Practice Solutions – VI

1. Determine the location of any asymptotes on the graph of $y = \dfrac{1}{x^2 + 4x - 7}$ and any intersections of the graphs of $y = x^2 + 4x - 7$ and $y = \dfrac{1}{x^2 + 4x - 7}$.

Asymptotes will occur when $f(x) = 0$.

$$f(x) = x^2 + 4x - 7$$
$$0 = x^2 + 4x - 7$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-7)}}{2(1)} \\ &= \frac{-4 \pm \sqrt{44}}{2} \\ &= -2 \pm \sqrt{11} \end{aligned}$$

Vertical asymptotes will occur at $x = -2 + \sqrt{11}$ and $x = -2 - \sqrt{11}$.

Intersections will occur when $f(x) = \pm 1$.

$$f(x) = x^2 + 4x - 7$$
$$1 = x^2 + 4x - 7$$
$$0 = x^2 + 4x - 8$$

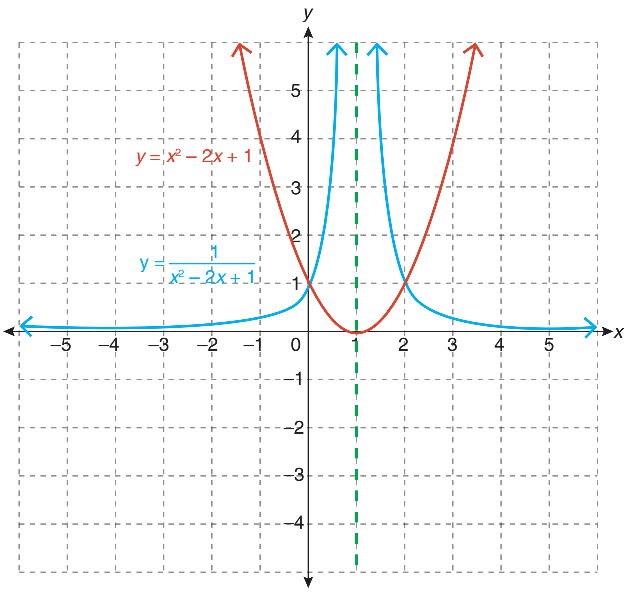
$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-8)}}{2(1)} \\ &= \frac{-4 \pm \sqrt{48}}{2} \\ &= -2 \pm 2\sqrt{3} \end{aligned}$$

$$f(x) = x^2 + 4x - 7$$
$$-1 = x^2 + 4x - 7$$
$$0 = x^2 + 4x - 6$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-6)}}{2(1)} \\ &= \frac{-4 \pm \sqrt{40}}{2} \\ &= -2 \pm \sqrt{10} \end{aligned}$$

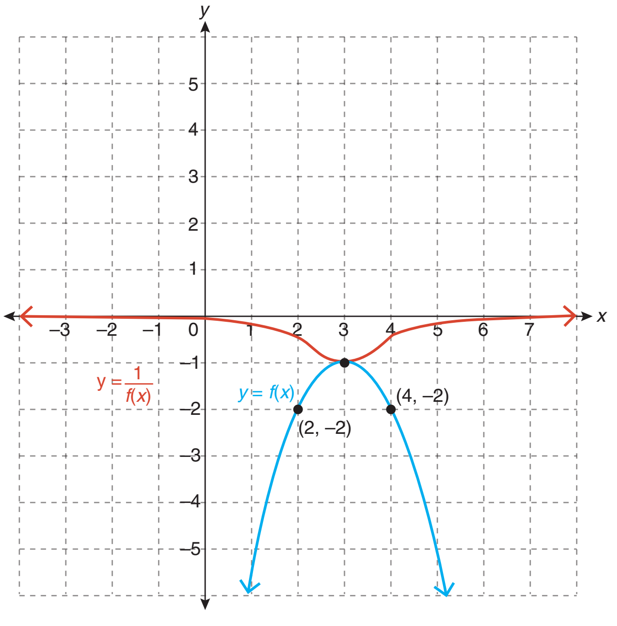
The graphs of the two functions will intersect at $(-2 + 2\sqrt{3}, 1)$, $(-2 - 2\sqrt{3}, 1)$, $(-2 + \sqrt{10}, -1)$, and $(-2 - \sqrt{10}, -1)$.

2. Sketch the graph of $y = \dfrac{1}{x^2 - 2x + 1}$.



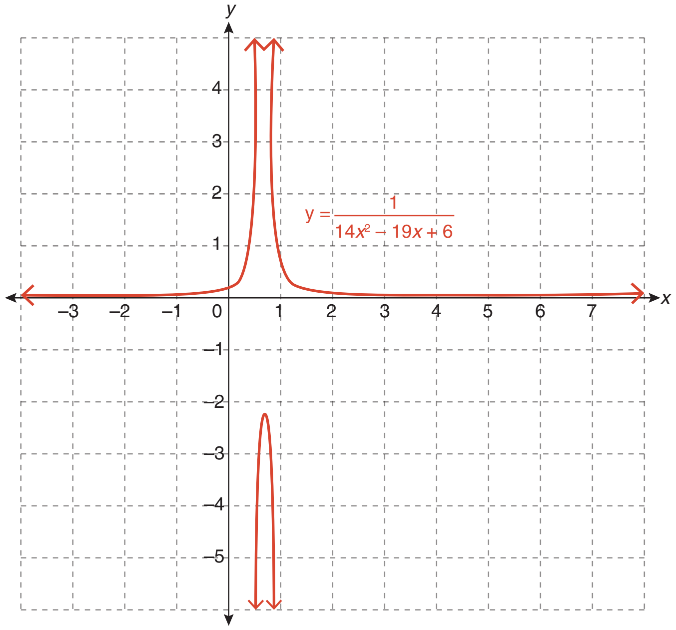
Start by graphing $y = x^2 - 2x + 1$. The curves will intersect at (0, 1) and (2, 1) because $y = 1$ at these points. The x-intercept of $y = x^2 - 2x + 1$ is 1, so $x = 1$ is a vertical asymptote. There is a horizontal asymptote at $y = 0$.

3. Use the graph of $y = \dfrac{1}{f(x)}$ to sketch the graph of $y = f(x)$.



There are no vertical asymptotes, so the graph of $y = f(x)$ has no x-intercepts. The point (3, –1) satisfies $y = -1$, so the graph of $y = f(x)$ must pass through this point. Two other definite points on the graph of $y = \dfrac{1}{f(x)}$ occur at $\left(2, -\dfrac{1}{2}\right)$ and $\left(4, -\dfrac{1}{2}\right)$, so the corresponding points on the graph of $y = f(x)$ are (2, –2) and (4, –2) respectively. These points are found by taking the reciprocal of each y-value.

4. Use technology to graph $y = \dfrac{1}{14x^2 - 19x + 6}$. Describe the steps used.

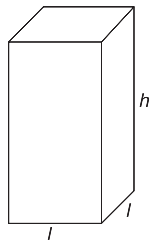


Steps will vary.

5. Explain why functions of the form $y = \dfrac{1}{f(x)}$ cannot equal zero. How does this relate to an asymptote?

For any fraction to equal zero, the numerator must be zero. However, the numerator is always 1 in $\dfrac{1}{f(x)}$. An asymptote occurs at $y = 0$ on the graph of the reciprocal of linear and quadratic functions because as $f(x)$ increases to a large positive number (or decreases to a large negative number), $\dfrac{1}{f(x)}$ approaches 0.

6. a. Write a reciprocal function that relates the length and height of a square prism that has a volume of 1 cubic unit.



$$l^2 h = 1$$
$$h = \frac{1}{l^2}$$

b. State the domain and range of the function.

Domain: $\{l \mid l > 0, l \in R\}$

Range: $\{h \mid h > 0, h \in R\}$

c. Sketch the graph of the function.