Practice Solutions – IV

1. Solve the following equations algebraically.

a. $2 = |-3x - 6| + x$

Case 1: $x \leq -2$

$$\begin{aligned} 2 &= |-3x - 6| + x \\ 2 &= -3x - 6 + x \\ 2x &= -8 \\ x &= -4 \end{aligned}$$

The interval $x \leq -2$ includes –4, so –4 is a solution.

Case 2: $x > -2$

$$\begin{aligned} 2 &= |-3x - 6| + x \\ 2 &= -(-3x - 6) + x \\ 2 &= 3x + 6 + x \\ -4 &= 4x \\ -1 &= x \end{aligned}$$

The interval $x > -2$ includes –1, so –1 is a solution.

b. $|-2x^2 - 6x + 2| = 6x^2 + 24x + 21$

Case 1: $-2x^2 - 6x + 2 \geq 0$

$$\begin{aligned} |-2x^2 - 6x + 2| &= 6x^2 + 24x + 21 \\ -2x^2 - 6x + 2 &= 6x^2 + 24x + 21 \\ 0 &= 8x^2 + 30x + 19 \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-30 \pm \sqrt{30^2 - 4(8)(19)}}{2(8)} \\ &= \frac{-30 \pm \sqrt{292}}{16} \\ &= \frac{-15 \pm \sqrt{73}}{8} \end{aligned}$$

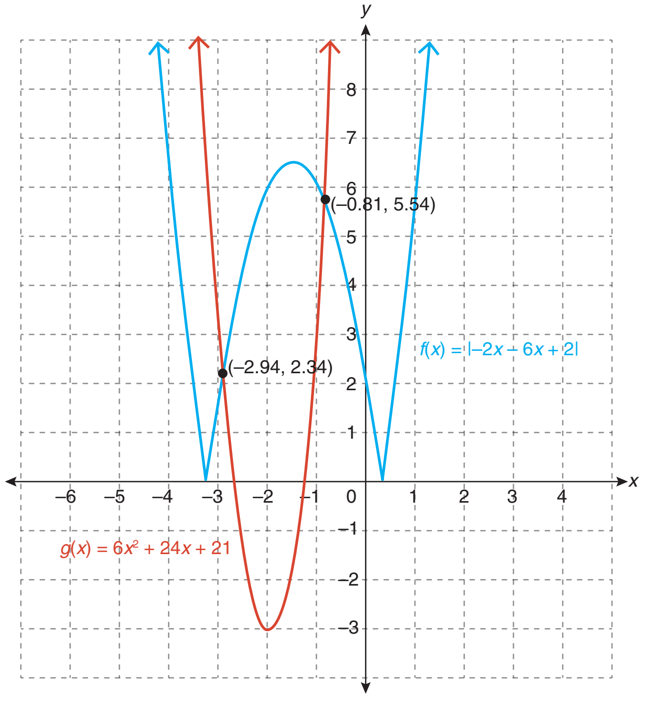
Case 2: $-2x^2 - 6x + 2 < 0$

$$\begin{aligned} |-2x^2 - 6x + 2| &= 6x^2 + 24x + 21 \\ -(-2x^2 - 6x + 2) &= 6x^2 + 24x + 21 \\ 2x^2 + 6x - 2 &= 6x^2 + 24x + 21 \\ 0 &= 4x^2 + 18x + 23 \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-18 \pm \sqrt{18^2 - 4(4)(23)}}{2(4)} \\ &= \frac{-18 \pm \sqrt{-44}}{8} \end{aligned}$$

The discriminant is negative, so there is no solution to this case.

The solutions $\frac{-15+\sqrt{73}}{8}$ and $\frac{-15-\sqrt{73}}{8}$ can be verified graphically.

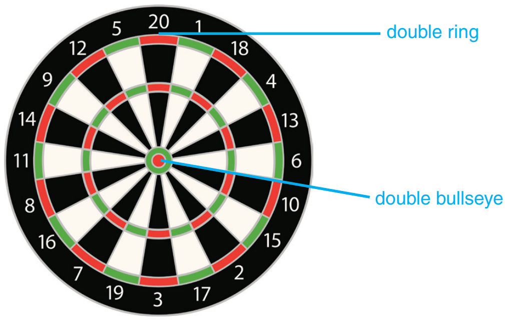


2. Explain why two equations are usually solved when solving an absolute value equation algebraically.

The absolute value operation is defined differently for positive and negative contents. Each possibility needs to be considered separately when solving an equation.

3. The double ring on a dartboard is approximately $6\frac{1}{2}$ inches from the double bullseye.

   For a particular throw, the height of the dart, h, from the double bullseye, and its distance, d, away from the thrower's hand can be modelled by the function $h(d) = d - 0.13d^2$, where both h and d are measured in feet.



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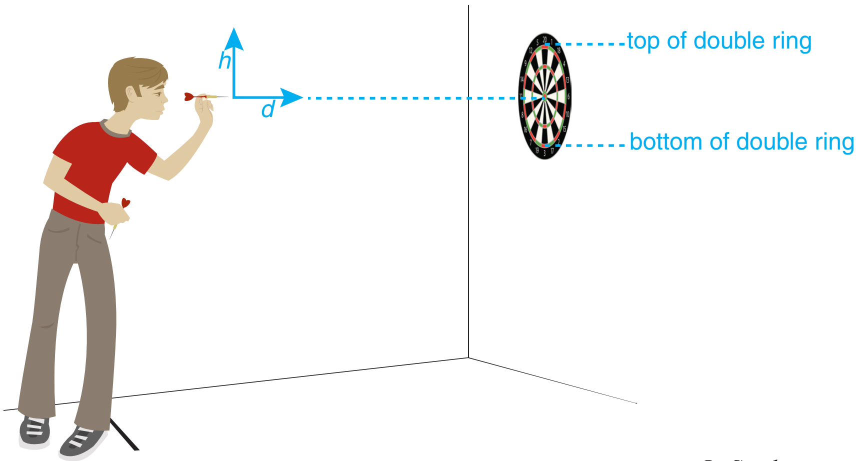

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a. Solve an absolute value equation to determine the distance(s) at which the dart is at the same height as the top or the bottom of the double ring.

$6\frac{1}{2}\text{ in} = \frac{13}{24}\text{ ft}$

$h(d) = d - 0.13d^2$

$\frac{13}{24} = |d - 0.13d^2|$

Case 1: $d - 0.13d^2 \geq 0$

$\frac{13}{24} = d - 0.13d^2$

$0.13d^2 - d + \frac{13}{24} = 0$

$d = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$d = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(0.13)\left(\frac{13}{24}\right)}}{2(0.13)}$

$d = \frac{1 \pm 0.847...}{0.26}$

$d \doteq 7.11 \text{ or } 0.59$

Case 2: $d - 0.13d^2 < 0$

$\frac{13}{24} = -(d - 0.13d^2)$

$0 = 0.13d^2 - d - \frac{13}{24}$

$d = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$d = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(0.13)\left(-\frac{13}{24}\right)}}{2(0.13)}$

$d = \frac{1 \pm 1.132...}{0.26}$

$d \doteq 8.20 \text{ or } -0.51$

Verify for $d \doteq 7.11$.

Left Side	Right Side
$\frac{13}{24}$ 0.541...	$\lvert d - 0.13d^2 \rvert$ $\lvert (7.11) - 0.13(7.11)^2 \rvert$ $\lvert 0.538227 \rvert$ 0.538227
LS $\doteq$ RS	

Verify for $d \doteq 8.20$.

Left Side	Right Side
$\frac{13}{24}$ 0.541...	$\lvert d - 0.13d^2 \rvert$ $\lvert (8.20) - 0.13(8.20)^2 \rvert$ $\lvert -0.5412 \rvert$ 0.5412
LS $\doteq$ RS	

Verify for $d \doteq 0.59$.

Left Side	Right Side
$\frac{13}{24}$ 0.541...	$\lvert d - 0.13d^2 \rvert$ $\lvert (0.59) - 0.13(0.59)^2 \rvert$ $\lvert 0.544747 \rvert$ 0.544747
LS $\doteq$ RS	

Verify for $d \doteq -0.51$.

Left Side	Right Side
$\frac{13}{24}$ 0.541...	$\lvert d - 0.13d^2 \rvert$ $\lvert (-0.51) - 0.13(-0.51)^2 \rvert$ $\lvert -0.543813 \rvert$ 0.543813
LS $\doteq$ RS	

The dart will be at the same height as the top or bottom of the double ring at approximately –0.51, 0.59, 7.11, and 8.20 ft.

b. If the dart board is 7 feet 9 inches away from the thrower, is each solution valid in this scenario?

The solutions of approximately –0.51, 0.59, 7.11, and 8.20 ft each represent the distance between the dart and the thrower when the dart is $6\frac{1}{2}$ in from the double ring. However, –0.51 ft is a negative distance, representing a place behind the thrower's hand, which is not reasonable in this context. Similarly, 8.20 ft represents a distance beyond the dart board, which is not reasonable in this context. The other two distances are reasonable.

4. Explain why the equation $|x^2 - 7x + 24| = -4$ has no solution.

   The absolute value of an expression is always positive, so there is no way it can equal –4.

5. Patrick attempted to solve the equation $|2x^2 - 4| = x^2 - 3$ as follows. Comment on Patrick's solution, and, if necessary, correct any errors.

Case 1:

$$|2x^2 - 4| = x^2 - 3$$
$$2x^2 - 4 = x^2 - 3$$
$$x^2 = 1$$
$$x = \pm 1$$

Left Side	Right Side
$\|2(\pm 1)^2 - 4\|$	$(\pm 1)^2 - 3$
2	-2
LS $\neq$ RS	

Case 2:

$$|2x^2 - 4| = x^2 - 3$$
$$2x^2 - 4 = -x^2 + 3$$
$$3x^2 = 7$$
$$x = \pm\sqrt{\frac{7}{3}}$$

Left Side	Right Side
$\left\|2\left(\pm\sqrt{\frac{7}{3}}\right)^2 - 4\right\|$	$\left(\pm\sqrt{\frac{7}{3}}\right)^2 - 3$
$\frac{2}{3}$	$-\frac{2}{3}$
LS $\neq$ RS	

There are no real solutions to

$|2x^2 - 4| = x^2 - 3$.

Patrick's solution is correct, but he could have better clarified what each case represented.

In addition, he took a shortcut when verifying the possible solution values. Recognizing that there were no degree-one terms on either side of the equation and that squaring a positive or a negative gives a positive, he was able to verify both solutions for each case at the same time. In addition, while it may seem incorrect that Patrick multiplied the right side of the equation in case 2 by –1 instead of the left side, the result would be the same.

Please complete *Lesson 6.2: Explore Your Understanding Assignment* located in *Workbook 6A* before proceeding to *Lesson 6.3*.