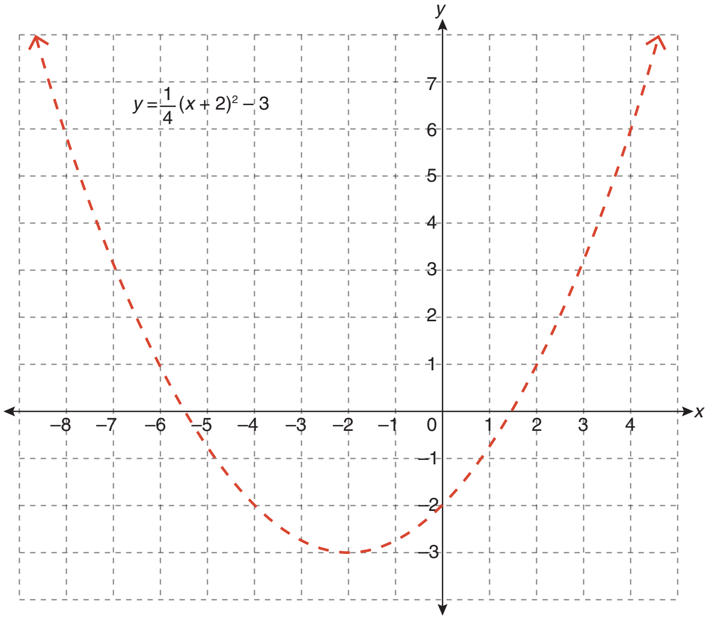
Practice Solutions – VI

1. Determine if the point (5, 17) is a solution to $y \leq x^2 - 2x + 4$.

Left Side	Right Side
y	$x^2 - 2x + 4$
17	$(5)^2 - 2(5) + 4$
	19
LS < RS	

The point (5, 17) satisfies the inequality, so it is a solution.

2. Graph the inequality $y < \frac{1}{4}(x+2)^2 - 3$.

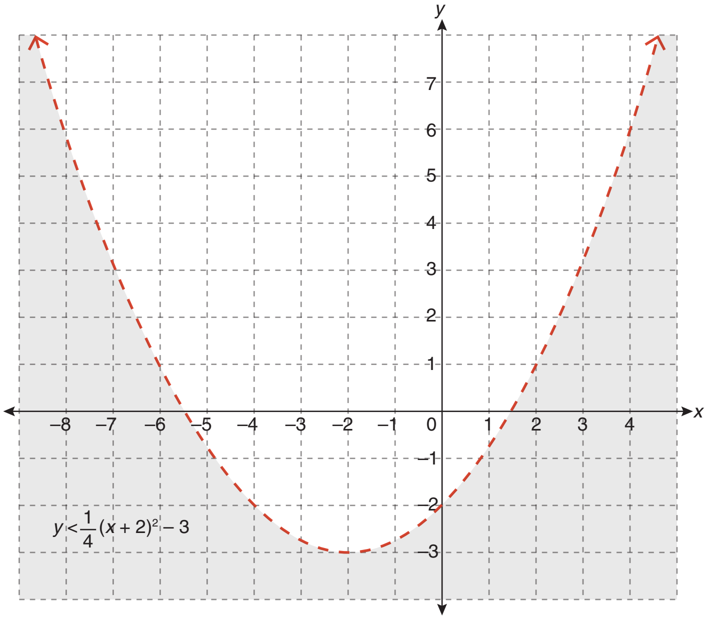


Start by graphing the corresponding function, $y = \frac{1}{4}(x+2)^2 - 3$. The inequality is strict, so graph the boundary using a dashed curve.

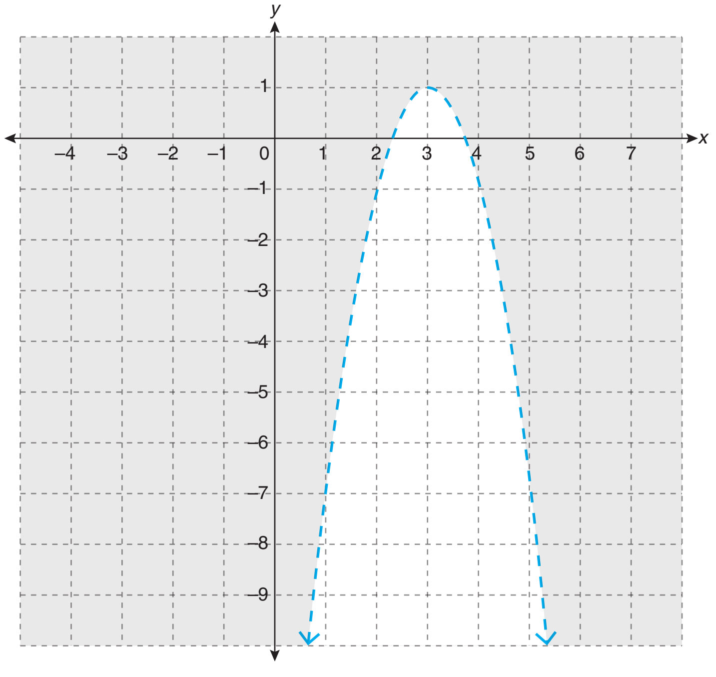
A test point, such as (0, 0), can be used to determine which region to shade.

Left Side	Right Side
y 0	$\frac{1}{4}(x+2)^2 - 3$ $\frac{1}{4}((0)+2)^2 - 3$ -2
LS > RS	

The point (0, 0) does not satisfy the inequality, so it and all other points on that side of the boundary are not solutions. Shade the region that does not include the test point.



3. Write an inequality to represent the given graph.



The vertex occurs at (3, 1), so the equation of the boundary will be of the form $y = a(x-3)^2 + 1$. The point (2, –1) is also a point on the boundary.

$$\begin{aligned} y &= a(x-3)^2 + 1 \\ -1 &= a((2)-3)^2 + 1 \\ -1 &= a + 1 \\ -2 &= a \end{aligned}$$

The equation of the boundary is $y = -2(x-3)^2 + 1$. The curve is dashed, so the inequality is strict. Use a test point from the solution region to determine the direction of the inequality symbol. The point (0, 0) is a point in the solution region.

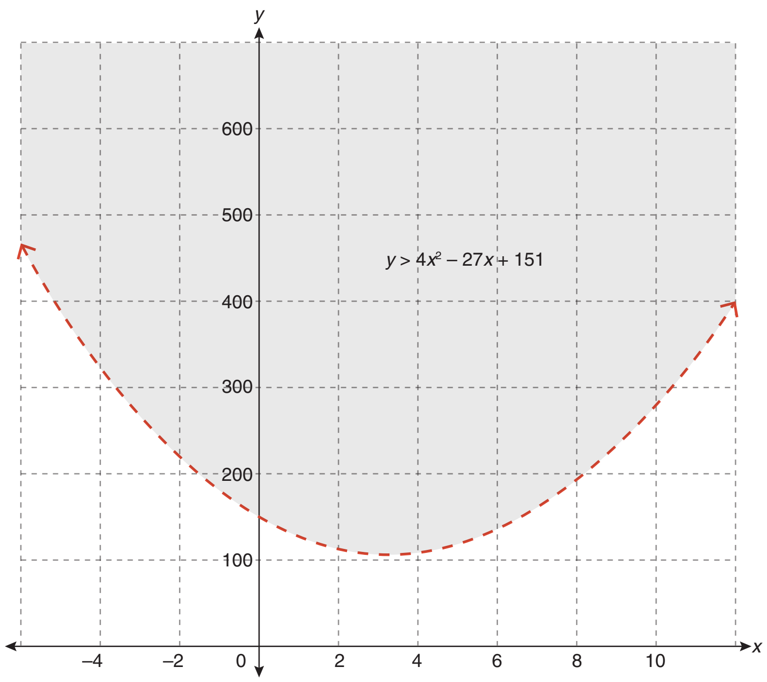
Left Side	Right Side
y	$-2(x-3)^2 + 1$
0	$-2(0-3)^2 + 1$
	-17
LS > RS	

An inequality that corresponds to the graph is $y > -2(x-3)^2 + 1$.

4. Suppose the points (3, 0) and (5, 0) are solutions to a quadratic inequality, but the point (4, 0) is not. Describe how the direction of opening of the parabolic boundary can be determined.

   The three points all lie on the *x*-axis. The middle point, (4, 0), is not a solution, so it must lie on a different side of the parabola than the two solution points. As such, the parabola must curve around (4, 0). Test points that are directly above or below (4, 0) can be checked until one is found that is a solution. If the test point is above (4, 0), the parabola opens down. If the test point is below (4, 0), the parabola opens up.

5. Use technology to graph the solution to $y > 4x^2 - 27x + 151$.



6. In one type of solar thermal power station, an array of parabolic troughs focuses sunlight onto a pipe to heat steam inside. The troughs are rotated throughout the day, so they always directly face the sun. Describe the region of sunlight captured by a trough with a width of 6 m and a maximum depth of 1 m.

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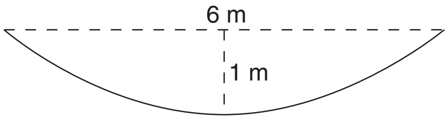
Start by sketching a diagram.



Let (0, 0) be the vertex of the parabola when the trough is pointing directly upwards. The points (–3, 1) and (3, 1) are also on the parabola.

$$y = a(x-h)^2 + k$$
$$y = a(x-0)^2 + 0$$
$$y = ax^2$$
$$1 = a(3)^2$$
$$\frac{1}{9} = a$$

The equation of the boundary is $y = \frac{1}{9}x^2$.

The region of sunlight captured by a trough occurs when the light is above the parabola and between x-values of –3 and 3, so the inequality is $y > \frac{1}{9}x^2, -3 < x < 3$.

Please complete *Lesson 7.5 Explore Your Understanding Assignment*, *Final Review Assignment*, and *Check Point* located in *Workbook 7B*.