Step 2: Determine $\angle A$.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin A}{14} = \frac{\sin 112^\circ}{25}$$
$$\sin A = \frac{14 \sin 112^\circ}{25}$$
$$\sin A = 0.519...$$
$$\angle A = \sin^{-1}(0.519...)$$
$$\angle A = 31.280...^\circ$$

Step 3: Determine $\angle B$.

$$\angle B = 180^\circ - 112^\circ - 31.280...^\circ$$
$$= 36.719...^\circ$$
$$\angle B \doteq 36.7^\circ$$

The angle between the guy wire and the pole is approximately 36.7°.

Please return to *Unit 4: Trigonometry Lesson 4.3* to continue your exploration.

Practice Solutions – VI

1. For each triangle, determine whether there is no solution, one solution, or two solutions.

   a. $\triangle ABC$, $\angle A = 25^\circ$, $a = 40$ m, and $b = 90$ m

   Because $\angle A$ is acute, and $a < b$, calculate $b \sin A$.

   $$b \sin A = 90 \sin 25^\circ$$
   $$= 38.035...$$

   Because $38.035... < 40 < 90$, there are two solutions possible.

   b. $\triangle ABC$, $\angle A = 95^\circ$, $a = 5$ mm, and $b = 7$ mm

   Because $\angle A$ is obtuse, and $a < b$, there is no solution.

c. $\triangle ABC$, $\angle A = 45°$, $a = 15$ in, and $b = 20$ in

Because $\angle A$ is acute, and $a < b$, calculate $b \sin A$.

$$\begin{aligned} b\sin A &= 20\sin 45° \\ &= 14.142... \end{aligned}$$

Because $14.142... < 15 < 20$, there are two solutions possible.

d. $\triangle ABC$, $\angle A = 145°$, $a = 49$ cm, and $b = 18$ cm

Because $\angle A$ is obtuse, and $a > b$, there is one solution.

2. For $\triangle ABC$, where $\angle A = 35°$ and $b = 100$ cm, determine the range of values of a for which there is(are)

a. One oblique triangle

For there to be one oblique triangle,

$$\begin{aligned} &a \geq b \\ &a \geq 100 \end{aligned}$$

b. One right triangle

For there to be one right triangle,

$$\begin{aligned} &a = h \\ &a = b\sin A \\ &a = 100\sin 35° \\ &a = 57.357... \end{aligned}$$

c. Two oblique triangles

For there to be two triangles,

$$\begin{aligned} b\sin A &< a < b \\ 100\sin 35° &< a < 100 \\ 57.357... &< a < 100 \end{aligned}$$

d. No triangle

For there to be no triangle,

$$\begin{aligned} &a < h \\ &a < b\sin A \\ &a < 100\sin 35° \\ &a < 57.357... \end{aligned}$$

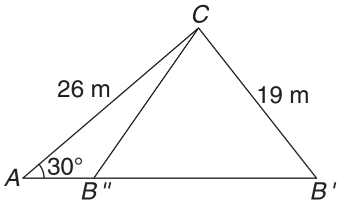
3. Solve the following triangles. If more than one solution is possible, give both. Round all answers to the nearest tenth.

   a. $\triangle ABC$, $\angle A = 30°$, $a = 19$ m, and $b = 26$ m

   Check to see how many triangles are possible.

   $$b \sin A < a < b$$
   $$26 \sin 30° < 19 < 26$$
   $$13 < 19 < 26$$

   There are two triangles possible.



   State what is given.

   - $\angle A = 30°$
   - $a = 19$ m
   - $b = 26$ m

   Triangle $AB'C$:

   Step 1: Determine $\angle B'$.

   $$\frac{\sin A}{a} = \frac{\sin B'}{b}$$
   $$\frac{\sin 30°}{19} = \frac{\sin B'}{26}$$
   $$\frac{26 \sin 30°}{19} = \sin B'$$
   $$0.684... = \sin B'$$
   $$43.173...° = \angle B'$$
   $$43.2° \doteq \angle B'$$

   Step 2: Determine $\angle C$.

   $$\angle C = 180° - 30° - 43.173...°$$
   $$\angle C = 106.826...°$$
   $$\angle C \doteq 106.8°$$

Step 3: Determine side c.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 30^\circ}{19} = \frac{\sin 106.826...^\circ}{c}$$
$$c = \frac{19 \sin 106.826...^\circ}{\sin 30^\circ}$$
$$c = 36.373...$$
$$c \doteq 36.4 \text{ m}$$

Triangle $AB"C$:

Step 1: Determine $\angle B''$.

To determine $\angle B''$, note that $\angle B'$ is the reference angle for $\angle B''$.

$$\angle B'' = 180^\circ - \angle B'$$
$$\angle B'' = 180^\circ - 43.173...^\circ$$
$$\angle B'' = 136.826...^\circ$$

Step 2: Determine $\angle C$.

$$\angle C = 180^\circ - 30^\circ - 136.826...^\circ$$
$$\angle C = 13.173...^\circ$$
$$\angle C \doteq 13.2^\circ$$

Step 3: Determine side c.

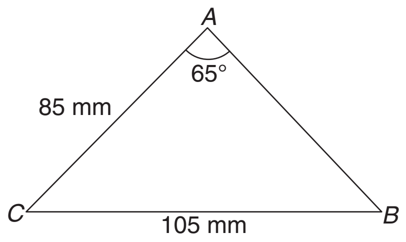
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 30^\circ}{19} = \frac{\sin 13.173...^\circ}{c}$$
$$c = \frac{19 \sin 13.173...^\circ}{\sin 30^\circ}$$
$$c = 8.660...$$
$$c \doteq 8.7 \text{ m}$$

b. $\triangle ABC, \angle A = 65^\circ, a = 105$ mm, and $b = 85$ mm

Check to see how many triangles are possible.

$$a \geq b$$
$$105 \geq 85$$

Therefore, there is only one solution possible.



State what is given.

- $\angle A = 65°$
- $a = 105$ mm
- $b = 85$ mm

Step 1: Determine $\angle B$.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 65°}{105} = \frac{\sin B}{85}$$
$$\frac{85 \sin 65°}{105} = \sin B$$
$$0.733... = \sin B$$
$$47.195...° = \angle B$$
$$47.2° \doteq \angle B$$

Step 2: Determine $\angle C$.

$$\angle C = 180° - 65° - 47.195...°$$
$$\angle C = 67.804...°$$
$$\angle C \doteq 67.8°$$

Step 3: Determine side c.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 65°}{105} = \frac{\sin 67.804...°}{c}$$
$$c = \frac{105 \sin 67.804...°}{\sin 65°}$$
$$c = 107.2698$$
$$c \doteq 107.3 \text{ mm}$$

4. The Fisgard Lighthouse, near Victoria, BC, was the first lighthouse built on Canada's west coast. Its role was to usher ships into the Esquimalt Harbour. (Today, it is an historical site.) Suppose the light on Fisgard Lighthouse reaches a maximum distance of 11 miles.

   A tourist boat is travelling at an angle of 25° to the lighthouse, 20 miles from the lighthouse. Determine the total distance, to the nearest tenth of a mile, that the tourists will first and last see the light from the lighthouse.

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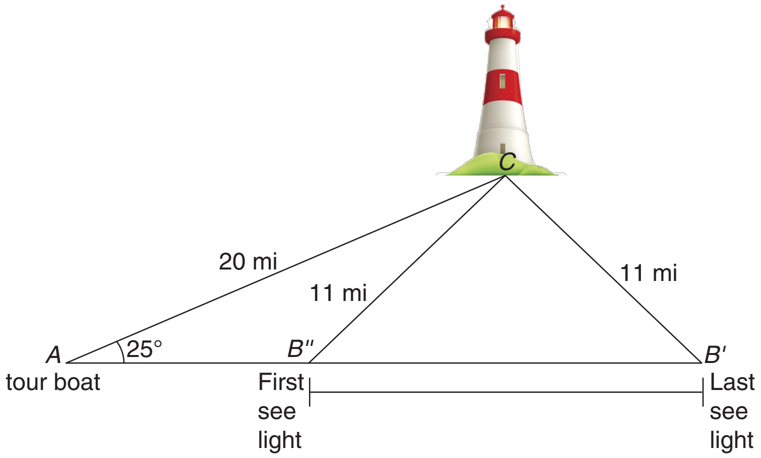

State what is known.

- $\angle A = 25^\circ$
- $\angle B = ?$
- $a = 11$ miles
- $b = 20$ miles

Step 1: Determine $\angle B$.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 25^\circ}{11} = \frac{\sin B}{20}$$
$$\frac{20 \sin 25^\circ}{11} = \sin B$$
$$0.768... = \sin B$$
$$\sin^{-1}(0.768...) = \angle B$$
$$50.210...^\circ = \angle B$$

Note this is the reference angle for $\angle B''$.

$$\angle B'' = 180^\circ - 50.210...^\circ$$
$$= 129.789...^\circ$$

Step 2: Determine $\angle C$.

$$\angle C = 180^\circ - 25^\circ - 129.789...^\circ$$
$$= 25.210...^\circ$$

Step 3: Determine side c_1 (length to first sight of the light).

$$\frac{\sin A}{a} = \frac{\sin C}{c_1}$$
$$\frac{\sin 25^\circ}{11} = \frac{\sin 25.210...^\circ}{c_1}$$
$$c_1 = \frac{11 \sin 25.210...^\circ}{\sin 25^\circ}$$
$$c_1 = 11.086...$$

Determine distance to last sight of the light.

Step 1: Determine $\angle B'$.

This time, $\angle B'$ is equal to the reference angle, so $\angle B' = 50.210...^\circ$.

Step 2: Determine $\angle C$.

$\angle C = 180^\circ - 25^\circ - 50.210...^\circ = 104.789...^\circ$

Step 3: Determine side c_2 (length to last sight of the light).

$$\frac{\sin A}{a} = \frac{\sin C}{c_2}$$
$$\frac{\sin 25^\circ}{11} = \frac{\sin 104.789...^\circ}{c_2}$$
$$c_2 = \frac{11 \sin 104.789...^\circ}{\sin 25^\circ}$$
$$c_2 = 25.165...$$

Determine the distance the tourists see the light.

$d = c_2 - c_1$

$d = 25.165... - 11.086...$

$d = 14.079...$

$d \doteq 14.1$

The total distance the tourists can see the light is approximately 14.1 miles.

Please complete *Lesson 4.3 Explore Your Understanding Assignment*, *Final Review Assignment*, and *Check Point* located in *Workbook 4B*.