Lesson 5.3: Rational Equations

Practice Solutions – IV

1. Solve the rational equations. Verify the solution(s).

   a. $\frac{3x+7}{4x-1} = \frac{27}{5}$

NPVs:

$$4x - 1 \neq 0$$
$$x \neq \frac{1}{4}$$

$$\frac{3x+7}{4x-1} = \frac{27}{5}$$
$$5(3x+7) = 27(4x-1)$$
$$15x + 35 = 108x - 27$$
$$62 = 93x$$
$$\frac{62}{93} = x$$
$$\frac{2}{3} = x$$

Note that because there are two fractions separated by an equal sign, cross multiplication can be used to solve the equation.

Verify for $x = \frac{2}{3}$.

Left Side	Right Side
$\frac{3x+7}{4x-1}$ $\frac{3\left(\frac{2}{3}\right)+7}{4\left(\frac{2}{3}\right)-1}$ $\frac{2+7}{\frac{8}{3}-1}$ $\frac{9}{\frac{5}{3}}$ $\frac{27}{5}$	$\frac{27}{5}$
LS = RS	

b. $\frac{8}{t}+\frac{t}{4}=3$

NPVs: $t \neq 0$

LCD: $4t$

$$\left[\frac{8}{t}\right]4t+\left[\frac{t}{4}\right]4t=(3)4t$$
$$32+t^2=12t$$
$$t^2-12t+32=0$$
$$(t-8)(t-4)=0$$
$$t=8 \text{ and } t=4$$

Verify for $t=8$.

Left Side	Right Side
$\frac{8}{t}+\frac{t}{4}$ $\frac{8}{(8)}+\frac{(8)}{4}$ $1+2$ 3	3
LS = RS	

Verify for $t=4$.

Left Side	Right Side
$\frac{8}{t}+\frac{t}{4}$ $\frac{8}{(4)}+\frac{(4)}{4}$ $2+1$ 3	3
LS = RS	

c. $\dfrac{3}{x-2}+\dfrac{2}{x+2}=\dfrac{17}{5}$

NPVs:

$x-2\neq 0 \qquad x+2\neq 0$

$x\neq 2 \qquad x\neq -2$

LCD: $5(x-2)(x+2)$

$$\left(\frac{3}{x-2}\right)(5)(x-2)(x+2)+\left(\frac{2}{x+2}\right)(5)(x-2)(x+2)=\left(\frac{17}{5}\right)(5)(x-2)(x+2)$$
$$3(5)(x+2)+2(5)(x-2)=17(x-2)(x+2)$$
$$15x+30+10x-20=17(x^2-4)$$
$$25x+10=17x^2-68$$
$$0=17x^2-25x-78$$

$a=17,\ b=-25,\ c=-78$

$$t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$t=\frac{-(-25)\pm\sqrt{(-25)^2-4(17)(-78)}}{2(17)}$$
$$t=\frac{25\pm\sqrt{5929}}{34}$$
$$t=\frac{25\pm 77}{34}$$
$$t=3 \text{ and } t=-\frac{26}{17}$$

Verify for $t=3$.

Left Side	Right Side
$\dfrac{3}{x-2}+\dfrac{2}{x+2}$	$\dfrac{17}{5}$
$\dfrac{3}{(3)-2}+\dfrac{2}{(3)+2}$	
$\dfrac{3}{1}+\dfrac{2}{5}$	
$\dfrac{15+2}{5}$	
$\dfrac{17}{5}$	
LS = RS	

Verify for $t=-\dfrac{26}{17}$.

Left Side	Right Side
$\dfrac{3}{x-2}+\dfrac{2}{x+2}$	$\dfrac{17}{5}$
$\dfrac{3}{\left(-\frac{26}{17}\right)-2}+\dfrac{2}{\left(-\frac{26}{17}\right)+2}$	
$\dfrac{3}{\frac{-26-34}{17}}+\dfrac{2}{\frac{-26+34}{17}}$	
$\dfrac{3}{\frac{-60}{17}}+\dfrac{2}{\frac{8}{17}}$	
$\dfrac{3(17)}{-60}+\dfrac{2(17)}{8}$	
$\dfrac{51}{-60}+\dfrac{34}{8}$	
$-\dfrac{102}{120}+\dfrac{510}{120}$	
$\dfrac{408}{120}$	
$\dfrac{17}{5}$	
LS = RS	

2. Solve the rational equations. Verify the solution(s).

a. $\frac{3}{x-2}-\frac{4}{x-3}=\frac{-6}{x^2-5x+6}$

$\frac{3}{x-2}-\frac{4}{x-3}=\frac{-6}{(x-2)(x-3)}$

NPVs: $x \neq 2, 3$

LCD: $(x-2)(x-3)$

$$\left[\frac{3}{x-2}\right](x-2)(x-3)-\left[\frac{4}{x-3}\right](x-2)(x-3)=\left[\frac{-6}{(x-2)(x-3)}\right](x-2)(x-3)$$

$$3(x-3)-4(x-2)=-6$$

$$3x-9-4x+8=-6$$

$$-x=-5$$

$$x=5$$

Verify for $x = 5$.

Left Side	Right Side
$\frac{3}{x-2}-\frac{4}{x-3}$ $\frac{3}{(5)-2}-\frac{4}{(5)-3}$ $\frac{3}{3}-\frac{4}{2}$ $1-2$ -1	$\frac{-6}{x^2-5x+6}$ $\frac{-6}{(5)^2-5(5)+6}$ $\frac{-6}{25-25+6}$ $\frac{-6}{6}$ -1
LS = RS	

b. $\frac{3c^2 - c - 2}{c^2 - 1} = \frac{4c - 1}{c - 1} + \frac{c - 3}{c + 1}$

$$\frac{(3c+2)(c-1)}{(c-1)(c+1)} = \frac{4c-1}{c-1} + \frac{c-3}{c+1}$$

NPVs:

$c - 1 \neq 0$ $\quad$ $c + 1 \neq 0$

$c \neq 1$ $\quad$ $c \neq -1$

LCD: $(c-1)(c+1)$

$$\left[\frac{3c^2 - c - 2}{(c-1)(c+1)}\right](c-1)(c+1) = \left[\frac{4c-1}{c-1}\right](c-1)(c+1) + \left[\frac{c-3}{c+1}\right](c-1)(c+1)$$

$$3c^2 - c - 2 = (4c-1)(c+1) + (c-3)(c-1)$$

$$3c^2 - c - 2 = 4c^2 + 3c - 1 + c^2 - 4c + 3$$

$$2c^2 + 4 = 0$$

$$c^2 = -2$$

$$c = \text{undefined}$$

Since the square root of a negative number is undefined, there is no solution to this rational equation.

3. Solve the rational equation $\frac{w}{w+3} - 3 = \frac{-6}{w^2-9}$. Round the answer to the nearest hundredth, and verify the solution(s).

$$\frac{w}{w+3} - 3 = \frac{-6}{w^2-9}$$
$$\frac{w}{w+3} - 3 = \frac{-6}{(w-3)(w+3)}$$

NPVs:

$w - 3 \neq 0$ $\quad$ $w + 3 \neq 0$

$w \neq 3$ $\quad$ $w \neq -3$

LCD: $(w-3)(w+3)$

$$\left[\frac{w}{w+3}\right](w-3)(w+3) - (3)(w-3)(w+3) = \left[\frac{-6}{(w-3)(w+3)}\right](w-3)(w+3)$$
$$w(w-3) - 3(w-3)(w+3) = -6$$
$$w^2 - 3w - 3(w^2-9) = -6$$
$$w^2 - 3w - 3w^2 + 27 = -6$$
$$0 = 2w^2 + 3w - 33$$

$a = 2,\ b = 3,\ c = -33$

$$w = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$w = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-33)}}{2(2)}$$
$$w = \frac{-3 \pm \sqrt{273}}{4}$$

$w = 3.380\ldots$ and $w = -4.880\ldots$

$w \doteq 3.38$ and $w \doteq -4.88$

Verify for $w \doteq 3.38$.

Left Side	Right Side
$\frac{w}{w+3} - 3$ $\frac{(3.38)}{(3.38)+3} - 3$ $0.529\ldots - 3$ $-2.470\ldots$	$\frac{-6}{w^2-9}$ $\frac{-6}{(3.38)^2-9}$ $\frac{-6}{2.4244}$ $-2.474\ldots$
LS $\doteq$ RS	

Verify for $w \doteq -4.88$.

Left Side	Right Side
$\frac{w}{w+3} - 3$ $\frac{(-4.88)}{(-4.88)+3} - 3$ $2.595\ldots - 3$ $-0.404\ldots$	$\frac{-6}{w^2-9}$ $\frac{-6}{(-4.88)^2-9}$ $\frac{-6}{14.8144}$ $-0.405\ldots$
LS $\doteq$ RS	

4. Misty solved the rational equation $\frac{4c+17}{c^2+c-6}=\frac{5}{c-2}$. She says the answer is $c=2$. Explain why Misty is right or wrong.

When solving rational equations, the first step is to determine the non-permissible values.

$$\frac{4c+17}{c^2+c-6}=\frac{5}{c-2}$$
$$\frac{4c+17}{(c-2)(c+3)}=\frac{5}{c-2}$$

NPVs: $c\neq -3, 2$

Because $c\neq 2$, Misty's solution is incorrect. Her answer is an extraneous root because when two is substituted for c, the denominators of both rational expressions will be undefined.

This rational equation has no solution.

Please return to *Unit 5: Rational Expressions and Equations Lesson 5.3* to continue your exploration.

Practice Solutions – V

1. The sum of two numbers is 30. The sum of their reciprocals is $\frac{3}{20}$. Determine the two numbers.

Let x represent the first number, and let $30 – x$ represent the second number.

$$\frac{1}{x}+\frac{1}{30-x}=\frac{3}{20}, x\neq 0, 30$$

LCD $= 20x(30-x)$

$$\left[\frac{1}{x}\right](20x)(30-x)+\left[\frac{1}{30-x}\right](20x)(30-x)=\left[\frac{3}{20}\right](20x)(30-x)$$
$$20(30-x)+20x=3x(30-x)$$
$$600-20x+20x=90x-3x^2$$
$$3x^2-90x+600=0$$
$$3(x^2-30x+200)=0$$
$$(x-20)(x-10)=0$$
$$x=10 \text{ and } x=20$$

The two numbers are 10 and 20. If $x = 10$, the second number is 20, and if $x = 20$, the second number is 10.

Verify.

$$\frac{1}{10}+\frac{1}{20}=\frac{2}{20}+\frac{1}{20}=\frac{3}{20}$$