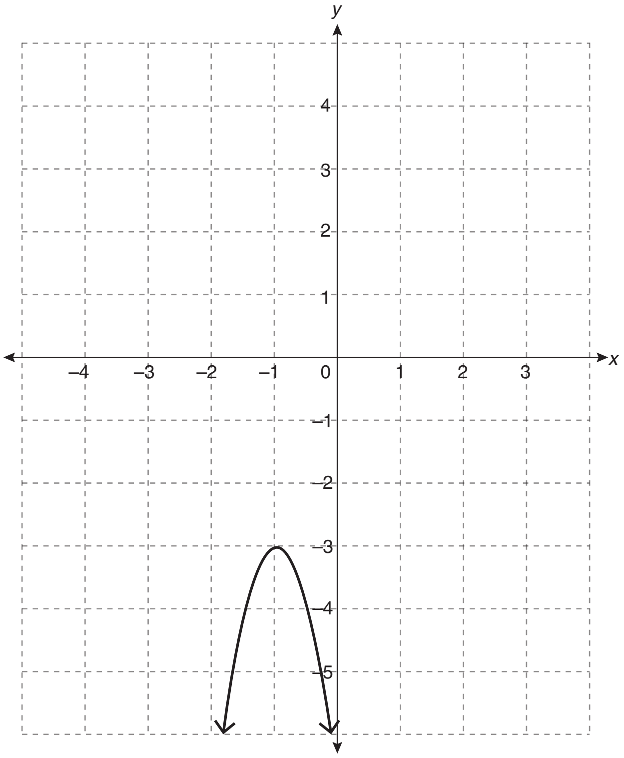
Appendix 2: Solutions

Lesson 2.1: Quadratic Functions Expressed in Vertex Form

Practice Solutions – I

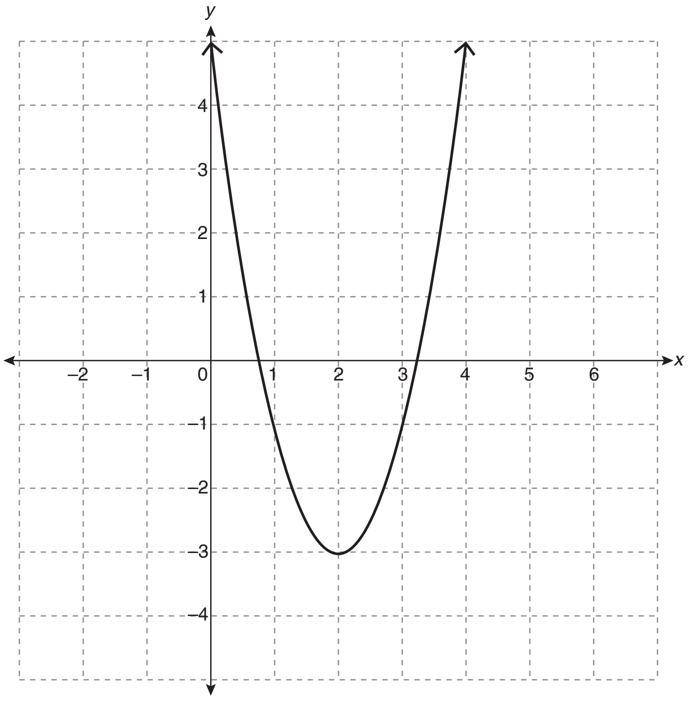
1. Given the graph of a quadratic function, determine the vertex.

a.



The vertex is at (–1, –3).

b.



The vertex is at (2, –3).

2. Without graphing, determine the equation of the axis of symmetry of the following quadratic functions. Verify by graphing.

a. $f(x) = -\frac{1}{2}(x-3)^2 + 4$

Recall that the vertex is (p, q). For this function, $p = 3$ and $q = 4$; therefore, the equation of the axis of symmetry is $x = p$ or $x = 3$.

Window Settings	
XMin	–1
XMax	7
XScale	1
YMin	–2
YMax	4
YScale	1

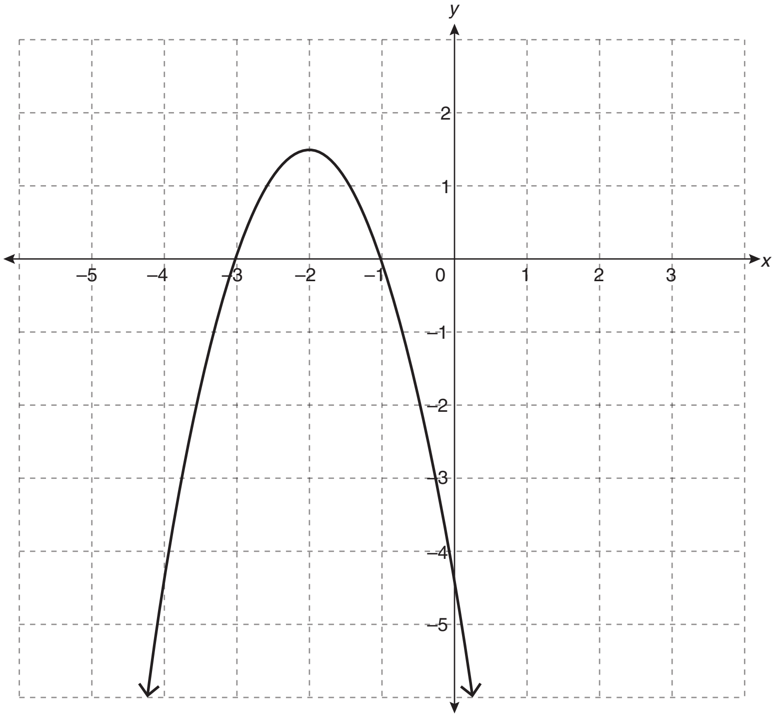
b. $g(x) = 3(x+5)^2 - 1$

For this function, $p = -5$ and $q = -1$; therefore, the equation of the axis of symmetry is $x = -5$.

Window Settings	
XMin	–7
XMax	1
XScale	1
YMin	–2
YMax	3
YScale	1

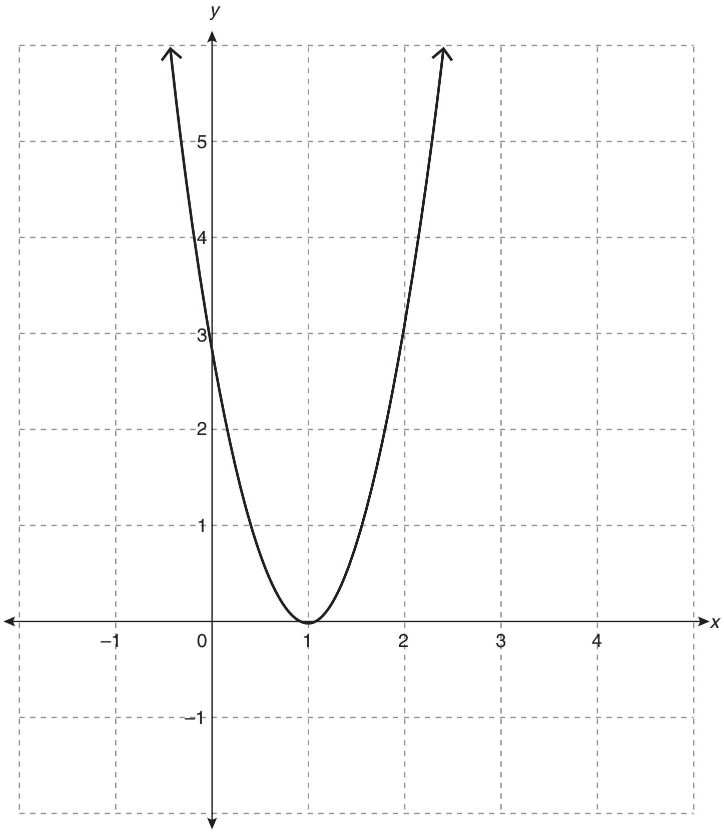
3. Given the following graphs of quadratic functions, determine the direction of opening and give the maximum or minimum value.

a.



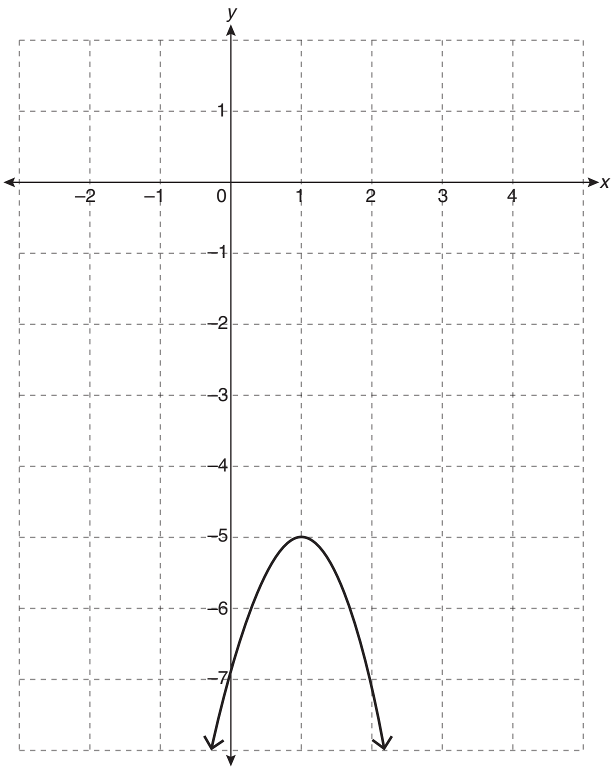
The graph of this function opens downward. The maximum value is 1.5.

b.



The graph of this function opens upward. The minimum value is 0.

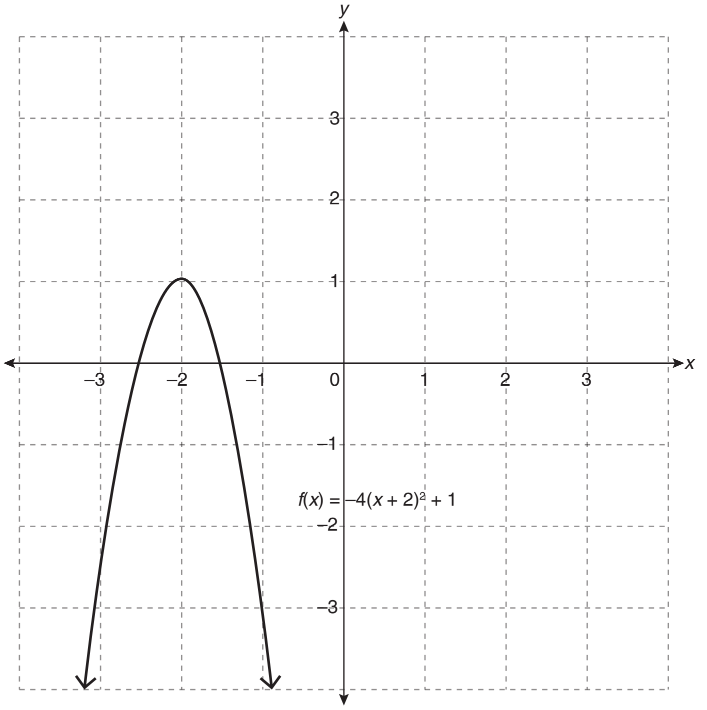
c.



The graph of this function opens downward. The maximum value is –5.

4. Without graphing, determine the domain, range, and the number of x-intercepts of the graph of the following quadratic functions. Verify by graphing.

a. $f(x) = -4(x+2)^2 + 1$

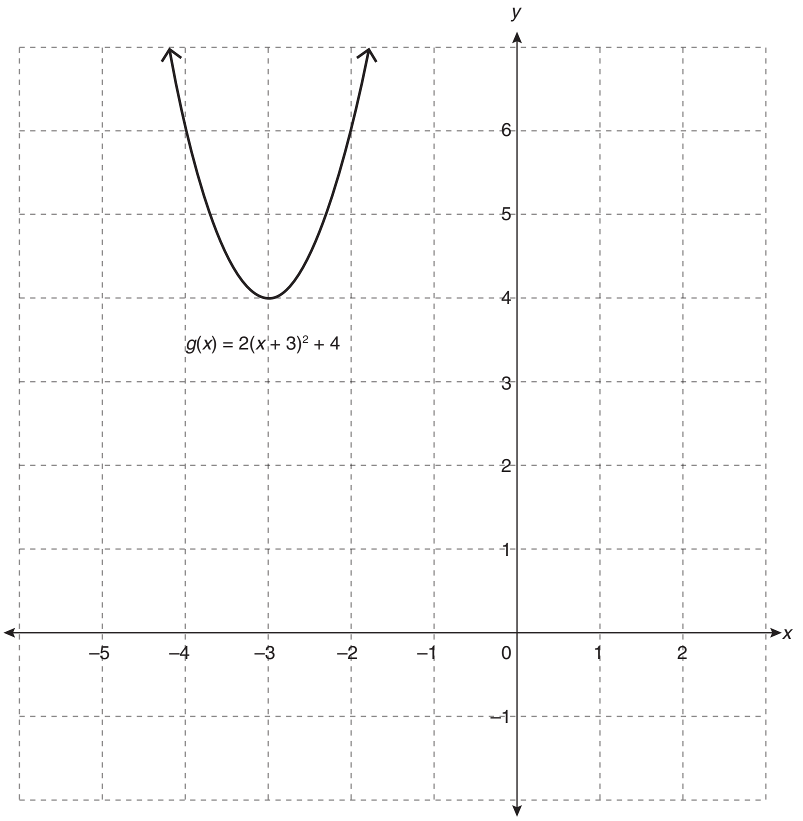


The domain is $\{x \mid x \in R\}$. The range is dependent upon the direction of opening and the value of q. For the given function, $a = -4$ and $q = 1$. Since $a < 0$, the graph opens downward; therefore, the range is $\{y \mid y \leq 1, y \in R\}$.

Because a and q have opposite signs, there will be two x-intercepts.

Window Settings	
XMin	–3
XMax	3
XScale	1
YMin	–3
YMax	3
YScale	1

b. $g(x) = 2(x+3)^2 + 4$

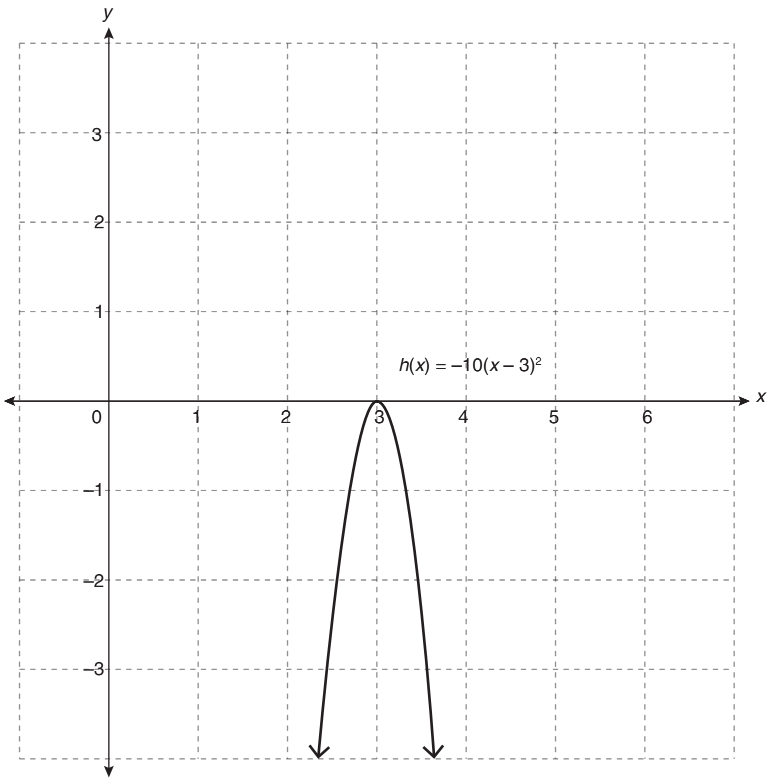


The domain is $\{x \mid x \in R\}$. The range is dependent upon the direction of opening and the value of q. For the given function, $a = 2$ and $q = 4$. Since $a > 0$, the graph opens upward; therefore, the range is $\{y \mid y \geq 4, y \in R\}$.

Because a and q have the same sign, there will be zero x-intercepts.

Window Settings	
XMin	–5
XMax	2
XScale	1
YMin	–1
YMax	6
YScale	1

c. $h(x) = -10(x-3)^2$



The domain is $\{x \mid x \in \mathrm{R}\}$. The range is dependent upon the direction of opening and the value of q. For the given function, $a = -10$ and $q = 0$. Since $a < 0$, the graph opens downward; therefore, the range is $\{y \mid y \le 0, y \in \mathrm{R}\}$.

Because $q = 0$, there will be one x-intercept.

Window Settings	
XMin	0
XMax	6
XScale	1
YMin	–3
YMax	3
YScale	1

5. The graph of the basic quadratic function $f(x) = x^2$ is translated 5 units to the right and 2 units down.

   a. What is the vertex of the graph of the transformed function?
   The vertex has moved from (0, 0) to (5, –2).

   b. What is the equation of the transformed function?
   A vertex of (5, –2) means $p = 5$ and $q = -2$. The new equation is $f(x) = (x-5)^2 - 2$.

   c. How many x-intercepts does the graph of the function have?
   Because a and q have opposite signs, the graph of the transformed function has 2 x-intercepts.

Please return to *Unit 2: Quadratic Functions and Equations Lesson 2.1* to continue your exploration.