Lesson 6.3: Reciprocal Functions

Practice Solutions – V

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1. A pizza is divided equally among p people.

   a. Write a reciprocal function, $A(p)$, that represents the amount of the pizza each person receives.

   $A(p) = \frac{1}{p}$

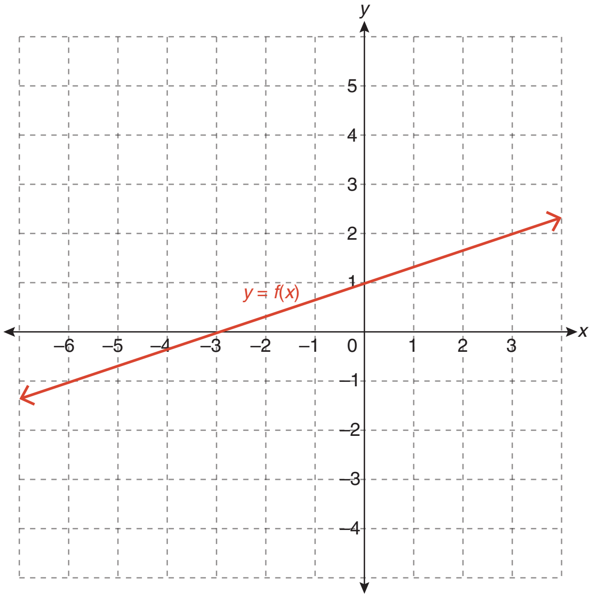
   b. Describe how the amount of pizza and the number of people are related. Is this relationship typical for reciprocal functions?

   As the number of people increases, the amount of pizza each person receives decreases. This is typical for reciprocal functions – the two variables, or expressions, multiply to one, so as one increases, the other decreases. This relationship is sometimes called inversely proportional.

2. Describe the relationships between the graphs of $y = f(x)$ and $y = \frac{1}{f(x)}$.

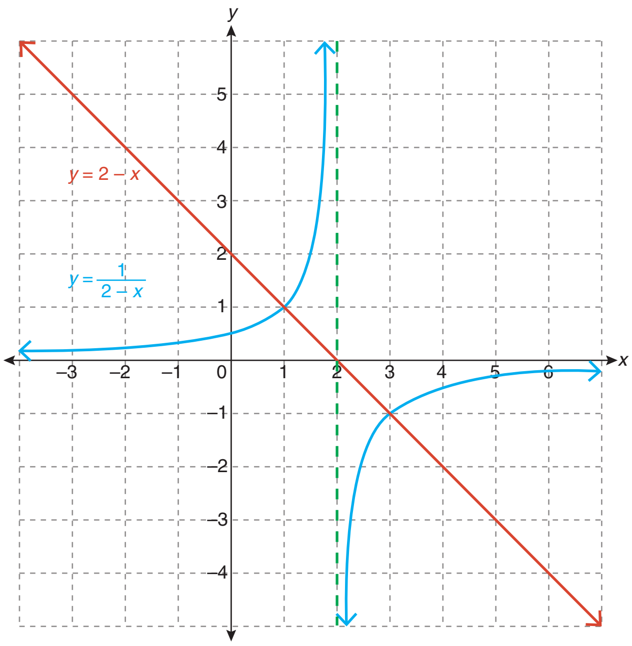
   - As $f(x)$ increases, $\frac{1}{f(x)}$ decreases, and vice versa.
   - When $f(x) < 1$, $\frac{1}{f(x)} > 1$, and vice versa.
   - If $f(x) = 1$, then $\frac{1}{f(x)} = 1$, and if $f(x) = -1$, then $\frac{1}{f(x)} = -1$.
   - Vertical asymptotes on the graph of $y = \frac{1}{f(x)}$ pass through x-values for which $f(x) = 0$.

3. For the given graph, determine the location of any asymptotes of $y = \frac{1}{f(x)}$ and the locations where the graphs of $y = f(x)$ and $y = \frac{1}{f(x)}$ will intersect.



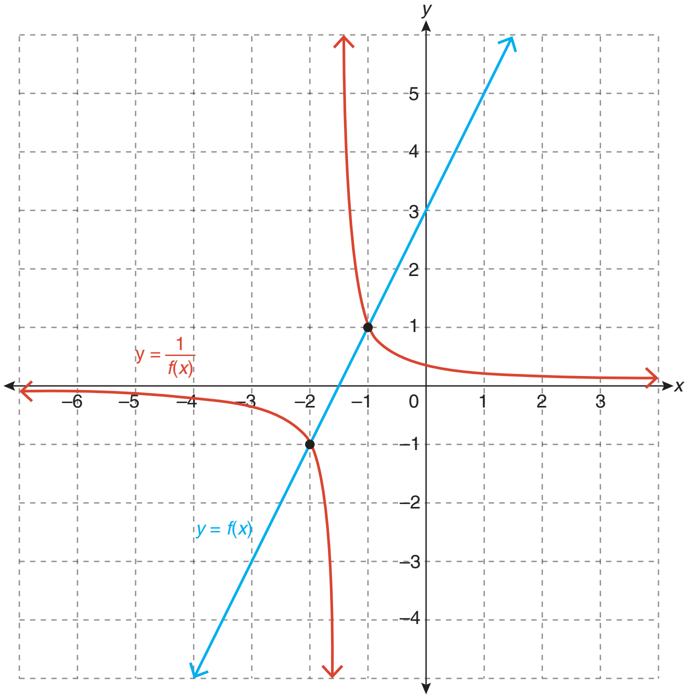
The graph of the function $y = \frac{1}{f(x)}$ will have a horizontal asymptote at $y = 0$. The graph of $y = f(x)$ has an x-intercept of –3, so there will be a vertical asymptote on the graph of $y = \frac{1}{f(x)}$ at $x = -3$. The ordered pairs (–6, –1) and (0, 1) satisfy $f(x) = \pm 1$, so these are the points of intersection of the graphs of $y = f(x)$ and $y = \frac{1}{f(x)}$.

4. Sketch the graph of $y = \frac{1}{2 - x}$.



Start by graphing $y = 2 - x$. The curves will intersect at (1, 1) and (3, –1) because $y = \pm 1$ at these points. The x-intercept of $y = 2 - x$ is 2, so $x = 2$ is a vertical asymptote on the graph of $y = \frac{1}{(2 - x)}$. There is a horizontal asymptote at $y = 0$.

5. Use the graph of $y = \dfrac{1}{f(x)}$ to sketch $y = f(x)$.



The points (–1, 1) and (–2, –1) satisfy $y = \pm 1$, so the graph of $y = f(x)$ must pass through these points.

6. In functions of the form $y = \dfrac{1}{f(x)}$, why do vertical asymptotes occur when $f(x) = 0$?

When $f(x) = 0$, $\dfrac{1}{f(x)}$ is not defined, so the corresponding x-value is not part of the domain of $y = \dfrac{1}{f(x)}$. However, as $f(x)$ approaches zero (as it becomes very small positive or negative), $\dfrac{1}{f(x)}$ becomes very large positive or negative. As a result, the graph of $\dfrac{1}{f(x)}$ includes curves that become nearly vertical as they approach the x-value that makes $f(x) = 0$.