Practice Solutions – VII

1. Solve the following equations using the quadratic formula, if possible.

a. $x^2 - 17x - 9 = 0$

$a = 1, b = -17, c = -9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{17 \pm \sqrt{(-17)^2 - 4(1)(-9)}}{2(1)}$$
$$x = \frac{17 \pm \sqrt{325}}{2}$$
$$x = \frac{17 \pm 5\sqrt{13}}{2}$$

The solutions to the equation are $x = \dfrac{17 + 5\sqrt{13}}{2}$ and $x = \dfrac{17 - 5\sqrt{13}}{2}$.

b. $4s^2 = -s - 14$

$4s^2 + s + 14 = 0$

$a = 4, b = 1, c = 14$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(4)(14)}}{2(4)}$$
$$x = \frac{-1 \pm \sqrt{-223}}{8}$$

Because the radicand is negative, there are no Real solutions to this equation.

c. $0 = x^2 - 7x + 7$

$a = 1, b = -7, c = 7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(7)}}{2(1)}$$
$$x = \frac{7 \pm \sqrt{21}}{2}$$

The solutions to the equation are $x = \frac{7 + \sqrt{21}}{2}$ and $x = \frac{7 - \sqrt{21}}{2}$.

2. Using the discriminant, determine the nature of the roots of each equation.

a. $4x^2 - 28x = -49$

$4x^2 - 28x + 49 = 0$
$a = 4, b = -28, c = 49$

$$\begin{aligned} b^2 - 4ac &= (-28)^2 - 4(4)(49) \\ &= 784 - 784 \\ &= 0 \end{aligned}$$

Because the discriminant is equal to zero, there are two equal Real roots.

b. $s^2 + 4 = -3s$

$s^2 + 3s + 4 = 0$
$a = 1, b = 3, c = 4$

$$\begin{aligned} b^2 - 4ac &= (3)^2 - 4(1)(4) \\ &= 9 - 16 \\ &= -7 \end{aligned}$$

Because the discriminant is less than zero, there are no Real roots.

c. $z^2 + 3z = 5$

$z^2 + 3z - 5 = 0$

$a = 1, b = 3, c = -5$

$$\begin{aligned} b^2 - 4ac &= (3)^2 - 4(1)(-5) \\ &= 9 + 20 \\ &= 29 \end{aligned}$$

Because the discriminant is greater than zero, there are two distinct Real roots.

3. June sells popcorn at the Nanton Candy Store. She sells each bag for \$2.00. At this price, she sells 60 bags per day. She did an experiment for a month, and found that every \$0.25 increase in price resulted in two fewer sales per day. At what price(s) can June sell the popcorn in order to earn \$150.00 per day?

Let p represent the number of price increases.
Let $R(p)$ represent the revenue dependent upon the number of price increases.
New price = \$2.00 + \$0.25p
Number of sales at new price = $60 - 2p$
Revenue an be expressed as:
$R(p) = (2.00 + 0.25p)(60 - 2p)$

The question gives the revenue as \$150.00. Substitute this value in for $R(p)$, and manipulate the equation until it equals zero.

$$\begin{aligned} R(p) &= (2.00 + 0.25p)(60 - 2p) \\ 150 &= (2.00 + 0.25p)(60 - 2p) \\ 150 &= 120 - 4p + 15p - 0.5p^2 \\ 0 &= -0.5p^2 + 11p - 30 \end{aligned}$$

Then, using the quadratic formula, solve for p.

$a = -0.5, b = 11, c = -30$

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$p = \frac{-11 \pm \sqrt{(11)^2 - 4(-0.5)(-30)}}{2(-0.5)}$$

$$p = \frac{-11 \pm \sqrt{121 - 60}}{-1}$$

$$p = \frac{-11 \pm \sqrt{61}}{-1}$$

$$p = \frac{-11 + \sqrt{61}}{-1} \text{ or } p = \frac{-11 - \sqrt{61}}{-1}$$

$p = 3.189...$ or $p = 18.810...$

New price possibilities are:
$\$2.00 + \$0.25(3.189...) = \$2.797...$
or
$\$2.00 + \$0.25(18.810...) = \$6.702...$

June can either sell the popcorn for \$2.80 or \$6.70 in order to earn \$150.00 per day. The lower price is more realistic because at \$6.70, the number of customers buying the popcorn is quite small. June will want to keep a large customer base so that news of her popcorn can spread more easily.

Please complete *Lesson 2.4 Explore Your Understanding Assignment*, *Final Review Assignment*, and *Check Point* located in *Workbook 2B*.