Practice Solutions – III

1. Multiply. Simplify and identify any non-permissible values.

a. $\frac{4ab^2c}{3} \cdot \frac{9bc}{2a^3}$

NPVs: $a \neq 0$

$$\frac{4ab^2c}{3} \cdot \frac{9bc}{2a^3} = \frac{36ab^3c^2}{6a^3}$$
$$= \frac{6b^3c^2}{a^2}, a \neq 0$$

b. $\frac{g^2 + 9g + 20}{2g} \cdot \frac{g}{g+4}$

NPVs: $g \neq 0, -4$

$$\frac{g^2 + 9g + 20}{2g} \cdot \frac{g}{g+4} = \frac{(g+5)\cancel{(g+4)}}{2\cancel{g}} \cdot \frac{\cancel{g}}{\cancel{g+4}}$$
$$= \frac{g+5}{2}, g \neq -4, 0$$

c. $\frac{2x^2 + 11x + 15}{3x^2 + 2x - 1} \cdot \frac{3x^2 - 10x + 3}{2x^2 + 3x - 5}$

$$\frac{2x^2 + 11x + 15}{3x^2 + 2x - 1} \cdot \frac{3x^2 - 10x + 3}{2x^2 + 3x - 5} = \frac{(2x+5)(x+3)}{(3x-1)(x+1)} \cdot \frac{(3x-1)(x-3)}{(2x+5)(x-1)}$$

NPVs: $x \neq -\frac{5}{2}, \pm 1, \frac{1}{3}$

$$= \frac{\cancel{(2x+5)}(x+3)}{\cancel{(3x-1)}(x+1)} \cdot \frac{\cancel{(3x-1)}(x-3)}{\cancel{(2x+5)}(x-1)}$$
$$= \frac{x+3}{x+1} \cdot \frac{x-3}{x-1}$$
$$= \frac{(x+3)(x-3)}{(x+1)(x-1)}, x \neq -\frac{5}{2}, \pm 1, \frac{1}{3}$$

2. Divide. Simplify and identify any non-permissible values.

a. $\frac{6xy^2}{w} \div \frac{2xy}{w^2}$

NPVs: $w \neq 0$

$$\frac{6xy^2}{w} \div \frac{2xy}{w^2} = \frac{6xy^2}{w} \cdot \frac{w^2}{2xy}, x \neq 0, y \neq 0$$
$$= \frac{6xw^2y^2}{2xwy}$$
$$= 3wy, w \neq 0, x \neq 0, y \neq 0$$

b. $\frac{2y^2 - 18}{4y + 4} \div \frac{y + 3}{y + 1}$

NPVs: $y \neq -1$

$$\frac{2y^2 - 18}{4y + 4} \div \frac{y + 3}{y + 1} = \frac{2(y^2 - 9)}{4(y + 1)} \div \frac{y + 3}{y + 1}$$
$$= \frac{2(y - 3)(y + 3)}{4(y + 1)} \div \frac{y + 3}{y + 1}$$
$$= \frac{\cancel{2}(y - 3)\cancel{(y + 3)}}{\cancel{4^2}\cancel{(y + 1)}} \cdot \frac{\cancel{y + 1}}{\cancel{y + 3}}, y \neq -3$$
$$= \frac{y - 3}{2}, y \neq -3, -1$$

c. $\frac{8x^2 + 14x + 3}{3x^2 - x - 2} \div \frac{2x^2 + 5x + 3}{x^2 - 4x + 3}$

$$\frac{8x^2 + 14x + 3}{3x^2 - x - 2} \div \frac{2x^2 + 5x + 3}{x^2 - 4x + 3} = \frac{(4x + 1)(2x + 3)}{(3x + 2)(x - 1)} \div \frac{(2x + 3)(x + 1)}{(x - 1)(x - 3)}$$

NPVs: $x \neq -\frac{2}{3}, 1, 3$

$$= \frac{(4x + 1)\cancel{(2x + 3)}}{(3x + 2)\cancel{(x - 1)}} \cdot \frac{\cancel{(x - 1)}(x - 3)}{\cancel{(2x + 3)}(x + 1)}, x \neq -1$$
$$= \frac{4x + 1}{3x + 2} \cdot \frac{x - 3}{x + 1}$$
$$= \frac{(4x + 1)(x - 3)}{(3x + 2)(x + 1)}, x \neq \pm 1, -\frac{2}{3}, 3$$

3. How does the division of $\frac{2s+1}{s-3} \div \frac{2s+1}{s^2-9}$ compare to the division of $\frac{2s+1}{s^2-9} \div \frac{2s+1}{s-3}$? Explain how order matters when dividing rational expressions.

$$\frac{2s+1}{s-3} \div \frac{2s+1}{s^2-9} = \frac{2s+1}{s-3} \div \frac{2s+1}{(s-3)(s+3)}$$
$$= \frac{2s+1}{s-3} \cdot \frac{(s-3)(s+3)}{2s+1}$$
$$= s+3,\ s \neq \pm 3, -\frac{1}{2}$$

$$\frac{2s+1}{s^2-9} \div \frac{2s+1}{s-3} = \frac{2s+1}{(s-3)(s+3)} \div \frac{2s+1}{s-3}$$
$$= \frac{2s+1}{(s-3)(s+3)} \cdot \frac{s-3}{2s+1}$$
$$= \frac{1}{s+3},\ s \neq \pm 3, -\frac{1}{2}$$

When dividing rational expressions, the first term must be multiplied by the reciprocal of the second term. If the order of the terms is reversed, the result will be the reciprocal of the original quotient.

4. A shipping box has a volume of $\frac{v^2+2v-3}{v-1}$. The length of the box is $v+3$, and the width is $v-5$. Write an expression for the height of the box. Indicate any non-permissible values.

$$V = lwh$$
$$h = \frac{V}{lw}$$
$$= V \div lw$$
$$h = \frac{v^2+2v-3}{v-1} \div (v+3)(v-5)$$
$$h = \frac{(v+3)(v-1)}{v-1} \cdot \frac{1}{(v+3)(v-5)}$$
$$h = \frac{1}{v-5},\ v \neq -3, 1, 5$$

5. Simplify the rational expressions. Identify any non-permissible values.

a. $\dfrac{a-5}{a-4}+\dfrac{3a^2-2a-1}{a^2-3a-4}\cdot\dfrac{a^2-4a-5}{3a^2-5a-2}$

$$\frac{a-5}{a-4}+\frac{3a^2-2a-1}{a^2-3a-4}\cdot\frac{a^2-4a-5}{3a^2-5a-2}=\frac{a-5}{a-4}+\frac{(3a+1)(a-1)}{(a+1)(a-4)}\cdot\frac{(a+1)(a-5)}{(3a+1)(a-2)}$$

$$\text{NPVs: } a\neq -1,-\frac{1}{3},2,4$$

$$=\frac{a-5}{a-4}+\frac{\cancel{(3a+1)}(a-1)}{\cancel{(a+1)}(a-4)}\cdot\frac{\cancel{(a+1)}(a-5)}{\cancel{(3a+1)}(a-2)}$$

$$=\frac{a-5}{a-4}+\frac{a-1}{a-4}\cdot\frac{a-5}{a-2}$$

$$=\frac{a-5}{a-4}+\frac{(a-1)(a-5)}{(a-4)(a-2)}$$

$$\text{LCD}=(a-4)(a-2)$$

$$=\frac{(a-5)(a-2)}{(a-4)(a-2)}+\frac{(a-1)(a-5)}{(a-4)(a-2)}$$

$$=\frac{(a-5)(a-2)}{(a-4)(a-2)}+\frac{(a-1)(a-5)}{(a-4)(a-2)}$$

$$=\frac{a^2-7a+10}{(a-4)(a-2)}+\frac{a^2-6a+5}{(a-4)(a-2)}$$

$$=\frac{2a^2-13a+15}{(a-4)(a-2)},a\neq -1,-\frac{1}{3},2,4$$

b. $3\div\left[\dfrac{-2}{s^2}-\dfrac{4}{s}+6\right]$

$$\text{NPVs: } s\neq 0$$

$$\text{LCD}=s^2$$

$$3\div\left[\frac{-2}{s^2}-\frac{4}{s}+6\right]=3\div\left[\frac{-2}{s^2}-\frac{4s}{s^2}+\frac{6s^2}{s^2}\right]$$

$$=3\div\left[\frac{-2-4s+6s^2}{s^2}\right]$$

$$=\frac{3}{1}\cdot\frac{s^2}{-2-4s+6s^2}$$

$$=\frac{3}{1}\cdot\frac{s^2}{2(3s^2-2s-1)}$$

$$=\frac{3}{1}\cdot\frac{s^2}{2(3s+1)(s-1)}$$

$$=\frac{3s^2}{2(3s+1)(s-1)},s\neq -\frac{1}{3},0,1$$

Please complete *Lesson 5.2 Explore Your Understanding Assignment* located in *Workbook 5A* before proceeding to *Lesson 5.3*.