Lesson 2.3: Quadratic Functions Expressed in Standard Form

Practice Solutions – IV

1. Determine the zeros of the function $y = 24x^2 - 10x - 6$, by factoring. Check your answers.

First look for a GCF.
The GCF = 2

$$y = 24x^2 - 10x - 6$$
$$y = 2(12x^2 - 5x - 3)$$

Then, factor by decomposition.

$$y = 2(12x^2 - 5x - 3)$$
$$y = 2(12x^2 - 9x + 4x - 3)$$
$$y = 2[3x(4x - 3) + (4x - 3)]$$
$$y = 2(4x - 3)(3x + 1)$$

Then, calculate the values of x that cause each factor to equal zero.

$$4x - 3 = 0 \qquad 3x + 1 = 0$$
$$4x = 3 \qquad 3x = -1$$
$$x = \frac{3}{4} \qquad x = -\frac{1}{3}$$

Check:

$x = \frac{3}{4}$

$$y = 24x^2 - 10x - 6$$
$$y = 24\left(\frac{3}{4}\right)^2 - 10\left(\frac{3}{4}\right) - 6$$
$$y = 24\left(\frac{9}{16}\right) - \frac{30}{4} - 6$$
$$y = \frac{54}{4} - \frac{30}{4} - \frac{24}{4}$$
$$y = 0$$

$x = -\frac{1}{3}$

$$y = 24x^2 - 10x - 6$$
$$y = 24\left(-\frac{1}{3}\right)^2 - 10\left(-\frac{1}{3}\right) - 6$$
$$y = 24\left(\frac{1}{9}\right) + \frac{10}{3} - 6$$
$$y = \frac{8}{3} + \frac{10}{3} - \frac{18}{3}$$
$$y = 0$$

The zeros of the function are $\frac{3}{4}$ and $-\frac{1}{3}$.

2. Convert the function $f(x) = -2x^2 + 12x - 6$ to vertex form by completing the square.

Group the first two terms, and factor a from both.

$f(x) = -2x^2 + 12x - 6$
$f(x) = -2(x^2 - 6x) - 6$

Make a perfect square trinomial in the brackets by adding and subtracting $\left(\frac{b}{2}\right)^2$.

$f(x) = -2\left(x^2 - 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2\right) - 6$
$f(x) = -2(x^2 - 6x + 3^2 - 3^2) - 6$

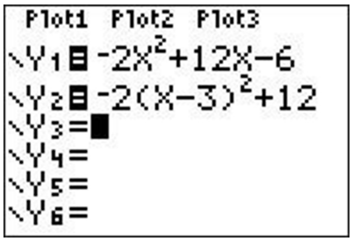
Remove the subtracted value by multiplying by a.

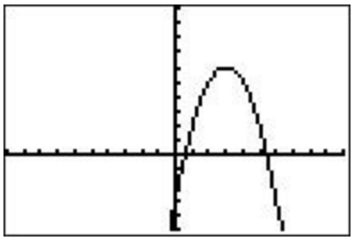
$f(x) = -2(x^2 - 6x + 3^2) - 6 + 2(3)^2$
$f(x) = -2(x^2 - 6x + 3^2) - 6 + 18$

Factor the perfect square trinomial and simplify the constants.

$f(x) = -2(x - 3)^2 + 12$

Check by using technology.

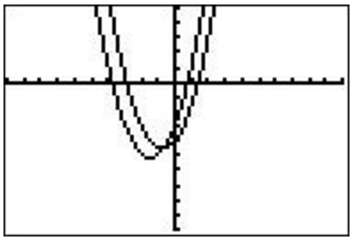




The two graphs overlap, therefore the process of completing the square was done correctly.

3. Kyle converts the function $y = 2x^2 + 3x - 8$ to vertex form by completing the square. When verifying his solution on his graphing calculator, the two graphs do not lie on top of each other. Look through Kyle's steps and find his error(s). Then, fix his error(s) and give the correct solution.

$$y = 2x^2 + 3x - 8$$
$$y = 2(x^2 + 3x) - 8$$
$$y = 2\left(x^2 + 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right) - 8$$
$$y = 2\left(x^2 + 3x + \left(\frac{3}{2}\right)^2\right) - 8 - \left(\frac{3}{2}\right)^2$$
$$y = 2\left(x + \frac{3}{2}\right)^2 - \frac{32}{4} - \frac{9}{4}$$
$$y = 2\left(x + \frac{3}{2}\right)^2 - \frac{41}{4}$$

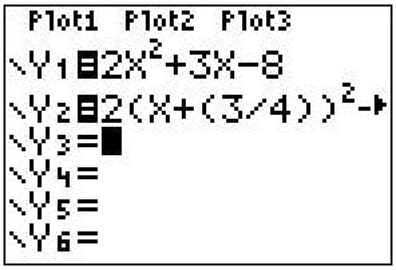


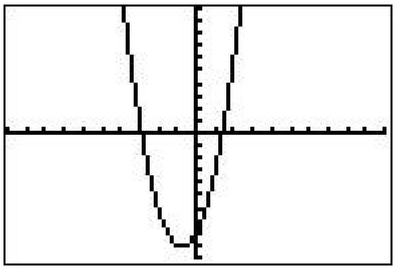
The first error was that Kyle forgot to divide *b* by *a* when he factored *a* from the first two terms.

Also, Kyle forgot to multiply by *a* when he moved the $-\left(\frac{3}{2}\right)^2$ out of the brackets. The correct solution is:

$$y = 2x^2 + 3x - 8$$
$$y = 2\left(x^2 + \frac{3}{2}x\right) - 8$$
$$y = 2\left(x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right) - 8$$
$$y = 2\left(x^2 + 3x + \left(\frac{3}{4}\right)^2\right) - 8 - 2\left(\frac{3}{4}\right)^2$$
$$y = 2\left(x + \frac{3}{4}\right)^2 - \frac{64}{8} - \frac{9}{8}$$
$$y = 2\left(x + \frac{3}{4}\right)^2 - \frac{73}{8}$$

Verify using technology.





The two graphs overlap, so completing the square has been done correctly.

Please return to *Unit 2: Quadratic Functions and Equations Lesson 2.3* to continue your exploration.