Practice Solutions – IV

1. The sum of the first four terms of an arithmetic series is 122 and the sum of the first nine terms of the same series is 612.

   a. Determine the common difference, d, of the series.

   Set up a system of linear equations to solve for d.

   Equation I

   $t_1 = ?$
   $d = ?$
   $S_4 = 122$
   $n = 4$

   $S_n = \frac{n}{2}[2t_1 + (n-1)d]$
   $122 = \frac{4}{2}[2t_1 + (4-1)d]$
   $61 = 2t_1 + 3d$

   Equation II

   $t_1 = ?$
   $d = ?$
   $S_9 = 612$
   $n = 9$

   $S_n = \frac{n}{2}[2t_1 + (n-1)d]$
   $612 = \frac{9}{2}[2t_1 + (9-1)d]$
   $136 = 2t_1 + 8d$

   Subtract Equation I from Equation II.

   $$\begin{array}{rrcl} & 136 & = & 2t_1 + 8d \\ -\ (& 61 & = & 2t_1 + 3d\ ) \\ \hline & 75 & = & 5d \\ & 15 & = & d \end{array}$$

   The common difference is 15.

   b. Determine the first term, t_1, of the series.

   Use one of the equations from part a. to solve for t_1.

   $61 = 2t_1 + 3d$
   $61 = 2t_1 + 3(15)$
   $16 = 2t_1$
   $8 = t_1$

   The first term is 8.

2. Jillian is a research assistant looking at the environmental and genetic factors related to a particular illness. Over the next two months, she will be reviewing patient files as background for further research. On the first day, she plans to reviews 8 patient files. The second day, she plans to review 10, and on the third day, she plans to review 12. Over the two months, Jillian plans to work 40 days.

   a. Determine the number of patient files Jillian should be able to review on the 40th day.

   The arithmetic sequence is 8, 10, 12, …
   Use the general formula to find the number of patient files Jillian will review on day 40.

   $t_1 = 8$
   $t_{40} = ?$
   $d = 10 - 8 = 2$
   $n = 40$

   $t_n = t_1 + (n-1)d$
   $t_{40} = 8 + (40-1)2$
   $t_{40} = 86$

   On the 40th day, Jillian should be able to review 86 files.

   b. Determine the total number of patient files Jillian will review in 40 days.

   This is asking for the sum of the series, 8 + 10 + 12 + … + 86.

   $t_1 = 8$
   $t_{40} = 86$
   $S_{40} = ?$
   $n = 40$

   $S_n = \frac{n}{2}(t_1 + t_n)$
   $S_{40} = \frac{40}{2}(8 + 86)$
   $S_{40} = 1\,880$

   After 40 days, Jillian will have reviewed a total of 1 880 patient files.

   c. What is assumed in order to answer part a. and part b.?

   The assumption is that Jillian continues to increase her daily number of patient files by 2 all the way until day 40. This may not be the case. There may be a limit to what Jillian is able to review.

   For example, suppose Jillian hits a maximum number of files she can review daily, such as 30. This limit would be hit on day 12. For the remaining 28 days of work, she would review 30 files each day. This would change the total number of files reviewed to 1 068 after 40 days.
   $\left(S_{40} = \left[\frac{12}{2}(8 + 30)\right] + (30)(28) = 1\,068\right)$

Please complete *Lesson 1.2 Explore Your Understanding Assignment* located in *Workbook 1A* before proceeding to *Lesson 1.3*.