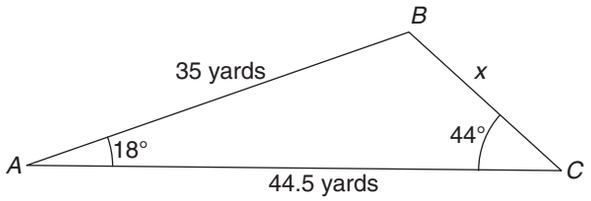
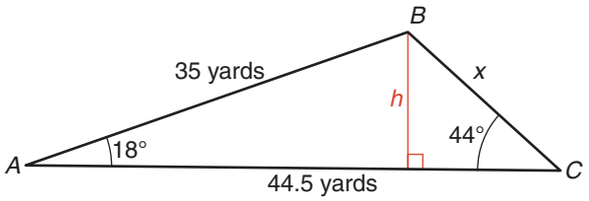
Lesson 4.3: Solving Non-Right Triangles

Practice Solutions – IV

1. Solve for x, to the nearest tenth, by making two right triangles.





State what is given.

- $\angle A = 18^\circ$
- $\angle C = 44^\circ$
- $a = x$
- $b = 44.5$ yards
- $c = 35$ yards

Step 1: Determine h, using the triangle on the left.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 18^\circ = \frac{h}{35}$$

$$35 \sin 18^\circ = h$$

$$10.815... = h$$

Step 2: Determine side x, using the triangle on the right.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 44^\circ = \frac{10.815...}{x}$$

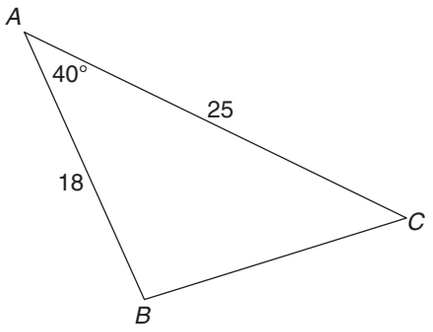
$$x = \frac{10.815...}{\sin 44^\circ}$$

$$x = 15.569...$$

$$x \doteq 15.6$$

Side x is approximately 15.6 yards in length.

2. Determine the unknown side and the two unknown angles in the triangle below. Round the answers to the nearest tenth.



State what is given.

- $\angle A = 40°$
- $\angle B = ?$
- $\angle C = ?$
- $a = ?$
- $b = 25$
- $c = 18$

Step 1: Determine side a.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 25^2 + 18^2 - 2(25)(18)\cos 40°$$
$$a^2 = 259.560...$$
$$a = 16.110...$$
$$a \doteq 16.1$$

Step 2: Determine $\angle B$.

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos B = \frac{(16.110...)^2 + 18^2 - 25^2}{2(16.110...)(18)}$$
$$\cos B = -0.0714...$$
$$\angle B = \cos^{-1}(-0.0714...)$$
$$\angle B = 94.097...°$$
$$\angle B \doteq 94.1°$$

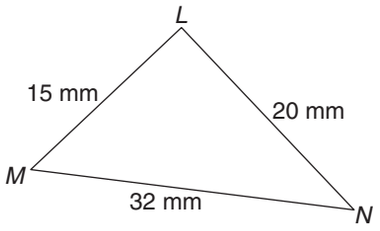
Step 3: Determine $\angle C$.

$$\angle C = 180° - 40° - 94.097...°$$
$$= 45.903...°$$
$$\angle C \doteq 45.9°$$

3. The triangle LMN has the following measurements:

   $LM = 15$ mm
   $MN = 32$ mm
   $NL = 20$ mm

   Draw a diagram of triangle LMN, and determine the angle measures, to the nearest tenth of a degree, for L, M, and N.



Angle L:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos L = \frac{15^2 + 20^2 - 32^2}{2(15)(20)}$$
$$\cos L = -0.665$$
$$\angle L = \cos^{-1}(-0.665)$$
$$\angle L = 131.682...^\circ$$
$$\angle L \doteq 131.7^\circ$$

Angle M:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos M = \frac{15^2 + 32^2 - 20^2}{2(15)(32)}$$
$$\cos M = 0.884\,375$$
$$\angle M = \cos^{-1}(0.884\,375)$$
$$\angle M = 27.825...^\circ$$
$$\angle M \doteq 27.8^\circ$$

Angle N:

$$\angle N = 180^\circ - 131.682...^\circ - 27.825...^\circ$$
$$= 20.493...^\circ$$
$$\angle N \doteq 20.5^\circ$$

4. Having two eyes is important for depth perception. One reason is that your eyes need to look more inward when looking at an object nearby than they do for an object far away. One of the clues your brain uses to determine the distance to an object is the convergence angle between the line of sight from each eye.

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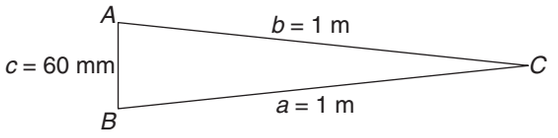
   a. Janet has a pupillary distance of 60 mm. This means the pupils of her eyes are 60 mm apart. Determine the convergence angles for objects that are 1 m, 2 m, 100 m, and 200 m from her eyes (assume each eye is the same distance from the objects).

   Start by drawing a diagram for an object that is 1 m away. In this diagram, A and B represent the eyes, and C represents the object the eyes converge on.



   Write down what is given.

   • $\angle C = ?$

   • $a = 1$ m

   • $b = 1$ m

   • $c = 60$ mm $= 0.06$ m

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
$$\cos C = \frac{1^2 + 1^2 - 0.06^2}{2(1)(1)}$$
$$\cos C = 0.998\,2$$
$$\angle C = \cos^{-1}(0.998\,2)$$
$$\angle C = 3.438...^\circ$$
$$\angle C \doteq 3.44^\circ$$

   The same calculation can be done when a and b are equal to 2 m, 100 m, and 200 m.

Distance to Eyes	1 m	2 m	100 m	200 m
Convergence Angle	3.44°	1.72°	0.034 4°	0.017 2°

   b. Based on the information determined in part a, do you expect it to be easier to distinguish the distance between objects 1 m and 2 m away or between objects 100 m and 200 m away? Explain.

   For objects 1 m and 2 m away, the convergence angles differ by $3.44^\circ - 1.72^\circ = 1.72^\circ$. For objects that are 100 m and 200 m away, the convergence angles differ by $0.034\,4^\circ - 0.017\,2^\circ = 0.017\,2^\circ$. Based on convergence angles, it should be easier to distinguish the distance between objects 1 m and 2 m away because the change in convergence angles is much larger.

Please return to *Unit 4: Trigonometry Lesson 4.3* to continue your exploration.