Lesson 1.4: Geometric Series

Practice Solutions – VII

1. Determine whether or not the following series are geometric. For those that are geometric, determine S_{10}.

   a. $25 + 50 + 75 + \ldots$

   $$r = \frac{t_n}{t_{n-1}}$$
   $$r = \frac{50}{25} = 2$$
   $$r = \frac{75}{50} = \frac{3}{2}$$

   Because the r values are not constant, this is not a geometric series and the sum cannot be determined using the geometric series formula. Note, this is an arithmetic series, and the sum could be found using one of the sum of an arithmetic series formula

   $$S_n = \frac{n}{2}[2t_1 + (n-1)d]$$
   $$S_{10} = \frac{10}{2}[2(25) + (10-1)25]$$
   $$S_{10} = 1375$$

   b. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

   $$r = \frac{t_n}{t_{n-1}}$$
   $$r = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$
   $$r = \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{1}{2}$$

   Because r is constant, this is a geometric series and the sum can be determined.

   $$t_1 = \frac{1}{2}$$
   $$r = \frac{1}{2}$$
   $$n = 10$$
   $$S_{10} = ?$$
   $$S_n = \frac{t_1(r^n - 1)}{r - 1}$$
   $$S_{10} = \frac{\frac{1}{2}\left[\left(\frac{1}{2}\right)^{10} - 1\right]}{\frac{1}{2} - 1}$$
   $$S_{10} = 0.999\ldots$$
   $$S_{10} \doteq 1.00$$

c. $1 + 3 + 9 + \ldots$

$r = \frac{t_n}{t_{n-1}}$

$r = \frac{3}{1} = 3$

$r = \frac{9}{3} = 3$

Because r is constant, this is a geometric series and the sum can be determined.

$t_1 = 1$
$r = 3$
$n = 10$
$S_{10} = ?$

$S_n = \frac{t_1(r^n - 1)}{r - 1}$

$S_{10} = \frac{1(3^{10} - 1)}{3 - 1}$

$S_{10} = 29\,524$

2. Determine the value of S_n for the following geometric series.

a. $64 - 16 + 4 - 1 + \ldots + \frac{1}{64}$

In this question, the first and last terms are given, and the value of r can be determined. As such, either of the sum of a geometric series formulas can be used. Use the alternate form of the sum formula.

$t_1 = 64$

$r = \frac{-16}{64} = -\frac{1}{4}$

$t_n = \frac{1}{64}$

$S_n = ?$

$S_n = \frac{rt_n - t_1}{r - 1}$

$S_n = \frac{\left(-\frac{1}{4}\right)\left(\frac{1}{64}\right) - 64}{-\frac{1}{4} - 1}$

$S_n = 51.203...$

$S_n = \frac{3\,277}{64}$

b. $0.2 - 0.6 + 1.8 - 5.4 + \ldots - 3\,936.6$

$t_1 = 0.2$
$r = \frac{-0.6}{0.2} = -3$
$t_n = -3\,936.6$
$S_n = ?$

$S_n = \frac{rt_n - t_1}{r - 1}$
$S_n = \frac{(-3)(-3\,936.6) - 0.2}{-3 - 1}$
$S_n = -2\,952.4$

3. Determine the value of t_1 for the following geometric series.

a. $S_n = 7\,812, r = 5, t_n = 6\,250$

$$S_n = \frac{rt_n - t_1}{r - 1}$$
$$7\,812 = \frac{(5)(6\,250) - t_1}{5 - 1}$$
$$31\,248 = 31\,250 - t_1$$
$$2 = t_1$$

b. $S_n = 3\,571, r = 0.5, n = 7$

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$
$$3\,571 = \frac{t_1[(0.5)^7 - 1]}{0.5 - 1}$$
$$-1\,785.5 = t_1\left[\left(\frac{1}{2}\right)^7 - 1\right]$$
$$-1\,785.5 = t_1\left(\frac{1}{128} - 1\right)$$
$$1\,799.559\ldots = t_1$$
$$1\,800 \doteq t_1$$

4. Determine the number of terms in the series $3 + 6 + 12 + \ldots + t_n$, given $S_n = 765$.

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$
$$765 = \frac{3(2^n - 1)}{2 - 1}$$
$$256 = 2^n$$
$$2^8 = 2^n$$
$$8 = n$$

There are 8 terms in the series.

5. Wade posted a video on the internet and shared the link with five of his best friends. Each of those friends shared the video with five of their friends. This pattern continued ten times.

   a. Ignoring Wade, how many people have viewed this video?

   $t_1 = 5$
   $t_2 = 5(5)^{2-1} = 25$
   $t_3 = 5(5)^{3-1} = 125$

   The sum of the first ten terms is:

   $$S_n = \frac{t_1(r^n - 1)}{r - 1}$$
   $$S_{10} = \frac{5(5^{10} - 1)}{5 - 1}$$
   $$S_{10} = 12\,207\,030$$

   After ten times, over 12 million people will have viewed the video!

   b. How many times would the pattern need to continue in order for 100 million people to have viewed the video?

   $S_n = 100\,000\,000$
   $t_1 = 5$
   $r = 5$
   $n = ?$

   $$S_n = \frac{t_1(r^n - 1)}{r - 1}$$
   $$100\,000\,000 = \frac{5(5^n - 1)}{5 - 1}$$
   $$400\,000\,000 = 5(5^n - 1)$$
   $$80\,000\,001 = 5^n$$

   $5^{11} = 48\,828\,125$ ...Not enough
   $5^{12} = 244\,140\,625$ ...Over and above!

   The pattern must continue 12 times for 100 million people to view the video.

   c. What assumption is made answering these questions?

   One assumption is that everyone told about the video watches it and passes it along to five new friends. Another assumption is that every set of five friends is new; in other words, no one who has already seen the video gets it sent to them a second, third, fourth time, etc.

Please return to *Unit 1: Sequences and Series Lesson 1.4* to continue your exploration.