Lesson 5.2: Operations on Rational Expressions

Practice Solutions – II

1. Add or subtract the rational expressions. Give the answer in simplest form, and identify any non-permissible values.

   a. $\frac{7x-1}{4}+\frac{4x+3}{4}$

   Because there are no variables in the denominator, there are no non-permissible values.

   $$\frac{7x-1}{4}+\frac{4x+3}{4}=\frac{7x-1+4x+3}{4}$$
   $$=\frac{11x+2}{4}$$

   b. $\frac{3m^2}{m-1}-\frac{3m}{m-1}$

   $$\frac{3m^2}{m-1}-\frac{3m}{m-1}=\frac{3m^2-3m}{m-1},m\neq 1$$
   $$=\frac{3m(m-1)}{m-1}$$
   $$=3m,m\neq 1$$

   c. $\frac{x^2}{x-3}+\frac{3x}{x-3}-\frac{18}{x-3}$

   $$\frac{x^2}{x-3}+\frac{3x}{x-3}-\frac{18}{x-3}=\frac{x^2+3x-18}{x-3},x\neq 3$$
   $$=\frac{(x-3)(x+6)}{x-3}$$
   $$=x+6,x\neq 3$$

2. Add or subtract the rational expressions. Give the answer in simplest form, and identify any non-permissible values.

   a. $\frac{3}{2s+1}-\frac{2}{(s+3)(2s+1)}$

   NPVs: $s\neq -3,-\frac{1}{2}$

   LCD $=(2s+1)(s+3)$

   $$\frac{3}{2s+1}-\frac{2}{(s+3)(2s+1)}=\frac{3(s+3)}{(2s+1)(s+3)}-\frac{2}{(s+3)(2s+1)}$$
   $$=\frac{3(s+3)}{(2s+1)(s+3)}-\frac{2}{(s+3)(2s+1)}$$
   $$=\frac{3s+9-2}{(2s+1)(s+3)}$$
   $$=\frac{3s+7}{(2s+1)(s+3)},s\neq -3,-\frac{1}{2}$$

b. $\dfrac{a+4}{a}+\dfrac{a-8}{a-4}$

NPVs: $a \neq 0, 4$

LCD $= a(a-4)$

$$\begin{aligned}\frac{a+4}{a}+\frac{a-8}{a-4} &= \frac{(a+4)(a-4)}{a(a-4)}+\frac{(a-8)a}{(a-4)a}\\ &= \frac{(a+4)(a-4)}{a(a-4)}+\frac{a(a-8)}{a(a-4)}\\ &= \frac{a^2-16+a^2-8a}{a(a-4)}\\ &= \frac{2a^2-8a-16}{a(a-4)}\\ &= \frac{2(a^2-4a-8)}{a(a-4)}, a \neq 0, 4\end{aligned}$$

c. $\dfrac{1}{y-4}-\dfrac{2y+1}{y^2-8y+16}+\dfrac{3y-2}{y^2-16}$

$$\frac{1}{y-4}-\frac{2y+1}{y^2-8y+16}+\frac{3y-2}{y^2-16} = \frac{1}{y-4}-\frac{2y+1}{(y-4)^2}+\frac{3y-2}{(y-4)(y+4)}$$

NPVs: $y \neq \pm 4$

LCD $= (y-4)^2(y+4)$

$$\begin{aligned}\frac{1}{y-4}-\frac{2y+1}{(y-4)^2}+\frac{3y-2}{(y-4)(y+4)} &= \frac{1(y-4)(y+4)}{(y-4)(y-4)(y+4)}-\frac{(2y+1)(y+4)}{[(y-4)^2](y+4)}+\frac{(3y-2)(y-4)}{[(y-4)(y+4)](y-4)}\\ &= \frac{(y-4)(y+4)}{(y-4)^2(y+4)}-\frac{(2y+1)(y+4)}{(y-4)^2(y+4)}+\frac{(3y-2)(y-4)}{(y-4)^2(y+4)}\\ &= \frac{y^2-16}{(y-4)^2(y+4)}-\frac{2y^2+9y+4}{(y-4)^2(y+4)}+\frac{3y^2-14y+8}{(y-4)^2(y+4)}\\ &= \frac{y^2-16-2y^2-9y-4+3y^2-14y+8}{(y-4)^2(y+4)}\\ &= \frac{2y^2-23y-12}{(y-4)^2(y+4)}, y \neq \pm 4\end{aligned}$$

3. Lara has made at least one error simplifying the rational expression $\frac{5}{x-3}+\frac{10}{x^2-9}-\frac{15}{x+3}$. Identify her error(s), and correct the answer.

$$\begin{aligned}\frac{5}{x-3}+\frac{10}{x^2-9}-\frac{15}{x+3} &= \frac{5(x-3)+10-15(x-3)}{(x-3)(x+3)}\\ &= \frac{5x-15+10-15x+45}{(x-3)(x+3)}\\ &= \frac{-10x+40}{(x-3)(x+3)}\\ &= \frac{-10(x-4)}{(x-3)(x+3)}\\ &= \frac{10(x-4)}{(x+3)^2}\end{aligned}$$

The first error was not listing the NPVs.

The second error was multiplying the first term in the first line by $x - 3$; it should have been multiplied by $x + 3$.

In the last line of the solution, errors were made by removing the negative from the 10 and changing the $x - 3$ factor to $x + 3$ in the denominator.

The correct solution is:

$$\begin{aligned}\frac{5}{x-3}+\frac{10}{x^2-9}-\frac{15}{x+3} &= \frac{5(x+3)+10-15(x-3)}{(x-3)(x+3)}\\ &= \frac{5x+15+10-15x+45}{(x-3)(x+3)}\\ &= \frac{-10x+70}{(x-3)(x+3)}\\ &= \frac{-10(x-7)}{(x-3)(x+3)},\ x\neq\pm 3\end{aligned}$$

4. Hue runs an average speed of x m/s on level ground. In training, he runs three distances, 100 m, 200 m, and 400 m. If his speed reduces by 2 m/s with each new distance, how long in total does it take him to run all three distances?

Recall,

$s = \frac{d}{t}$

OR

$t = \frac{d}{s}$

Time for each distance:

100 m: $t_{100} = \frac{100}{x}$

200 m: $t_{200} = \frac{200}{x-2}$

400 m: $t_{400} = \frac{400}{x-4}$

For total time, add the times together.

$$\begin{aligned}
t_{100} + t_{200} + t_{400} &= \frac{100}{x} + \frac{200}{x-2} + \frac{400}{x-4}, x \neq 0, 2, 4 \\
&= \frac{100(x-2)(x-4)}{x(x-2)(x-4)} + \frac{200x(x-4)}{(x-2)x(x-4)} + \frac{400x(x-2)}{(x-4)x(x-2)} \\
&= \frac{100(x-2)(x-4)}{x(x-2)(x-4)} + \frac{200x(x-4)}{x(x-2)(x-4)} + \frac{400x(x-2)}{x(x-2)(x-4)} \\
&= \frac{100(x^2-6x+8)+200x^2-800x+400x^2-800x}{x(x-2)(x-4)} \\
&= \frac{100x^2-600x+800+600x^2-1600x}{x(x-2)(x-4)} \\
&= \frac{700x^2-2200x+800}{x(x-2)(x-4)} \\
&= \frac{100(7x^2-22x+8)}{x(x-2)(x-4)}
\end{aligned}$$

The total time it takes Hue to run all three distances is $\frac{100(7x^2-22x+8)}{x(x-2)(x-4)}, x \neq 0, 2, 4$ seconds.

Please return to *Unit 5: Rational Expressions and Equations Lesson 5.2* to continue your exploration.