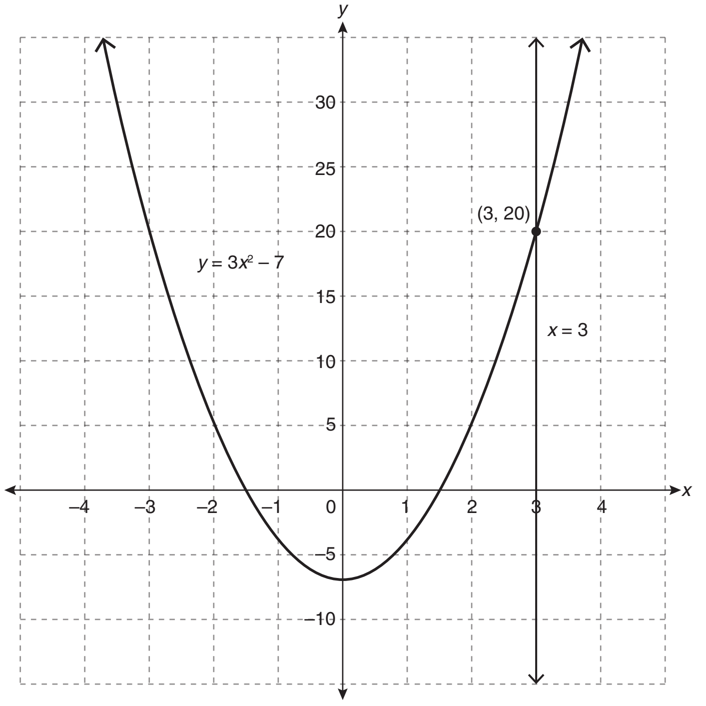
Appendix 2: Solutions

Lesson 7.1: Solving Systems of Equations Graphically

Practice Solutions – I

1. Solve the following systems of equations graphically. Verify the solution(s) by substitution.

a. $y = 3x^2 - 7$ and $x = 3$



The solution to this system is (3, 20).

Verification:

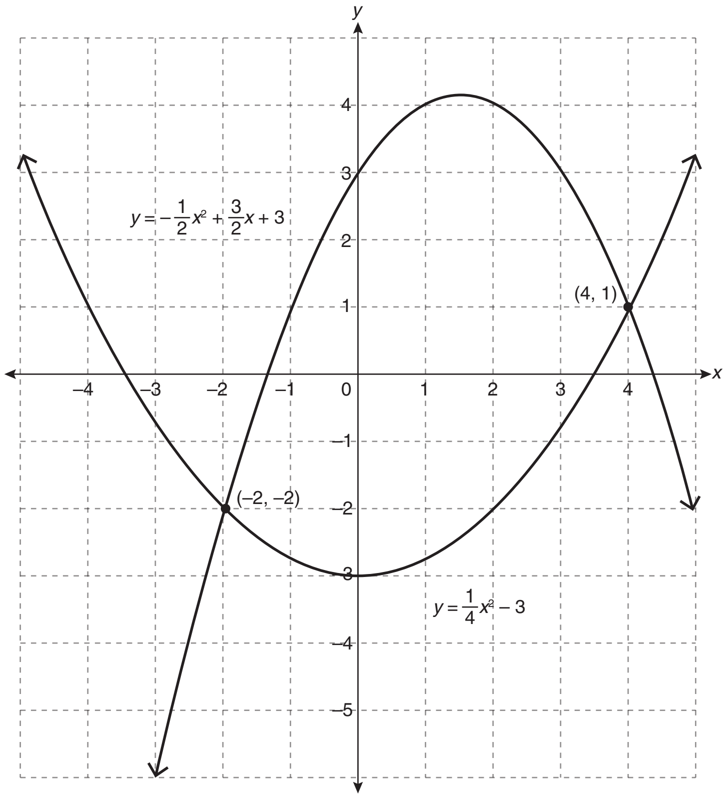
$y = 3x^2 - 7$

Left Side	Right Side
y	$3x^2 - 7$
20	$3(3)^2 - 7$
	20
LS = RS	

$x = 3$

Left Side	Right Side
x	3
3	
LS = RS	

b. $y = -\frac{1}{2}x^2 + \frac{3}{2}x + 3$ and $y = \frac{1}{4}x^2 - 3$



The solutions to this system are (–2. –2) and (4, 1).

Verify (–2, –2):

$y = -\frac{1}{2}x^2 + \frac{3}{2}x + 3$

Left Side	Right Side
y	$-\frac{1}{2}x^2 + \frac{3}{2}x + 3$
-2	$-\frac{1}{2}(-2)^2 + \frac{3}{2}(-2) + 3$
	-2
LS = RS	

$y = \frac{1}{4}x^2 - 3$

Left Side	Right Side
y	$\frac{1}{4}x^2 - 3$
-2	$\frac{1}{4}(-2)^2 - 3$
	-2
LS = RS	

Verify (4, 1):

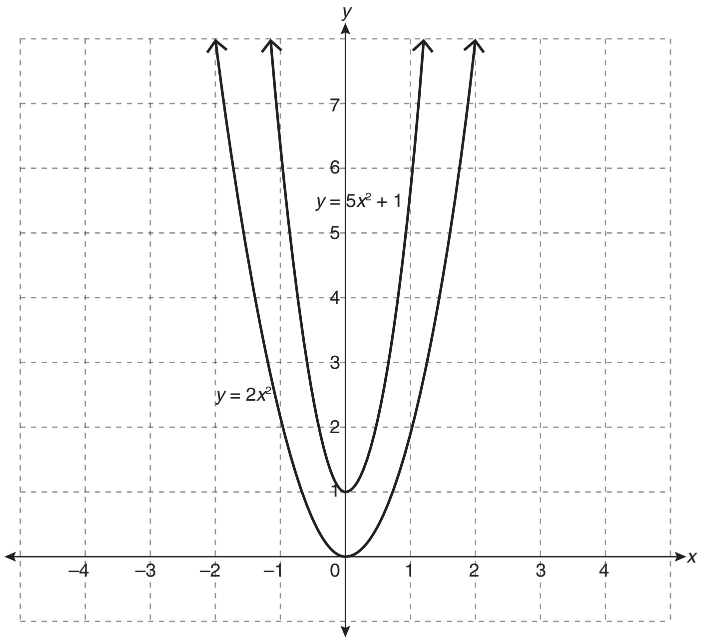
$y = -\frac{1}{2}x^2 + \frac{3}{2}x + 3$

Left Side	Right Side
y	$-\frac{1}{2}x^2 + \frac{3}{2}x + 3$
1	$-\frac{1}{2}(4)^2 + \frac{3}{2}(4) + 3$
	1
LS = RS	

$y = \frac{1}{4}x^2 - 3$

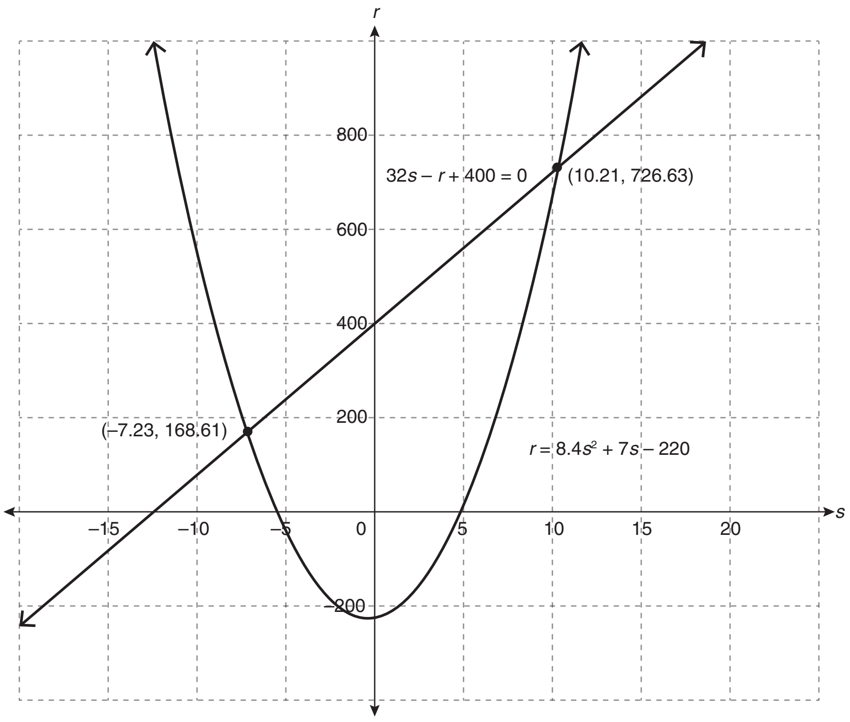
Left Side	Right Side
y	$\frac{1}{4}x^2 - 3$
1	$\frac{1}{4}(4)^2 - 3$
	1
LS = RS	

c. $y = 5x^2 + 1$ and $y = 2x^2$



The curves do not intersect, so there is no solution to this system.

d. $32s - r + 400 = 0$ and $r = 8.4s^2 + 7s - 220$



The solutions to this system are approximately:

- $s = -7.23$ and $r = 168.61$
- $s = 10.21$ and $r = 726.63$

Verify $s = -7.23$ and $r = 168.61$:

$-(x-2)^2 + 5$ y

Left Side	Right Side
$32s - r + 400$	0
$32(-7.23) - (168.61) + 400$	
0.03	
$LS \doteq RS$	

Left Side	Right Side
r	$8.4s^2 + 7s - 220$
168.61	$8.4(-7.23)^2 + 7(-7.23) - 220$
	$168.48...$
$LS \doteq RS$	

Verify $s = 10.21$ and $r = 726.63$:

$-x + 5$ $y = (x-1)^2 + 1$

Left Side	Right Side
$32s - r + 400$	0
$32(10.21) - (726.63) + 400$	
0.09	
$LS \doteq RS$	

Left Side	Right Side
r	$8.4s^2 + 7s - 220$
726.63	$8.4(10.21)^2 + 7(10.21) - 220$
	$727.12...$
$LS \doteq RS$	

2. Describe advantages and disadvantages associated with using technology to solve a system of equations graphically.

Some advantages of graphically solving a system of equations using technology include the ability to quickly produce an accurate graph and accuracy in determining the points of intersection of curves.

Some disadvantages of graphically solving a system of equations using technology are that the technology needs to be available and the user needs to understand how to graph relations using that technology.

3. An archer is standing at the base of an incline and shoots an arrow uphill. If the archer is standing at the point (0, 0), the path of the arrow can be modeled by $y = -0.002x^2 + x + 1.5$. If the slope of the incline is 0.3, at which coordinates will the arrow land?

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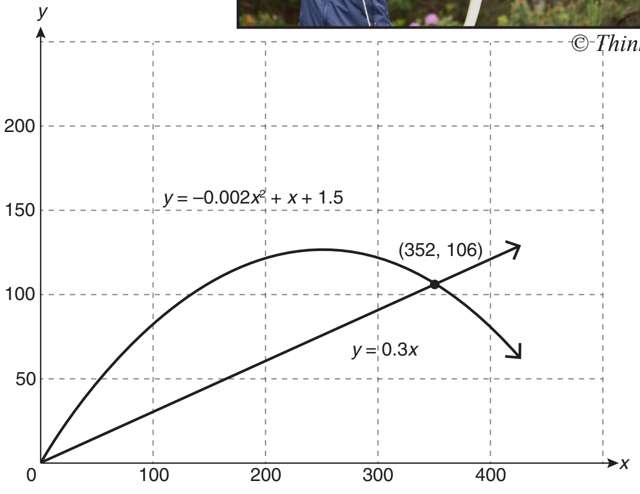
The base of the slope passes through (0, 0), so the y-intercept of the incline is 0. The equation corresponding to the incline is $y = 0.3x + 0$ or $y = 0.3x$. The system can be solved graphically. Negative values do not make sense in this scenario, so the positive solution represents the landing point of the arrow.



The solution to the system is approximately (352, 106). This represents the location the arrow will land.

4. The equations $y = -3(x-2)^2 + k$ and $y = 2$ form a system. State the value(s) of k that will provide a system with

a. no solution

The equation $y = 2$ corresponds to a horizontal line passing through (0, 2). The equation $y = -3(x-2)^2 + k$ is written in vertex form, so the corresponding graph can be readily determined. The parabola will have a vertex at (2, k) and will open downwards.

If the horizontal line is above the vertex, there will be no solution to the system. This will occur if $k < 2$.

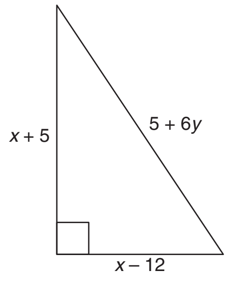
b. one solution

There will be one solution if the horizontal line passes through the vertex. This will occur if $k = 2$.

c. two solutions

There will be two solutions if the horizontal line is below the vertex. This will occur if $k > 2$.

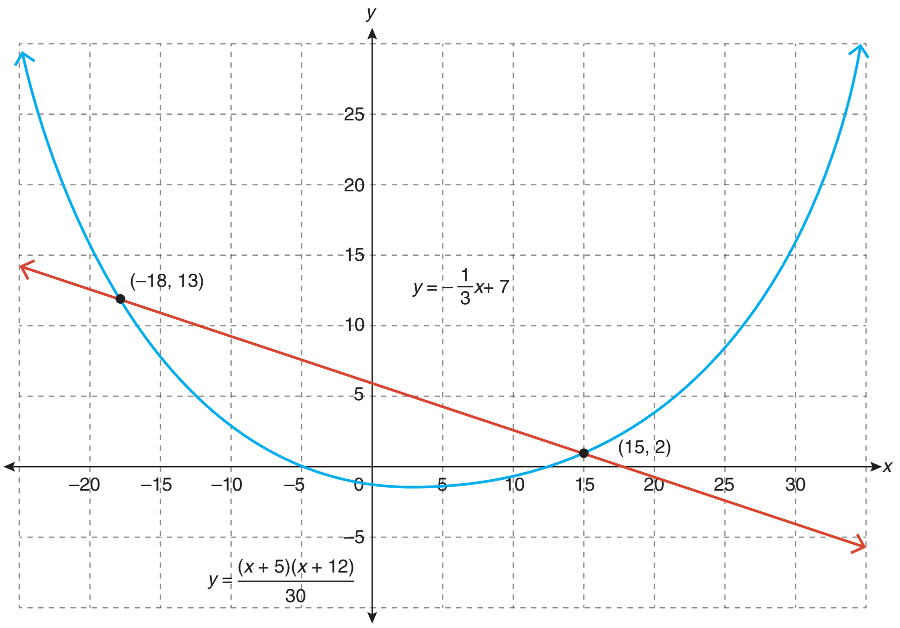
5. The perimeter of the triangle shown is 40 and the area is $15y$. Determine the value(s) of x and y.



Two equations that represent this situation are $x+5+x-12+5+6y=40$ and $\frac{1}{2}(x+5)(x-12)=15y$. Isolate y in each equation and solve by graphing.

$$\begin{aligned} x+5+x-12+5+6y &= 40 \\ 2x+6y-2 &= 40 \\ 6y &= -2x+42 \\ y &= -\frac{1}{3}x+7 \end{aligned}$$

$$\frac{1}{2}(x+5)(x-12)=15y$$
$$\frac{(x+5)(x-12)}{30}=y$$



The solutions to the system appear to be (–18, 13) and (15, 2). However, the solution (–18, 13) does not make sense in the context of the problem because the lengths of the sides of the triangle cannot be negative. As such, x could be 15 and y could be 2.

6. The equations $y = ax$ and $y = x^2$ form a system that intersects at the point (0, 0) for any a value.

   a. Explain why there is only one solution when $a = 0$.

   When $a = 0$, the first equation becomes $y = 0$. This represents a horizontal line passing through (0, 0). The vertex of the graph of $y = x^2$ also passes through the point (0, 0). This is the only intersection of these two graphs.

   b. Predict whether or not there are other a values for which the system has only one solution. (Hint: Try graphing the system using different values of a to get a feel for its effect.)

   Predictions will vary.

   c. Describe a method that can be used to test the prediction.

   Methods will vary. A sample is shown.

   As a increases (or decreases), the line corresponding to $y = ax$ becomes more vertical. A vertical line will only yield one solution, so a nearly vertical line may also only yield one solution. By selecting large values of a, graphing each system using a calculator, and zooming out, such an a-value may be searched for.

Please complete *Lesson 7.1 Explore Your Understanding Assignment* located in *Workbook 7A* before proceeding to *Lesson 7.2*.