## Practice Solutions – II

1. Given $t_5 = 64$ and $t_9 = 112$ of an arithmetic sequence, determine the values of the common difference, $d$ and the first term, $t_1$.

Equation I

$t_1 = ?$
$d = ?$
$t_5 = 64$
$n = 5$

$$t_n = t_1 + (n-1)d$$
$$64 = t_1 + (5-1)d$$
$$64 = t_1 + 4d$$

Equation II

$t_1 = ?$
$d = ?$
$t_9 = 112$
$n = 9$

$$t_n = t_1 + (n-1)d$$
$$112 = t_1 + (9-1)d$$
$$112 = t_1 + 8d$$

Subtract Equation I from Equation II.

$$\begin{array}{rrcl} & 112 &=& t_1 + 8d \\ -\ ( & 64 &=& t_1 + 4d\ ) \\ \hline & 48 &=& 4d \\ & 12 &=& d \end{array}$$

Solve for $t_1$.

$$112 = t_1 + 8d$$
$$112 = t_1 + 8(12)$$
$$16 = t_1$$

2. Kyle is saving money to buy a new electric guitar. He had $225.00 saved by the fourth week and $360.00 by the 13th week.

a. Assuming his savings amount increases in an arithmetic sequence, write the general formula that relates the amount of money saved to the number of weeks of saving. Be sure to define the variables.

Let $t_n$ be the amount in savings, $n$ be the number of weeks, and $d$ be the common difference.

Set up two equations, and solve the system of equations.

Equation I

$t_1 = ?$

$d = ?$

$t_4 = 225$

$n = 4$

$t_n = t_1 + (n-1)d$

$225 = t_1 + (4-1)d$

$225 = t_1 + 3d$

Equation II

$t_1 = ?$

$d = ?$

$t_{13} = 360$

$n = 13$

$t_n = t_1 + (n-1)d$

$360 = t_1 + (13-1)d$

$360 = t_1 + 12d$

Subtract Equation I from Equation II.

$$\begin{aligned} 360 &= t_1 + 12d \\ -(225 &= t_1 + 3d) \\ \hline 135 &= 9d \\ 15 &= d \end{aligned}$$

Next, determine the value of $t_1$.

$360 = t_1 + 12d$

$360 = t_1 + 12(15)$

$180 = t_1$

The general formula is

$t_n = t_1 + (n-1)d$

$t_n = 180 + (n-1)15$

$t_n = 180 + 15n - 15$

$t_n = 165 + 15n$

b. The electric guitar Kyle is looking to buy costs $1 250.00. For how many weeks does Kyle need to save to achieve this goal?

Use the formula from part a. to solve for $n$.

$t_n = 165 + 15n$

$1\,250 = 165 + 15n$

$72.\bar{3} = n$

It will take Kyle 73 weeks to have at least $1 250.00 to purchase the guitar. Note the answer is rounded up because at 72 weeks, Kyle will still not have $1 250.00.

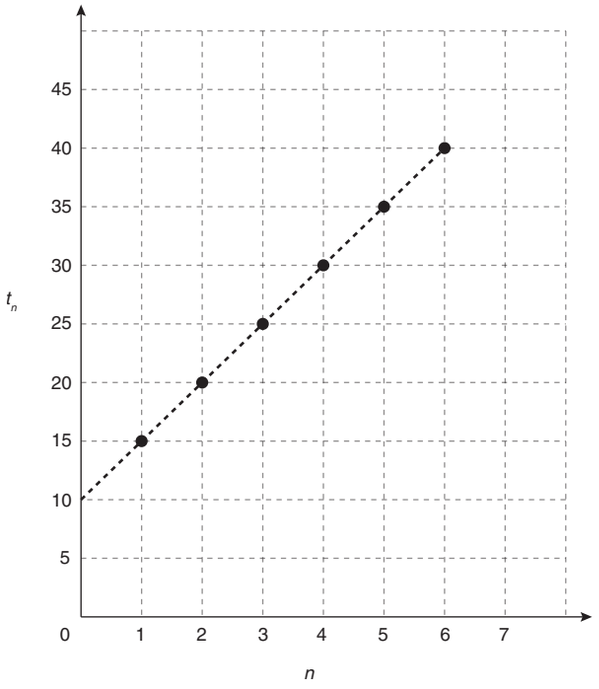
c. What assumption is made in order to answer part b.?

The assumption is that Kyle will continue to save $15.00 per week, no more and no less.

d. How much money did Kyle have in his savings account before he started saving for the guitar?

Kyle's initial savings is the value of $t_1$. Therefore, Kyle had $180 in the account before starting to save for the guitar.

3. Use the graph below to answer the following question.




a. List the first five terms of the sequence in the table.

| $n$ | $t_n$ |
|---|---|
| 1 | 15 |
| 2 | 20 |
| 3 | 25 |
| 4 | 30 |
| 5 | 35 |

b. Write the general term of this sequence.

$t_n = t_1 + (n-1)d$

$t_n = 15 + (n-1)(5)$

$t_n = 15 + 5n - 5$

$t_n = 10 + 5n$

c. Calculate the slope of the graph. How is it related to the general term?

Take two points on the graph to calculate slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{25 - 15}{3 - 1}$$

$$m = 5$$

Looking at the formula from part b., the slope is equal to the coefficient of $n$, which also corresponds to the common difference.

d. Determine the $y$-intercept of the graph. How is it related to the general term?

Extend the graph to the $y$-axis to see that it will pass through the point (0, 10). The $y$-intercept is equal to the constant term in the general formula.

Please complete *Lesson 1.1 Explore Your Understanding Assignment* located in *Workbook 1A* before proceeding to *Lesson 1.2*.