Lesson 7.4: Quadratic Inequalities in One Variable

Practice Solutions – IV

1. Solve the inequality $(x+3)(x-5)<0$. Represent the solution symbolically and on a number line. Verify the solution.

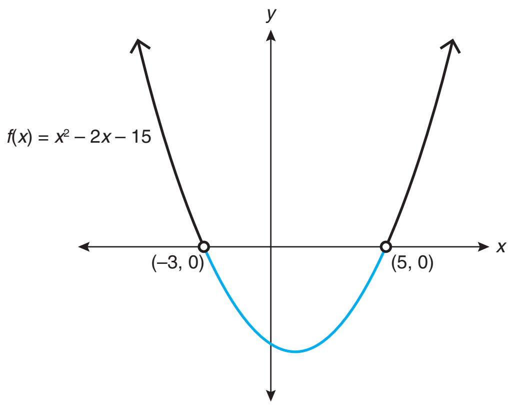
Solution strategies will vary. A graphical solution with a test point verification is shown.

$$(x+3)(x-5)=0$$

$$x+3=0 \qquad x-5=0$$

$$x=-3 \qquad x=5$$

The zeros of the corresponding function are –3 and 5. The expression $(x+3)(x-5)$ can be expanded to $x^2-2x-15$, which has a positive coefficient on the x^2 term, so the graph of the corresponding function opens up.

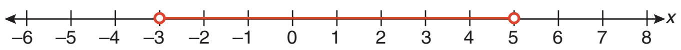


Verify $x<-3$ (test point –5):

Left Side	Right Side
$(x+3)(x-5)$ $((-5)+3)((-5)-5)$ 20	0
LS > RS	

The test point –5 does not satisfy the inequality, so $x<-3$ is not part of the solution set.

The solution set is $\{x \mid -3<x<5, x\in \mathrm{R}\}$.



Verify $-3<x<5$ (test point 0):

Left Side	Right Side
$(x+3)(x-5)$ $((0)+3)((0)-5)$ -15	0
LS < RS	

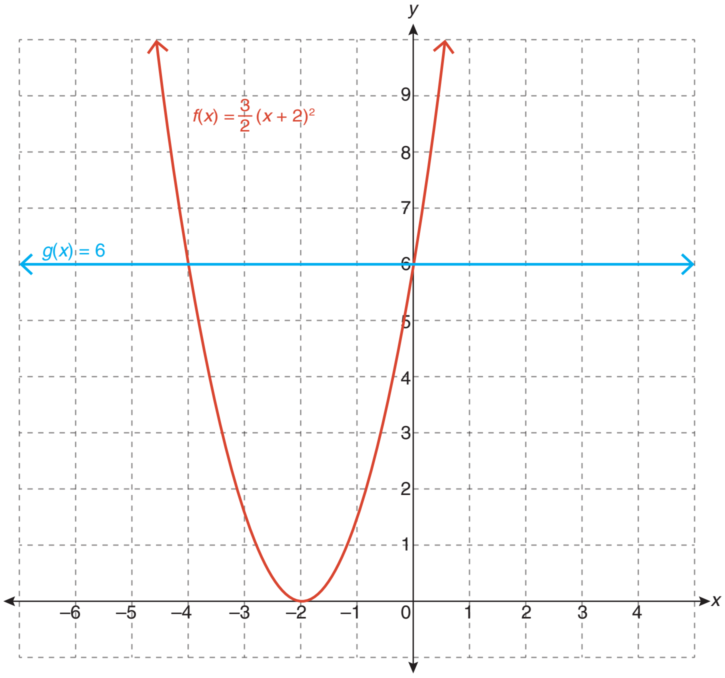
The test point 0 satisfies the inequality, so $-3<x<5$ is part of the solution set.

Verify $x>5$ (test point 6):

Left Side	Right Side
$(x+3)(x-5)$ $((6)+3)((6)-5)$ 9	0
LS > RS	

The test point 6 does not satisfy the inequality, so $x>5$ is not part of the solution set.

2. While trying to solve the inequality $\frac{3}{2}(x+2)^2 \geq 6$, Abagail drew the following graph.



a. Explain how this graph can be used to solve the inequality.

The solution to the inequality is the set of x-values that makes $\frac{3}{2}(x+2)^2$ greater than or equal to 6. This corresponds to the regions on the graph where $f(x)$ is greater than or equal to $g(x)$.

b. Solve the inequality.

By inspecting the graph, it can be seen that $f(x) \geq g(x)$ for $x \leq -4$ or $x \geq 0$. The solution to the system is $\{x \mid x \leq -4 \text{ or } x \geq 0, x \in \mathrm{R}\}$.

3. Solve $x^2 < 2x + 6$. Represent the solution symbolically and on a number line.

Solutions methods may vary. The test point method is shown.

$$x^2 < 2x + 6$$
$$x^2 - 2x - 6 < 0$$

$$x^2 - 2x - 6 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-6)}}{2(1)}$$
$$x = \frac{2 \pm \sqrt{28}}{2}$$
$$x = \frac{2 \pm 2\sqrt{7}}{2}$$
$$x = 1 \pm \sqrt{7}$$

The intervals that may be part of the solution are:

- $x < 1 - \sqrt{7}$
- $1 - \sqrt{7} < x < 1 + \sqrt{7}$
- $x > 1 + \sqrt{7}$

The zeros are not part of the solution set because the inequality is strict.

Test $x < 1 - \sqrt{7}$ (test point –5):

Left Side	Right Side
x^2	$2x + 6$
$(-5)^2$	$2(-5) + 6$
25	-4
LS > RS	

The test point –5 does not satisfy the original inequality, so $x < 1 - \sqrt{7}$ is not part of the solution set.

Test $1 - \sqrt{7} < x < 1 + \sqrt{7}$ (test point 0):

Left Side	Right Side
x^2	$2x + 6$
$(0)^2$	$2(0) + 6$
0	6
LS < RS	

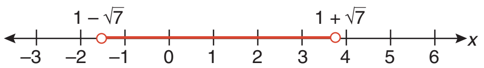
The test point 0 satisfies the original inequality, so $1 - \sqrt{7} < x < 1 + \sqrt{7}$ is part of the solution set.

Test $x > 1 + \sqrt{7}$ (test point 5):

Left Side	Right Side
x^2	$2x + 6$
$(5)^2$	$2(5) + 6$
25	16
LS > RS	

The test point 5 does not satisfy the original inequality, so $x > 1 + \sqrt{7}$ is not part of the solution set.

The solution set is $\{x \mid 1 - \sqrt{7} < x < 1 + \sqrt{7}, x \in \mathrm{R}\}$.



4. If the quadratic function corresponding to a quadratic inequality has no real zeros, the solution set for the inequality will either be empty or include all real numbers. Explain why this is true.

   For a continuous quadratic function to include both positive and negative values, it must have a value of zero at some point. If the function is continuous and does not have a value of zero, either all values of the function are positive or all values are negative. This means that an inequality with zero on one side will either always be satisfied for any input value or will not be satisfied for any input value.

   Graphically, a quadratic function with no real zeros is a parabola that either lies entirely above or entirely below the x-axis. As such, all input values of the function may satisfy the inequality or no input values of the function may satisfy the inequality, depending on the inequality symbol involved.

Please return to *Unit 7: Equations and Inequalities Lesson 7.4* to continue your exploration.