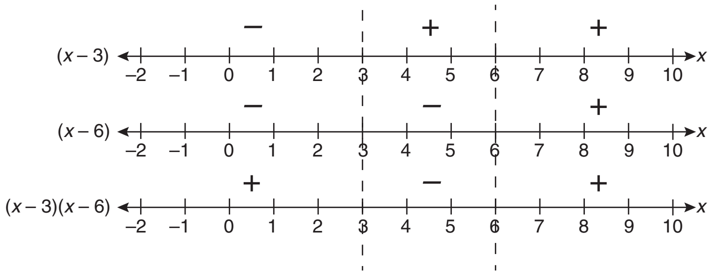
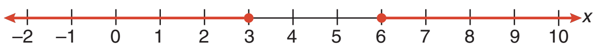
Practice Solutions – V

1. Solve $x^2 - 9x + 18 \geq 0$ using sign analysis. Represent the solution set symbolically and on a number line.

$$x^2 - 9x + 18 \geq 0$$
$$(x-3)(x-6) \geq 0$$



The solution set is $\{x \mid x \leq 3 \text{ or } x \geq 6, x \in \mathrm{R}\}$.



2. The ‘case analysis’ method for solving a quadratic inequality is shown on p. 479 of *Pre-Calculus 11*.

   a. Explain the case analysis strategy.

      1. Rearrange the inequality such that one side is zero.
      2. Factor the non-zero side of the inequality.
      3. The factors will need to multiply to either a positive or a negative value to satisfy the inequality. Determine the possible cases that will produce the appropriate sign.
      4. Combine the appropriate intervals for each factor to determine the solution set.

   b. Describe how the case analysis strategy is similar to the sign analysis strategy.

      Both strategies focus on determining whether a product of two factors will be positive or negative using the signs of the factors in particular intervals.

c. Solve $x(x+1) < 0$ using the case analysis strategy.

The product $x(x+1)$ must be negative in order to satisfy the inequality. This will happen when either

- x is positive and $x+1$ is negative

OR

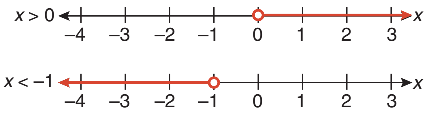
- x is negative and $x+1$ is positive

Determine the values of x that will make each case true.

Case 1: x is positive and $x+1$ is negative

$x > 0$ $\qquad x+1 < 0$

$\qquad\qquad x < -1$

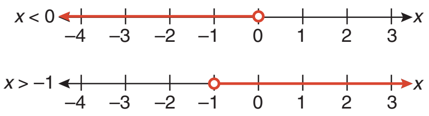


There are no x-values that satisfy both conditions, so there are no solutions from this case.

Case 2: x is negative and $x+1$ is positive

$x < 0$ $\qquad x+1 < 0$

$\qquad\qquad x > -1$



The interval $-1 < x < 0$ satisfies both inequalities, so this is the solution set.

The solution set is $\{x \mid -1 < x < 0, x \in \mathrm{R}\}$.

3. A golfer makes a chip shot that travels 40 feet over flat terrain before landing. If the ball follows a parabolic path, and the maximum height reached by the ball is 33 feet, during what interval is the ball above 25 feet?

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Let the ball's initial position be (0, 0). The maximum height occurs halfway through the flight, so the vertex of the parabola is at (20, 33). Substitute this information into the vertex form to determine a quadratic function that represents the ball's height, *h*, at distance *d*.

$$h(d) = a(d-20)^2 + 33$$
$$0 = a(0-20)^2 + 33$$
$$-33 = 400a$$
$$-\frac{33}{400} = a$$

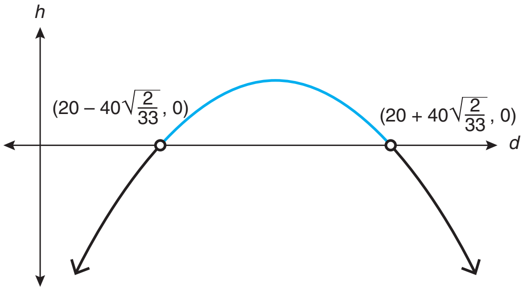
The trajectory of the ball is represented by $h(d) = -\frac{33}{400}(d-20)^2 + 33$. The interval for which the ball is above 25 feet can be determined by solving $25 < -\frac{33}{400}(d-20)^2 + 33$. This can be done partially algebraically and partially graphically. Begin by making one side of the inequality 0.

$$0 < -\frac{33}{400}(d-20)^2 + 8$$

Determine the zeros of the corresponding function, and then graph the function.

$$-8 = -\frac{33}{400}(d-20)^2$$
$$\frac{3\,200}{33} = (d-20)^2$$
$$\pm\sqrt{\frac{3\,200}{33}} = d-20$$
$$20 \pm 40\sqrt{\frac{2}{33}} = d$$

The *a*-value of the corresponding function is negative, so the parabola opens downward.



The solution set is $\left\{d \mid 20 - 40\sqrt{\frac{2}{33}} < d < 20 + 40\sqrt{\frac{2}{33}}, d \in \mathrm{R}\right\}$. The ball will be above 25 feet from approximately 10.2 feet to 29.8 feet.

Please complete *Lesson 7.4 Explore Your Understanding Assignment* located in *Workbook 7B* before proceeding to *Lesson 7.5*.