Lesson 7.2: Solving Systems of Equations Algebraically

Practice Solutions – II

1. Solve each system of equations algebraically. Verify the solution(s).

a. $\begin{cases} y = x^2 - 4 \\ 22 + y = 3x^2 \end{cases}$

$$\begin{aligned} 22 + y &= 3x^2 \\ 22 + (x^2 - 4) &= 3x^2 \\ 18 + x^2 &= 3x^2 \\ 18 &= 2x^2 \\ 9 &= x^2 \\ \pm 3 &= x \end{aligned}$$

$y = x^2 - 4$
$y = (3)^2 - 4$
$y = 5$

$y = x^2 - 4$
$y = (-3)^2 - 4$
$y = 5$

The solutions are (3, 5) and (–3, 5).

Verify (3, 5):

$y = x^2 - 4$

Left Side	Right Side
y	$x^2 - 4$
5	$(3)^2 - 4$
	5
LS = RS	

$22 + y = 3x^2$

Left Side	Right Side
$22 + y$	$3x^2$
$22 + 5$	$3(3)^2$
27	27
LS = RS	

Verify (–3, 5):

$y = x^2 - 4$

Left Side	Right Side
y	$x^2 - 4$
5	$(-3)^2 - 4$
	5
LS = RS	

$22 + y = 3x^2$

Left Side	Right Side
$22 + y$	$3x^2$
$22 + 5$	$3(-3)^2$
27	27
LS = RS	

b. $\begin{cases} y = x^2 + 2 \\ y = x^2 + 3 \end{cases}$

$$\begin{array}{rrcrcr} & y & = & x^2 & + & 2 \\ -\ (& y & = & x^2 & + & 3\ ) \\ \hline & 0 & = & & - & 1 \end{array}$$

There are no real solutions to this system.

c. $\begin{cases} y = (x - 12)^2 - 5 \\ y + 14x = 114 \end{cases}$

$$\begin{aligned} y + 14x &= 114 \\ ((x - 12)^2 - 5) + 14x &= 114 \\ x^2 - 24x + 144 - 5 + 14x &= 114 \\ x^2 - 10x + 25 &= 0 \\ (x - 5)^2 &= 0 \\ x &= 5 \end{aligned}$$

$y = (x - 12)^2 - 5$
$y = (5 - 12)^2 - 5$
$y = 44$

The solution is (5, 44).

Verification:

$y = (x - 12)^2 - 5$

Left Side	Right Side
y	$(x - 12)^2 - 5$
44	$(5 - 12)^2 - 5$
	44
LS = RS	

$y + 14x = 114$

Left Side	Right Side
$y + 14x$	114
$44 + 14(5)$	
114	
LS = RS	

d. $\begin{cases} 0 = 5x^2 - 20x - y - 231 \\ y = 23 - 4x \end{cases}$

$$y = 23 - 4x$$
$$0 = -4x - y + 23$$

$$\begin{array}{rrcrrrrrrrr} & 0 & = & 5x^2 & - & 20x & - & y & - & 231 & \\ -\ (& 0 & = & 0x^2 & - & 4x & - & y & + & 23 &) \\ \hline & 0 & = & 5x^2 & - & 16x & & & - & 254 & \end{array}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-16) \pm \sqrt{(-16)^2 - 4(5)(-254)}}{2(5)} \\ &= \frac{16 \pm \sqrt{5\,336}}{10} \\ &= \frac{16 \pm 2\sqrt{1\,334}}{10} \\ &= \frac{8 \pm \sqrt{1\,334}}{5} \end{aligned}$$

$$x = \frac{8 + \sqrt{1\,334}}{5} \text{ or } x = \frac{8 - \sqrt{1\,334}}{5}$$

$$\begin{aligned} y &= 23 - 4x \\ y &= 23 - 4\left(\frac{8 + \sqrt{1\,334}}{5}\right) \\ y &= 23 - \frac{32 + 4\sqrt{1\,334}}{5} \\ y &= \frac{83 - 4\sqrt{1\,334}}{5} \end{aligned}$$

$$\begin{aligned} y &= 23 - 4x \\ y &= 23 - 4\left(\frac{8 - \sqrt{1\,334}}{5}\right) \\ y &= 23 - \frac{32 - 4\sqrt{1\,334}}{5} \\ y &= \frac{83 + 4\sqrt{1\,334}}{5} \end{aligned}$$

The solutions are $\left(\frac{8 + \sqrt{1\,334}}{5}, \frac{83 - 4\sqrt{1\,334}}{5}\right)$ and $\left(\frac{8 - \sqrt{1\,334}}{5}, \frac{83 + 4\sqrt{1\,334}}{5}\right)$.

Verify $\left(\frac{8+\sqrt{1334}}{5}, \frac{83-4\sqrt{1334}}{5}\right)$:

$0 = 5x^2 - 20x - y - 231$

Left Side	Right Side
0	$5x^2 - 20x - y - 231$ $5\left(\frac{8+\sqrt{1334}}{5}\right)^2 - 20\left(\frac{8+\sqrt{1334}}{5}\right) - \left(\frac{83-4\sqrt{1334}}{5}\right) - 231$ $\frac{64}{5} + \frac{16\sqrt{1334}}{5} + \frac{1334}{5} - \frac{160}{5} - \frac{20\sqrt{1334}}{5} - \frac{83}{5} + \frac{4\sqrt{1334}}{5} - \frac{1155}{5}$ 0
LS = RS	

$y = 23 - 4x$

Left Side	Right Side
y $\frac{83-4\sqrt{1334}}{5}$	$23 - 4x$ $23 - 4\left(\frac{8+\sqrt{1334}}{5}\right)$ $23 - \frac{32+4\sqrt{1334}}{5}$ $\frac{83-4\sqrt{1334}}{5}$
LS = RS	

Verify $\left(\frac{8-\sqrt{1334}}{5}, \frac{83+4\sqrt{1334}}{5}\right)$:

$0 = 5x^2 - 20x - y - 231$

Left Side	Right Side
0	$5x^2 - 20x - y - 231$ $5\left(\frac{8-\sqrt{1334}}{5}\right)^2 - 20\left(\frac{8-\sqrt{1334}}{5}\right) - \left(\frac{83+4\sqrt{1334}}{5}\right) - 231$ $\frac{64}{5} - \frac{16\sqrt{1334}}{5} + \frac{1334}{5} - \frac{160}{5} + \frac{20\sqrt{1334}}{5} - \frac{83}{5} - \frac{4\sqrt{1334}}{5} - \frac{1155}{5}$ 0
LS = RS	

$y = 23 - 4x$

Left Side	Right Side
y $\frac{83+4\sqrt{1334}}{5}$	$23 - 4x$ $23 - 4\left(\frac{8-\sqrt{1334}}{5}\right)$ $23 - \frac{32-4\sqrt{1334}}{5}$ $\frac{83+4\sqrt{1334}}{5}$
LS = RS	

2. Explain how the solution to a system of equations can be verified graphically using technology.

The solution(s) to a system can be verified by using a calculator or graphing program to graph the system. Determining where the curves intersect will give the solution, and this solution should match the algebraic solution.

3. Verona is designing a pair of fireworks. She would like the user to be able to light the pair two seconds apart, and have them explode in the sky at approximately the same time and height. To do this, she will put a larger lifting charge in one firework and use different delay fuses. The height, *h*, of each shell at *t* seconds after the first shell is launched is modelled by the following system:

Shell 1: $h = -4.9t^2 + 65t$
Shell 2: $h = -4.9t^2 + 90t - 160$

a. For how long should Verona set the delay fuse on each shell? (How long after each shell is launched should it explode?) Round your answer to the nearest tenth of a second.

$$\begin{aligned} h &= -4.9t^2 + 90t - 160 \\ -4.9t^2 + 65t &= -4.9t^2 + 90t - 160 \\ -25t &= -160 \\ t &= 6.4 \end{aligned}$$

Verona should set the first fuse for 6.4 seconds and the second fuse for 4.4 seconds.

b. At what height, rounded to the nearest metre, will the shells explode?

$$\begin{aligned} h &= -4.9t^2 + 65t \\ h &= -4.9(6.4)^2 + 65(6.4) \\ h &\doteq 215.3 \end{aligned}$$

The shells will explode at a height of approximately 215 m.

4. Recall, from *Practice I* (question 6), the equations $y = ax$ and $y = x^2$, which form a system that intersects at the point (0, 0) for any a-value. The question asked for a prediction of whether or not any non-zero a-value would lead to a single solution.

   a. Explain why it may be easier to use algebra than a graph to determine the number of solutions to the system for different a-values.

   A graph can only show a portion of the system, so it is possible that the two lines intersect outside the part of the system shown. Using algebra, it is possible to account for the entire system.

   b. Solve the system algebraically.

$$y = ax$$
$$x^2 = ax$$
$$x^2 - ax = 0$$
$$x(x - a) = 0$$

$$x = 0 \qquad x - a = 0$$
$$x = a$$

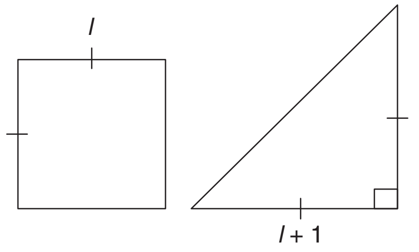
$$y = ax \qquad y = ax$$
$$y = a(0) \qquad y = (a)(a)$$
$$y = 0 \qquad y = a^2$$

   The solutions to the system are $(0, 0)$ and (a, a^2).

   c. Describe the number of solutions the system will have for different a-values.

   When $a = 0$, the system will have one solution. For any non-zero value of a, the system will have two solutions. So there are no non-zero a-values that will give exactly one solution.

5. The two shapes shown have the same area. Determine the value of *l*.



The areas can be represented by:

$$A_{\text{square}} = s^2 = l^2$$

$$\begin{aligned} A_{\text{triangle}} &= \frac{1}{2}bh \\ &= \frac{1}{2}(l+1)(l+1) \\ &= \frac{1}{2}(l+1)^2 \end{aligned}$$

The two areas must be equal so

$$\begin{aligned} l^2 &= \frac{1}{2}(l+1)^2 \\ l^2 &= \frac{1}{2}(l^2+2l+1) \\ l^2 &= \frac{1}{2}l^2 + l + \frac{1}{2} \\ \frac{1}{2}l^2 - 1 - \frac{1}{2} &= 0 \\ l^2 - 2l - 1 &= 0 \\ l &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} \\ &= \frac{2 \pm \sqrt{8}}{2} \\ &= \frac{2 \pm 2\sqrt{2}}{2} \\ &= 1 \pm \sqrt{2} \end{aligned}$$

The negative solution is not possible, so the length is $1 + \sqrt{2}$.

Please complete *Lesson 7.2 Explore Your Understanding Assignment* located in *Workbook 7A* before proceeding to *Lesson 7.3*.