$
After the sixth bounce, the ball reaches a height of approximately 0.53 m.

d. $t_n = 0.40$

$$0.40 = 3.0(0.75)^{n-1}$$
$$\frac{0.40}{3.0} = (0.75)^{n-1}$$
$$0.13333... = (0.75)^{n-1}$$

Using guess and check, determine the value of n that will satisfy the equation above.

$n = 6, (0.75)^{6-1} = (0.75)^5 = 0.237...$ Too big
$n = 8, (0.75)^{8-1} = (0.75)^7 = 0.13348...$ Very close, only slightly too big
$n = 9, (0.75)^{9-1} = (0.75)^8 = 0.100...$ Too small

Recall that although $n = 8$, the number of bounces is one less. So, after 7 bounces, the ball will reach a height of approximately 40 cm.

Question 10, page 40

a. If 5% of the color is lost, 95% remains after one washing.

b. $t_1 = 100$

$t_2 = 100(0.95) = 95$
$t_3 = 95(0.95) = 90.25$
$t_4 = 90.25(0.95) = 85.7375$

c. $r = 0.95$

d. $t_1 = 100, r = 0.95$

For 10 washings, $n = 11$ because t_1 was the original colour, prior to any washing. Therefore t_2 is the first wash, and it follows that the 10th wash will be when $n = 11$.

$t_n = t_1(r)^{n-1}$
$t_n = 100(0.95)^{n-1}$
$t_{11} = 100(0.95)^{11-1}$
$t_{11} = 100(0.95)^{10}$
$t_{11} = 59.8736...$
After 10 washings, approximately 60% of the colour remains.

e. $t_n = 25$

$t_n = 100(0.95)^{n-1}$
$25 = 100(0.95)^{n-1}$
$0.25 = (0.95)^{n-1}$
Using guess and check, determine the value of n that will satisfy the equation above.

$n = 17, (0.95)^{17-1} = (0.95)^{16} = 0.440...$Too big
$n = 26, (0.95)^{26-1} = (0.95)^{25} = 0.277...$Close, but a little too big
$n = 28, (0.95)^{28-1} = (0.95)^{27} = 0.250...$Ok
Remember to subtract one from n to find the number of washings. After 27 washings, only 25% of the original color remains. An assumption is that washing conditions remain the same, and the jeans don't get faded another way, such as being out in the sun a lot.

Question 14, page 41

a. The cell growth of yeast follows the sequence 1, 2, 4, 8, 16, 32.

b. $t_1 = 1, r = 2$

$t_n = t_1(r)^{n-1}$
$t_n = 1(2)^{n-1}$
$t_n = 2^{n-1}$

c. $n = 26$ for 25 doublings because t_1 is the original one bacteria before any doubling.

$$t_n = 2^{n-1}$$
$$t_{26} = 2^{26-1}$$
$$t_{26} = 2^{25}$$
$$t_{26} = 33\,554\,432$$

After 25 doublings, there would be 33 554 432 cells.

d. The assumption is that all cells continue to live and continue to reproduce.

Question 19, page 43

a. $t_1 = 250, r = 100\% - 18\% = 82\% = 0.82$

Since the amount of medicine is decreasing every 2 hours, there have been 6 time periods in 12 hours. Because t_1 is the original amount of the medicine before any is used up, use $n = 7$.

$$t_n = t_1(r)^{n-1}$$
$$t_7 = 250(0.82)^{7-1}$$
$$t_7 = 250(0.82)^{6}$$
$$t_7 = 76.001...$$
$$t_7 \doteq 76.0$$

After 12 hours, approximately 76.0 mL of the medicine remains.

b. $t_n = 20$

$$t_n = 250(0.82)^{n-1}$$
$$20 = 250(0.82)^{n-1}$$
$$\frac{20}{250} = (0.82)^{n-1}$$
$$0.08 = 0.82^{n-1}$$

Using guess and check, determine the value of n that will satisfy the equation above.

$n = 20, (0.82)^{20-1} = 0.023...$ Too low

$n = 10, (0.82)^{10-1} = 0.167...$ Too high

$n = 13, (0.82)^{13-1} = 0.0924...$ Close, but a little too high

$n = 14, (0.82)^{14-1} = 0.0757...$ Is less than the target

Therefore, $n = 14$. Remember to reduce the number of 2 hour periods by one because t_1 was the original amount of medicine. Then, multiply by two to find the total time that has passed. The amount will be less than 20 mL after approximately $2(13) = 26$ hours.

Question 20, page 43

a.

Time, *d* (days)	Charge Level, C (%)
0	100
1	$100(0.98) = 98$
2	$100(0.98)^2 = 96.04$
3	$100(0.98)^3 = 94.1192$

b. $t_n = 100(0.98)^{n-1}$

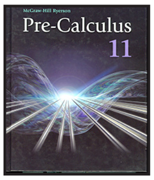
c. In the formula in part b., n represents the number of times the 2% reduction has occurred plus one…i.e. $n = 2$ for the first 2% reduction. We could say $n - 1$ represents the number of reductions. In the formula $C = 100(0.98)^d$, d is the day number. So, $d = n - 1$ or $n = d + 1$.

d. $t_{11} = 100(0.98)^{10}$

$t_{11} = 81.7072...$

After 10 days, approximately 81.7% of the battery's charge remains.

Lesson 1.4: Geometric Series



Refer to Pages 53 and 63 in *Pre-Calculus 11* for more practice.

- Page 53, #1, 2ab, 3bd, 4ac, 5a, 6, 8, 10, 13, and 22
- Page 63, #1, 2, 6, 8, 15, 16, and 17

Question 1, page 53

a. $t_1 = 4$

$r = \frac{t_n}{t_{n-1}}$

$r = \frac{24}{4} = 6$

$r = \frac{144}{24} = 6$

$r = \frac{864}{144} = 6$

Because r is constant, this is a geometric series.

b. $t_1 = -40$

$$r = \frac{t_n}{t_{n-1}}$$
$$r = \frac{20}{-40} = -0.5$$
$$r = \frac{-10}{20} = -0.5$$
$$r = \frac{5}{-10} = -0.5$$

Because *r* is constant, this is a geometric series.

c. $t_1 = 3$

$$r = \frac{t_n}{t_{n-1}}$$
$$r = \frac{9}{3} = 3$$
$$r = \frac{18}{9} = 2$$
$$r = \frac{54}{18} = 3$$

Because the *r* values are not constant, this is not a geometric series.

d. $t_1 = 10$

$$r = \frac{t_n}{t_{n-1}}$$
$$r = \frac{11}{10} = 1.1$$
$$r = \frac{12.1}{11} = 1.1$$
$$r = \frac{13.31}{12.1} = 1.1$$

Because *r* is constant, this is a geometric series.

Question 2, page 53

a $t_1 = 6, r = \frac{9}{6} = \frac{3}{2}, n = 10$

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$S_{10} = \frac{6\left[\left(\frac{3}{2}\right)^{10} - 1\right]}{\frac{3}{2} - 1}$$

$$S_{10} = \frac{6\left[\frac{59\,049}{1\,024} - 1\right]}{\frac{1}{2}}$$

$$S_{10} = \frac{174\,075}{256}$$

$$S_{10} = 679.980...$$

$$S_{10} \doteq 679.98$$

b. $t_1 = 18, r = \frac{-9}{18} = \frac{-1}{2}, n = 12$

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$S_{12} = \frac{18\left[\left(-\frac{1}{2}\right)^{12} - 1\right]}{-\frac{1}{2} - 1}$$

$$S_{12} = \frac{18\left[\frac{1}{4\,096} - 1\right]}{-\frac{3}{2}}$$

$$S_{12} = 18\left[-\frac{4\,095}{4\,096}\right]\left(-\frac{2}{3}\right)$$

$$S_{12} = \frac{12\,285}{1\,024}$$

$$S_{12} = 11.997...$$

$$S_{12} \doteq 12.00$$

Question 3, page 53

b. $t_1 = 27, r = \frac{1}{3}, n = 8$

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$
$$S_8 = \frac{27\left[\left(\frac{1}{3}\right)^8 - 1\right]}{\frac{1}{3} - 1}$$
$$S_8 = \frac{27\left[\frac{1}{6\,561} - 1\right]}{-\frac{2}{3}}$$
$$S_8 = 27\left[-\frac{6\,560}{6\,561}\right]\left(-\frac{3}{2}\right)$$
$$S_8 = \frac{3\,280}{81}$$

d. $t_1 = 72, r = \frac{1}{2}, n = 12$

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$
$$S_{12} = \frac{72\left[\left(\frac{1}{2}\right)^{12} - 1\right]}{\frac{1}{2} - 1}$$
$$S_{12} = \frac{72\left[\frac{1}{4\,096} - 1\right]}{-\frac{1}{2}}$$
$$S_{12} = 72\left[-\frac{4\,095}{4\,096}\right](-2)$$
$$S_{12} = \frac{36\,855}{256}$$

Question 4, page 53

a $t_1 = 27, t_n = \frac{1}{243}, r = \frac{9}{27} = \frac{1}{3}$

$$S_n = \frac{rt_n - t_1}{r - 1}$$
$$S_n = \frac{\left(\frac{1}{3}\right)\left(\frac{1}{243}\right) - 27}{\frac{1}{3} - 1}$$
$$S_n = \frac{\left(\frac{1}{729}\right) - 27}{-\frac{2}{3}}$$
$$S_n = \left(-\frac{19\,682}{729}\right)\left(-\frac{3}{2}\right)$$
$$S_n = \frac{9\,841}{243}$$
$$S_n = 40.497...$$
$$S_n \doteq 40.50$$

c. $t_1 = 5, t_n = 81\,920, r = 4$

$$S_n = \frac{rt_n - t_1}{r - 1}$$
$$S_n = \frac{(4)(81\,920) - 5}{4 - 1}$$
$$S_n = \frac{327\,675}{3}$$
$$S_n = 109\,225$$

Question 5, page 54

a. $S_n = 33, t_n = 48, r = -2$

$$S_n = \frac{rt_n - t_1}{r - 1}$$
$$33 = \frac{(-2)(48) - t_1}{-2 - 1}$$
$$(-3)(33) = -96 - t_1$$
$$-99 + 96 = -t_1$$
$$-3 = -t_1$$
$$3 = t_1$$

Question 6, page 54

$S_n = 4\,372, t_1 = 4, r = 3$

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$
$$4\,372 = \frac{4(3^n - 1)}{3 - 1}$$
$$4\,372 = 2(3^n - 1)$$
$$2\,186 = 3^n - 1$$
$$2\,187 = 3^n$$

Note: $3^7 = 2\,187$, this is very close to $2\,188$

Use 3^7 to approximate $2\,188$.

$$3^7 \doteq 3^n$$
$$7 \doteq n$$

There are seven terms in the series.

Question 8, page 54

$t_3 = \frac{9}{4}, t_6 = -\frac{16}{81}$

$$t_n = t_1(r)^{n-1}$$
$$t_3 = t_1(r)^{3-1}$$
$$\frac{9}{4} = t_1 r^2$$
$$\frac{9}{4r^2} = t_1$$

$$t_n = t_1(r)^{n-1}$$
$$t_6 = \frac{9}{4r^2}(r)^{6-1}$$
$$-\frac{16}{81} = \frac{9}{4r^2}(r)^5$$
$$-\frac{16}{81} = \frac{9}{4}r^3$$
$$\left(\frac{4}{9}\right)\left(-\frac{16}{81}\right) = r^3$$
$$-\frac{64}{729} = r^3$$
$$\sqrt[3]{-\frac{64}{729}} = r$$
$$-\frac{4}{9} = r$$

Then, $t_1 = \frac{9}{4r^2}$

$$t_1 = \frac{9}{4\left(-\frac{4}{9}\right)^2}$$
$$t_1 = \frac{9}{\left(\frac{64}{81}\right)}$$
$$t_1 = \frac{729}{64}$$

Multiply $t_1 = \frac{729}{64}$ by r to determine the second term: $t_2 = \frac{729}{64}\left(-\frac{4}{9}\right) = -\frac{81}{16}$.

Substitute known values to find S_6 using $S_n = \frac{rt_n - t_1}{r - 1}$.

$$S_6 = \frac{\left(-\frac{4}{9}\right)\left(-\frac{16}{81}\right) - \left(\frac{729}{64}\right)}{\left(-\frac{4}{9}\right) - 1}$$

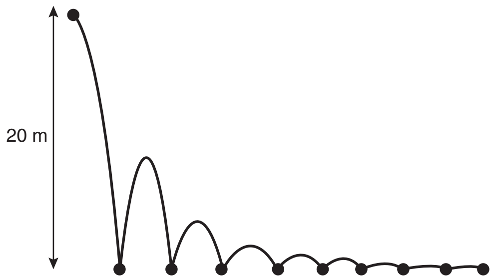
$$S_6 = \frac{\left(\frac{64}{729} - \frac{729}{64}\right)}{-\frac{13}{9}}$$

$$S_6 = \left(\frac{64}{729} - \frac{729}{64}\right)\left(-\frac{9}{13}\right)$$

$S_6 = 7.825...$

The sum of the first six terms is approximately 7.8.

Question 10, page 54



$t_1 = 16, r = 0.4, n = 5$

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$S_5 = \frac{16(0.4^5 - 1)}{0.4 - 1}$$

$S_5 = 26.393\,6$

The total vertical distance travelled when the ball hits the ground for the sixth time is approximately $20 \text{ m} + 26.3936 \text{ m} \doteq 46.4 \text{ m}$.

Question 13, page 55

$t_1 = 1000(1.4) = 1400, r = 1.4, n = 10$

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$S_{10} = \frac{1\,400(1.4^{10} - 1)}{1.4 - 1}$$

$S_{10} = 97\,739.129...$

After 100 days, approximately 97 739 + 1 000 = 98 739 people will be aware of the product.

Question 22, page 57

Answers may vary.

a. Tom is assuming that all 400 eggs do in fact produce a butterfly.

b. Tom's assumption is very optimistic. Some eggs would not survive due to weather, predators, or other unfavorable circumstances. Also he has calculated the total number in the first to fifth generations, not the fifth generation only.

c. Tom's estimate is probably too high based on the reasons in part b.

d. Answers will vary. For example, research the life span of butterflies and the normal number of successful hatchings expected from 400 eggs. Then, use these more accurate values and the general term formula for geometric series to calculate the fifth generation's population only.

Question 1, page 63

a. Since $r > 1$, the series is divergent.

b. Since $-1 < r < 1$, the series is convergent.

c. $r = \frac{25}{125} = \frac{1}{5}$; since $-1 < r < 1$, the series is convergent.

d. $r = \frac{-4}{-2} = 2$; since $r > 1$, the series is divergent.

e. $r = \left(-\frac{9}{5}\right) \div \left(\frac{27}{25}\right)$

$$r = \left(-\frac{9}{5}\right)\left(\frac{25}{27}\right)$$

$$r = -\frac{5}{3}$$

Since $r < -1$, the series is divergent.

Question 2, page 63

a. $t_1 = 8, r = -\frac{1}{4}$

$$S_\infty = \frac{t_1}{1-r}$$

$$S_\infty = \frac{8}{1-\left(-\frac{1}{4}\right)}$$

$$S_\infty = 6.4$$

b. Since $r > 1$, the series is divergent and a sum cannot be determined.

c. Since $r = 1$, the series is not geometric, and it has no infinite sum.

d. $t_1 = 1, r = \frac{0.25}{0.5} = 0.5$

$$S_\infty = \frac{t_1}{1-r}$$

$$S_\infty = \frac{1}{1-0.5}$$

$$S_\infty = 2$$

e. $t_1 = 4, r = -\frac{12}{5} \div 4 = -\frac{3}{5}$

$$S_\infty = \frac{t_1}{1-r}$$

$$S_\infty = \frac{4}{1-\left(-\frac{3}{5}\right)}$$

$$S_\infty = 2.5$$

Question 6, page 63

$S_\infty = 81, r = \frac{2}{3}$

$$S_\infty = \frac{t_1}{1-r}$$
$$81 = \frac{t_1}{1-\frac{2}{3}}$$
$$81 = \frac{t_1}{\frac{1}{3}}$$
$$81\left(\frac{1}{3}\right) = t_1$$
$$27 = t_1$$
$$t_2 = 27\left(\frac{2}{3}\right) = 18$$
$$t_3 = 18\left(\frac{2}{3}\right) = 12$$

The first three terms of the series are 27, 18, and 12.

Question 8, page 63

a. $t_1 = 24\,000, r = 0.94$

$$S_\infty = \frac{t_1}{1-r}$$
$$S_\infty = \frac{24\,000}{1-0.94}$$
$$S_\infty = 400\,000$$

If the trend continues, the lifetime production would be 400 000 barrels of crude.

b. The assumption is that the trend does continue and that the well is kept operational until it runs dry. This is not reasonable: once production is low, the well would not be profitable to operate and would probably be closed.

Question 15, page 64

The sum of all vertical heights is equal to the sum of the downward distances and the sum of the upward distances. The downward sum has one extra term, 16.

For downward sum: $t_1 = 16, r = \frac{1}{2}$

$$S_\infty = \frac{t_1}{1-r}$$
$$S_\infty = \frac{16}{1-\frac{1}{2}}$$
$$S_\infty = 32$$

For upward sum: $t_1 = 8, r = \frac{1}{2}$

$$S_\infty = \frac{t_1}{1-r}$$
$$S_\infty = \frac{8}{1-\frac{1}{2}}$$
$$S_\infty = 16$$

The total vertical distance that the ball travels is $32 + 16 = 48$ m.

Question 16, page 64

a. $t_1 = 30, r = \frac{27}{30} = \frac{9}{10}, n = 8$

$$S_n = \frac{t_1(r^n - 1)}{r-1}$$
$$S_8 = \frac{30\left[\left(\frac{9}{10}\right)^8 - 1\right]}{\frac{9}{10} - 1}$$
$$S_8 = 170.859...$$

After 8 times, the post will be driven approximately 170.9 cm into the ground.

b. $t_1 = 30, r = \frac{27}{30} = \frac{9}{10}$

$$S_\infty = \frac{t_1}{1-r}$$
$$S_\infty = \frac{30}{1-\frac{9}{10}}$$
$$S_\infty = 300$$

If the post is pounded indefinitely, it will be driven 300 cm into the ground.

Question 17, page 64

a. Rita is correct.

b. $r = \frac{4}{9} \div -\frac{1}{3} = -\frac{4}{3}$ Since $r < -1$, the series is divergent and has no sum.