4. Misty solved the rational equation $\frac{4c+17}{c^2+c-6}=\frac{5}{c-2}$. She says the answer is $c=2$. Explain why Misty is right or wrong.

   When solving rational equations, the first step is to determine the non-permissible values.

   $$\frac{4c+17}{c^2+c-6}=\frac{5}{c-2}$$
   $$\frac{4c+17}{(c-2)(c+3)}=\frac{5}{c-2}$$

   NPVs: $c\neq -3,2$

   Because $c\neq 2$, Misty's solution is incorrect. Her answer is an extraneous root because when two is substituted for c, the denominators of both rational expressions will be undefined.

   This rational equation has no solution.

Please return to *Unit 5: Rational Expressions and Equations Lesson 5.3* to continue your exploration.

Practice Solutions – V

1. The sum of two numbers is 30. The sum of their reciprocals is $\frac{3}{20}$. Determine the two numbers.

   Let x represent the first number, and let $30-x$ represent the second number.

   $$\frac{1}{x}+\frac{1}{30-x}=\frac{3}{20}, x\neq 0,30$$

   $$\text{LCD}=20x(30-x)$$

   $$\left[\frac{1}{x}\right](20x)(30-x)+\left[\frac{1}{30-x}\right](20x)(30-x)=\left[\frac{3}{20}\right](20x)(30-x)$$
   $$20(30-x)+20x=3x(30-x)$$
   $$600-20x+20x=90x-3x^2$$
   $$3x^2-90x+600=0$$
   $$3(x^2-30x+200)=0$$
   $$(x-20)(x-10)=0$$
   $$x=10 \text{ and } x=20$$

   The two numbers are 10 and 20. If $x=10$, the second number is 20, and if $x=20$, the second number is 10.

   Verify.

   $$\frac{1}{10}+\frac{1}{20}=\frac{2}{20}+\frac{1}{20}=\frac{3}{20}$$

2. Jill is filling a water trough for her horse. Using pump A, the trough is filled up in half an hour. Using pump B, the trough is filled up in three quarters of an hour. How long does it take to fill the trough using both pumps?

Let t represent the time it takes to fill the trough using both pumps.

$$\frac{1}{\text{time taken by pump } A} + \frac{1}{\text{time taken by pump } B} = \frac{1}{\text{time taken by both pumps } A \text{ and } B \text{ together}}$$

$$\frac{1}{\left(\frac{1}{2}\right)} + \frac{1}{\left(\frac{3}{4}\right)} = \frac{1}{t}, t \neq 0$$

$$2 + \frac{4}{3} = \frac{1}{t}$$

LCD $= 3t$

$$(2)(3t) + \left(\frac{4}{3}\right)(3t) = \left(\frac{1}{t}\right)(3t)$$

$$6t + 4t = 3$$

$$10t = 3$$

$$t = \frac{3}{10}$$

$$t = 0.3 \text{ hours}$$

Using both pumps, the trough will be filled in 0.3 hours.

3. Quentin refurbishes old furniture. He has just purchased a pallet of old dining room chairs at a cost of $100.00 total. Quentin has $100.00 worth of supplies to refurbish the chairs. His mother has requested four refurbished chairs as a gift for Mother's Day. Quentin will earn $1 000.00 after selling the lot of chairs. Quentin wants to profit $50.00 per chair. How many chairs must he refurbish in order to achieve this goal?

Let c represent the number of chairs refurbished.

Note that Quentin gives four of the chairs to his mother.

$$\text{Cost of each chair} = \frac{\$100 + \$100}{c} = \frac{\$200.00}{c}$$

$$\text{Earnings from each chair} = \frac{\$1\ 000.00}{c-4}$$

Profit from each chair = earnings from each chair – cost of each chair

$$50 = \frac{1\ 000}{c-4} - \frac{200}{c},\ c \neq 0,\ 4$$

$$\text{LCD} = c(c-4)$$

$$(50)(c)(c-4) = \left[\frac{1\ 000}{c-4}\right](c)(c-4) - \left[\frac{200}{c}\right](c)(c-4)$$

$$50c(c-4) = 1\ 000c - 200(c-4)$$

$$50c^2 - 200c = 1\ 000c - 200c + 800$$

$$50c^2 - 1\ 000c - 800 = 0$$

$$50(c^2 - 20c - 16) = 0$$

$$a = 1,\ b = -20,\ c = -16$$

$$c = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$c = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(-16)}}{2(1)}$$

$$c = \frac{20 \pm \sqrt{464}}{2}$$

$$c = \frac{20 \pm 4\sqrt{29}}{2}$$

$$c = 10 \pm 2\sqrt{29}$$

$$c = 20.770... \text{ and } c = -0.770...$$

$$c \doteq 21$$

A negative number of chairs is impossible and can be ignored in this context. Quentin must refurbish 21 chairs in order to achieve his goal.

4. A plane flying from Miami, Florida to Calgary, Alberta experienced headwinds, and took 1 hour longer than when it flew with tailwinds on the return flight. The distance between the two airports is 4 000 km, and the average headwind/tailwind is 95km/h. What is the speed of the plane in calm air?

   Let x represent the speed of the plane in calm air.

Direction	Distance (km)	Speed (km/h)	Time (h)
To Calgary	4 000	$x - 95$	$\frac{4\,000}{x-95}$
To Miami	4 000	$x + 95$	$\frac{4\,000}{x+95}$

How do you know which side to add one to? Note that both sides of the equation must be equal; therefore, the shorter time (the trip to Miami) needs to have one hour added to make it equal to the longer time (the trip to Calgary).

$$\frac{4\,000}{x+95} + 1 = \frac{4\,000}{x-95},\ x \neq \pm 95$$

$$\text{LCD} = (x+95)(x-95)$$

$$\left[\frac{4\,000}{x+95}\right](x-95)(x+95) + (1)(x-95)(x+95) = \left[\frac{4\,000}{x-95}\right](x-95)(x+95)$$

$$4\,000(x-95) + (x-95)(x+95) = 4\,000(x+95)$$

$$4\,000x - 380\,000 + x^2 - 9\,025 = 4\,000x + 380\,000$$

$$x^2 = 769\,025$$

$$x = \pm\sqrt{769\,025}$$

$$x = \pm 876.940...$$

$$x \doteq 877$$

Because the plane cannot go a negative speed, ignore the negative answer. The speed of the plane in calm air is approximately 877 km/h.

Please complete *Lesson 5.3 Explore Your Understanding Assignment*, *Final Review Assignment*, and *Check Point* located in *Workbook 5B*.