Math 20-1 Final Exam Review

Ch1. Sequences and Series

1. $d = t_2 - t_1$
$= 23 - 27$
$= -4$

2. This is an arithmetic sequence since $t_2 - t_1 = t_3 - t_2$

$t_1 = 32 \quad d = 9 \quad n = 23 \quad t_{23} = ?$

$t_{23} = t_1 + (n-1)d$
$= 32 + (23-1)9$
$= 32 + (22)(9)$
$t_{23} = 230.$

3. First find d.
$t_n = t_1 + (n-1)d$
$10 = (-5) + (6-1)d$
$15 = 5d$
$d = 3$

Next find the sum
$S_n = \frac{n}{2}(2t_1 + (n-1)d)$
$S_{14} = \frac{14}{2}(2(-5) + (14-1)3)$
$= 7(-10 + 39)$
$= 7(29)$
$S_{14} = 203$

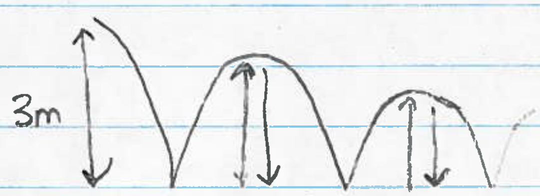
4. This in an arithmetic sequence since $t_2 - t_1 = t_3 - t_2 = 4$.

From Jan 2012 to Dec 2012 there are 12 months, and from Jan 2013 to Jun 2013 there are 6 months, so $n = 18$

$t_1 = 1 \quad n = 18 \quad d = 4 \quad S_{18} = ?$

$$S_{18} = \frac{n}{2}(2t_1 + (n-1)d)$$
$$= 18/2(2(1) + (18-1)4)$$
$$= 9(2 + 68)$$
$$S_{18} = \$630$$

5.



$2(2.55) + 2(2.1675) \ldots \quad 0$

$t_1 = 5.1 \quad t_n = 0 \quad r = 0.85$

Sum of "up" and "down" distances:

$$S_n = \frac{rt_n - t_1}{r-1}$$

$$= \frac{0.85(0) - 5.1}{0.85 - 1}$$

$= 34$ plus 3m of initial fall $= 37$m

6. $t_1 = 25\ 000$

$r = 1.027$

$n = 2025 - 2012 + 1 = 14.$

$t_n = t_1 \cdot r^{n-1}$

$= 25000(1.027)^{14-1}$

$= \$ 35\ 347.26$

7. The sequence is geometric since $r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = -2.5$

$r = -2.5$

$t_1 = 4$

$t_{36} = ?$

$n = 36$

$t_n = t_1 r^{n-1}$

$= 4(-2.5)^{36-1}$

$= -3.388 \times 10^{14}$

8. $t_1 = 250$

$t_n = 2/625$

$r = \frac{50}{250} = \frac{1}{5}$

$S_n = \frac{r t_n - t_1}{r - 1}$

$= \frac{\frac{1}{5}\left(\frac{2}{625}\right) - 250}{\frac{1}{5} - 1}$

$= \frac{-249.99936}{-0.8}$

$= 312.4992$

$= \frac{195312}{625}$

9. Step 1: Find d

Assume t_{12} is t_1

Then t_{17} is t_6

$\therefore t_1 = 73,\ t_6 = 38,\ n = 6$

$$t_n = t_1 + (n-1)d$$
$$38 = 73 + (6-1)d$$
$$-35 = 5d$$
$$d = -7$$

Step 2: Find the real t_1.

$t_{12} = 73$

$d = -7$

$t_1 = ?$

$n = 12$

$$t_n = t_1 + (n-1)d$$
$$73 = t_1 + (12-1)(-7)$$
$$73 = t_1 - 77$$
$$t_1 = 150.$$

Step 3: Find the number of terms, n.

$t_1 = 150$

$d = -7$

$t_n = -46$

$n = ?$

$$t_n = t_1 + (n-1)d$$
$$-46 = 150 + (n-1)(-7)$$
$$-196 = -7n + 7$$
$$\frac{-203}{-7} = \frac{-\cancel{7}n}{-\cancel{7}}$$
$$n = 29$$

10. Step 1: Find t_1

$S_{15} = 690$

$d = 6$

$n = 15$

$$S_n = \frac{n}{2}(2t_1 + (n-1)d)$$
$$690 = 15/2\,(2t_1 + (15-1)6)$$
$$92 = 2t_1 + 84$$
$$8 = 2t$$
$$t_1 = 4.$$
$$t_2 = 4 + 6 = 10$$
$$t_3 = 10 + 6 = 16$$

11. $d = 11 - 4 = 7$

$t_1 = 4$
$t_n = 116$
$d = 7$
$n = ?$

$t_n = t_1 + (n-1)d$
$116 = 4 + (n-1)7$
$112 = 7n - 7$
$119 = 7n$
$n = 17.$

12. The sequence is geometric since $r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = 3$

$t_1 = 6$
$r = 3$
$t_n = 28697814$
$n = ?$

$t_n = t_1 r^{n-1}$
$28\,697\,814 = 6(3)^{n-1}$
$4782969 = 3^{n-1}$
$3^3 \cdot 177147 = 3^{n-1}$
$3^3 \cdot 3^3 \cdot 6561 = 3^{n-1}$
$3^3 \cdot 3^3 \cdot 3^3 \cdot 243 = 3^{n-1}$
$3^3 \cdot 3^3 \cdot 3^3 \cdot 3^3 \cdot 9 = 3^{n-1}$
$3^3 \cdot 3^3 \cdot 3^3 \cdot 3^3 \cdot 3^2 = 3^{n-1}$
$3^{14} = 3^{n-1}$
$14 = n - 1$
$n = 15$

13. $\frac{t_2}{t_1} = \frac{t_3}{t_1}$

$\frac{x+4}{4x+1} = \frac{10-x}{x+4}$

$(x+4)(x+4) = (4x+1)(10-x)$
$x^2 + 8x + 16 = 40x - 4x^2 + 10 - x$
$5x^2 - 31x + 6 = 0$
$5x^2 - 30x - x + 6 = 0$

$$5x(x-6)-1(x-6)=0$$
$$(x-6)(5x-1)=0$$

$x-6=0$ $\quad\quad$ $5x-1=0$

$x=6$ $\quad\quad$ $x=1/5$

14. $r=\frac{50}{200}=0.25$ $\quad\quad$ $S_\infty=\frac{t_1}{1-r}$

$$=\frac{200}{1-0.25}$$
$$=266.\overline{6} \text{ or } {}^{800}/3.$$

15. a) $t_1=65$ $\quad d=7$

$$t_n=t_1+(n-1)d$$
$$t_n=65+(n-1)7$$
$$=65+7n-7$$
$$t_n=7n+58.$$

b) $t_{12}=7(12)+58$

$t_{12}=142$

It will cost \$142 in 12 years.

c) $t_n=170$

$$170=7n+58$$
$$112=7n$$
$$n=16$$

16 years will have passed.

1(a) $t_1 = 7500$ $\quad t_n = t_1 r^{n-1}$

$r = 0.97$ $\quad t_n = 7500(0.97)^{n-1}$

b) $t_5 = 7500(0.97)^{5-1}$

$t_5 = 6639$ barrels.

c) $t_1 = 7500$

$r = 0.97$

$n = 4 \times 12 = 48$

$S_{48} = ?$

$$S_n = \frac{t_1(r^n - 1)}{r-1}$$

$$= \frac{7500(0.97^{48} - 1)}{0.97 - 1}$$

$S_{48} = 192059$ barrels

17. $t_1 = 1.2$

$r = 0.75$

$t_n = 0.02138154$

$$S_n = \frac{rt_n - t_1}{r-1}$$

$$= \frac{0.75(0.02138154) - 1.2}{0.75 - 1}$$

$S_n = 4.736$ metres

18 Option 1:

Pay = \$5 × 100 = $\boxed{\$500}$

Second option is better.

Option 2:

$0.05 + 0.10 + 0.20 \ldots$

$t_1 = 0.05$

$r = 2$

$n = 100$

$S_n = ?$

$$S_n = \frac{t_1(r^n - 1)}{r-1}$$

$$= \frac{0.05(2^{100} - 1)}{2-1}$$

$\boxed{= \$6.338 \times 10^{28}}$

b) $n = ?$

$S_n = 100000$

$$100000 = \frac{0.05(2^n - 1)}{2-1}$$

$2000000 = 2^n - 1$

$2000001 = 2^n$

$2^{20} = 1048576$ and $2^{21} = 2097152$, so 20 days

Ch 2. Radicals Final Exam Review

1. $\sqrt[5]{32m^7n^{11}}$

$= \sqrt[5]{\underline{2^5}\,\underline{m^5}m^2\,\underline{n^5}\,\underline{n^5}n^1}$

$= 2mn^2\sqrt{m^2n}$

2.

$\dfrac{3+4\sqrt{c}}{5\sqrt{c}} \cdot \dfrac{\sqrt{c}}{\sqrt{c}}$ Step A

$= \dfrac{\sqrt{c}\,(3+4\sqrt{c})}{\sqrt{c}\,(5\sqrt{c})}$ Step B

$= \dfrac{3\sqrt{c}+4\sqrt{c^2}}{5\sqrt{c^2}}$ Step C

$= \dfrac{3\sqrt{c}+4c}{5c}$ Step D.

The error is in step C, $\sqrt{c} \times 4\sqrt{c}$ is $4\sqrt{c^2}$.

3. $7\sqrt{3}$

$= \sqrt{7^2\cdot 3}$

$= \sqrt{147}$.

4. $\sqrt{\dfrac{45}{96}} = \sqrt{\dfrac{3^2\cdot 5}{4^2\cdot 6}} = \dfrac{3}{4}\sqrt{\dfrac{5}{6}}$

5. $3\sqrt{294} - 2\sqrt{180} + 2\sqrt{486} + 4\sqrt{45}$

$= 3\sqrt{7^2 \cdot 6} - 2\sqrt{6^2 \cdot 5} + 2\sqrt{9^2 \cdot 6} + 4\sqrt{3^2 \cdot 5}$

$= 3 \cdot 7\sqrt{6} - 2 \cdot 6\sqrt{5} + 2 \cdot 9\sqrt{6} + 4 \cdot 3\sqrt{5}$

$= 21\sqrt{6} - \cancel{12\sqrt{5}} + 18\sqrt{6} + \cancel{12\sqrt{5}}$

$= 39\sqrt{6}$

6. $\frac{7}{2}\sqrt[3]{56} + \frac{4}{3}\sqrt[3]{108}$

$= \frac{7}{2}\sqrt[3]{2^3 \cdot 7} + \frac{4}{3}\sqrt[3]{3^3 \cdot 4}$

$= \frac{7}{\cancel{2}} \cdot \cancel{2}\sqrt[3]{7} + \frac{4}{\cancel{3}} \cdot \cancel{3}\sqrt[3]{4}$

$= 7\sqrt[3]{7} + 4\sqrt[3]{4}$

7. $4\sqrt{2}(3\sqrt{2} - 5\sqrt{10})$

$= (4 \cdot \sqrt{2})(3\sqrt{2}) - (4\sqrt{2})(5\sqrt{10})$

$= 12\sqrt{4} - 20\sqrt{20}$

$= 24 - 40\sqrt{5}$

8. $(\sqrt{11}+3)(\sqrt{11}+3)$

$=(\sqrt{11})(\sqrt{11})+(3)(\sqrt{11})+(3)(\sqrt{11})+(3)(3)$

$=11+3\sqrt{11}+3\sqrt{11}+9$

$=20+6\sqrt{11}$

9. $(4\sqrt{6}-3\sqrt{3})(\sqrt{6}+7\sqrt{3})$

$=(4\sqrt{6})(\sqrt{6})+(4\sqrt{6})(7\sqrt{3})-(3\sqrt{3})(\sqrt{6})-(3\sqrt{3})(7\sqrt{3})$

$=4\cdot 6+28\sqrt{18}-3\sqrt{18}-21\cdot 3$

$=24+25\sqrt{18}-63$

$=25\sqrt{3^2\cdot 2}-39$

$=75\sqrt{2}-39$

10. $\sqrt{3}(5-4\sqrt{3})-\sqrt{8}(4\sqrt{3}+2)$

$=\sqrt{3}(5)-(\sqrt{3})(4\sqrt{3})-(\sqrt{8})(4\sqrt{3})-(\sqrt{8})(2)$

$=5\sqrt{3}-4\cdot 3-4\sqrt{24}-2\sqrt{8}$

$=5\sqrt{3}-12-8\sqrt{6}-4\sqrt{2}$

11. $\frac{6\sqrt{3}}{5\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}}$

$= \frac{6\sqrt{45}}{5 \cdot 15}$

$= \frac{6\sqrt{3^2 \cdot 5}}{75}$

$= \frac{18\sqrt{5}}{75}$

$= \frac{6\sqrt{5}}{25}$

12. $\frac{4\sqrt{27} + 2\sqrt{3}}{\sqrt{8}} \cdot \frac{\sqrt{8}}{\sqrt{8}}$

$= \frac{(4\sqrt{27})(\sqrt{8}) + (2\sqrt{3})(\sqrt{8})}{8}$

$= \frac{4\sqrt{216} + 2\sqrt{24}}{8}$

$= \frac{4\sqrt{6^2 \cdot 6} + 2\sqrt{2^2 \cdot 6}}{8}$

$= \frac{24\sqrt{6} + 4\sqrt{6}}{8}$

$= \frac{28\sqrt{6}}{8} = \frac{7\sqrt{6}}{2}$

13. $\dfrac{(3\sqrt{5}+\sqrt{3})(4\sqrt{5}+\sqrt{3})}{(4\sqrt{5}-\sqrt{3})(4\sqrt{5}+\sqrt{3})}$

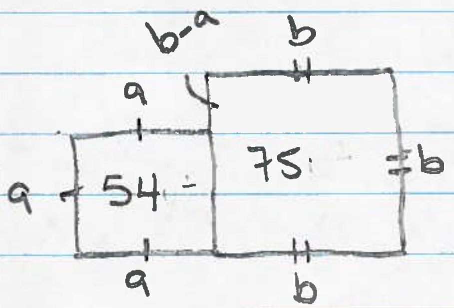
$= \dfrac{12(5)+3\sqrt{15}+4\sqrt{15}+3}{16(5)+\cancel{4\sqrt{15}}-\cancel{4\sqrt{15}}-3}$

$= \dfrac{63+7\sqrt{15}}{77}$

$= \dfrac{\cancel{7}(9)+\cancel{7}\sqrt{15}}{\cancel{7}(11)}$

$= \dfrac{9+\sqrt{15}}{11}$

14.



Let a = side of small square and b = side of large square

Since $A = l \cdot w$, the

$A_{small} = a^2$

$54 = a^2$

$a = \sqrt{54}$

$a = 3\sqrt{6}$

$A_{big} = b^2$

$75 = b^2$

$b = \sqrt{75}$

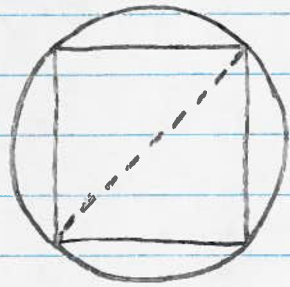
$b = 5\sqrt{3}$

Perimeter $= 3(a) + 3(b) + (b-a)$

$= 3(3\sqrt{6}) + 3(5\sqrt{3}) + (5\sqrt{3} - 3\sqrt{6})$

$= 9\sqrt{6} + 15\sqrt{3} + 5\sqrt{3} - 3\sqrt{6}$

$= 6\sqrt{6} + 20\sqrt{3}$

15. 

$A_O = 42\pi \ m^2$

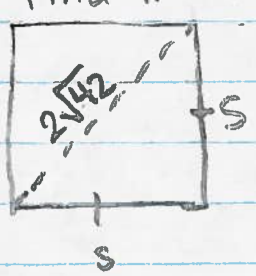
The diagonal of the square is the diameter of the circle. Find the radius using $A = \pi r^2$ and multiply it by 2 to get the diameter.

$$A = \pi r^2$$
$$\frac{42\pi}{\pi} = \frac{\pi r^2}{\pi}$$
$$r = \sqrt{42}$$
$$d = 2\sqrt{42}$$

The length of the diagonal is $2\sqrt{42}$ m.

The area of the circular portion $= A_O - A_\square$

$A_\square = lw$ → since its a square the sides are equal, use Pythagorean theorem to find the sides of the square, then find the area of the square, $A_\square$.



$$a^2 + b^2 = c^2$$
$$s^2 + s^2 = (2\sqrt{42})^2$$
$$\frac{2s^2}{2} = \frac{4 \cdot 42}{2}$$
$$\sqrt{s^2} = \sqrt{84}$$
$$s = \sqrt{84} \text{ units.}$$

$$A_\square = l \cdot w$$
$$\sqrt{84} \cdot \sqrt{84}$$
$$A_\square = 84 \text{ units}$$

$A_{circular\ portion} = A_O - A_\square$

$$(42\pi - 84) \, m^2$$

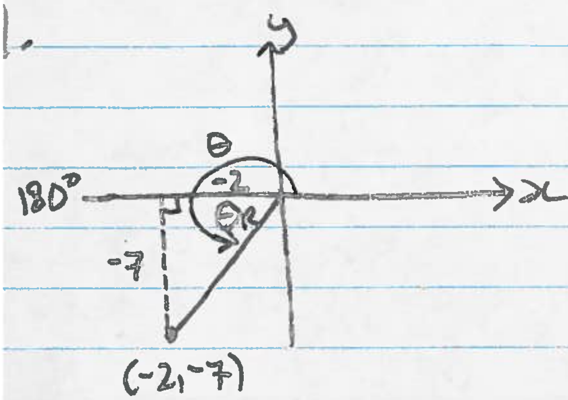
16. $\dfrac{(5\sqrt{x} + 4\sqrt{y})}{(3\sqrt{x} - 7\sqrt{y})} \cdot \dfrac{(3\sqrt{x} + 7\sqrt{y})}{(3\sqrt{x} + 7\sqrt{y})}$

$$= \frac{15x + 35\sqrt{xy} + 12\sqrt{xy} + 28y}{9x + 21\sqrt{xy} - 21\sqrt{xy} - 49y}$$

$$= \frac{15x + 47\sqrt{xy} + 28y}{9x - 49y}$$

Ch 3 Trigonometry

1.



$\tan \theta_R = \frac{opp}{adj}$

$\tan \theta_R = {}^{-7}/_{-2}$

$\theta_R = \tan^{-1}({}^{-7}/_{-2})$

$\theta_R = 74°$

$\theta = 180° + 74° = 254°$

2.

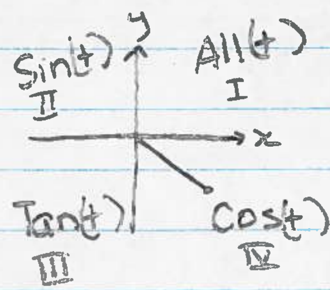
$\tan \theta = \frac{opp}{adj}$

$\tan 63° = \frac{12}{x}$

$x = \frac{12}{\tan 63°}$

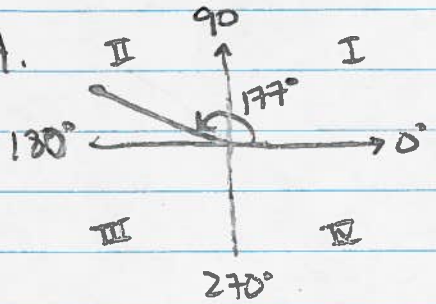
$x = 6.1\,m$

3.



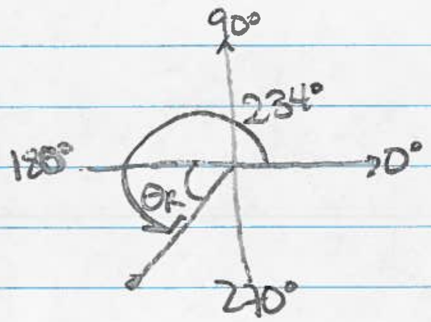
The only trig ratio that is positive (or greater than 0) in the 4th quadrant is the cosine ratio.

4.



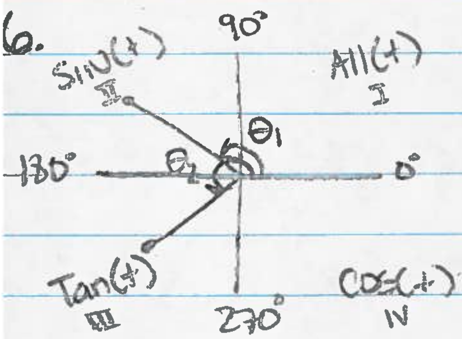
The terminal arm is in quadrant II

5.



$\theta_R = 234° - 180° = 54°$

6.

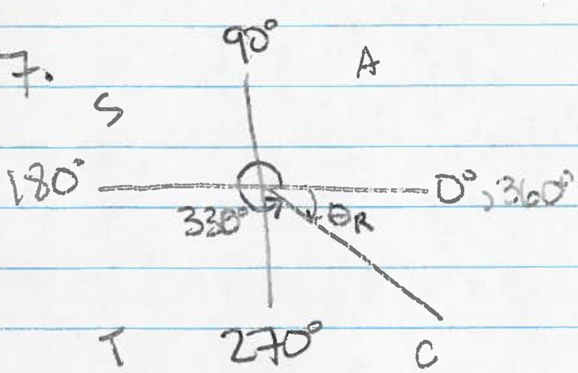


Cosine is negative in the 2nd + 3rd quadrant. Then, the terminal arm is in quadrant II or III

θ_R for $\cos\theta = \frac{-\sqrt{2}}{2}$ is 45°

$\theta_1 = 180° - 45° = 135°$
$\theta_2 = 180° + 45° = 225°$

7.



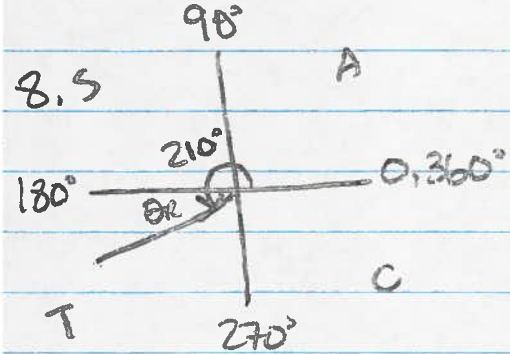
$\theta_R = 360° - 330° = 30°$

Cosine is positive in the 4th quadrant.

$\cos 30° = \frac{\sqrt{3}}{2}$

$\therefore \cos 330° = \frac{\sqrt{3}}{2}$

8.5



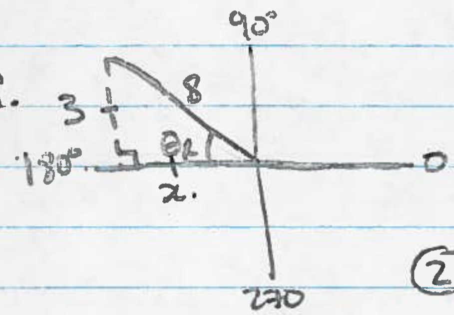
$\theta_R = 210° - 180° = 30°$

Tan is positive in the 3rd quadrant

$\tan 30° = \frac{\sqrt{3}}{3}$

$\therefore \tan 210° = \frac{\sqrt{3}}{3}$

9.



① $\sin \theta_R = \frac{3}{8}$ - opp / - hyp.

② Find side x: $a^2 + b^2 = c^2$

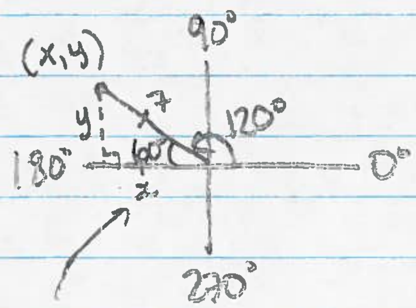
$3^2 + x^2 = 8^2$

$x^2 = 64 - 9$

$x = \sqrt{55}$

③ $\cos \theta = \frac{adj}{hyp} = \frac{\sqrt{55}}{8}$

10. (x,y)



*x is negative in the 2nd quadrant

$\cos \theta = \frac{adj}{hyp.}$

$\cos 60° = \frac{x}{7}$

$x = (\cos 60°)(7)$

$x = (\frac{1}{2})(7)$

$x = \frac{-7}{2}$

$\sin \theta = \frac{opp}{hyp}$

$\sin 60° = \frac{y}{7}$

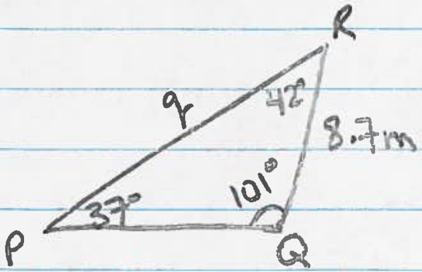
$y = \sin 60° (7)$

$y = \frac{\sqrt{3}}{2}(7)$

$y = \frac{7\sqrt{3}}{2}$

$\therefore \left(\frac{-7}{2}, \frac{7\sqrt{3}}{2}\right)$

11.

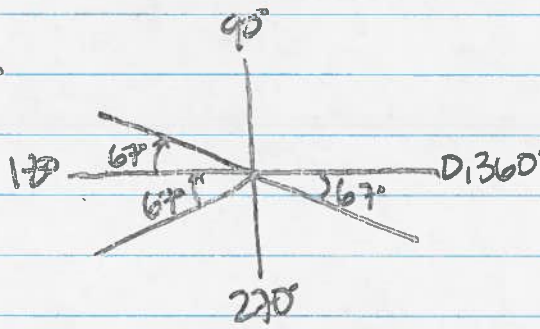


$\angle Q = 180° - 37° - 42° = 101°$

$$\frac{p}{\sin P} = \frac{q}{\sin Q}$$

$$\frac{8.7}{\sin 37°} = \frac{q}{\sin 101°}$$

12.

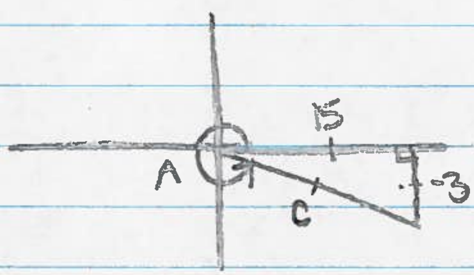


$\theta_1 = 180° - 67° = 113°$

$\theta_2 = 180° + 67° = 247°$

$\theta_3 = 360° - 67° = 293°$

13.



① Find third side of the triangle

$a^2 + b^2 = c^2$

$15^2 + (-3)^2 = c^2$

$\sqrt{234} = \sqrt{c^2}$

$c = \sqrt{234}$

$= 3\sqrt{26}$

② $\tan A = \frac{opp}{adj}$

$\tan A = \frac{-3}{15}$

③ $\sin A = \frac{opp}{hyp}$

$$\sin A = \frac{-\cancel{3}}{\cancel{3}\sqrt{26}} \cdot \frac{\sqrt{26}}{\sqrt{26}}$$

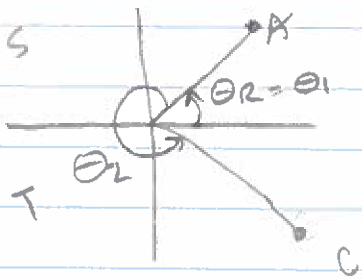
$$\sin A = \frac{-\sqrt{26}}{26}$$

③ $\cos A = \frac{adj}{hyp}$

$$= \frac{\overset{5}{\cancel{15}}}{\cancel{3}\sqrt{26}} \cdot \frac{\sqrt{26}}{\sqrt{26}}$$

$$\cos A = \frac{5\sqrt{26}}{26}$$

14. Since the cosine ratio is postive, the terminal arm of angle θ can be in quadrant I or IV



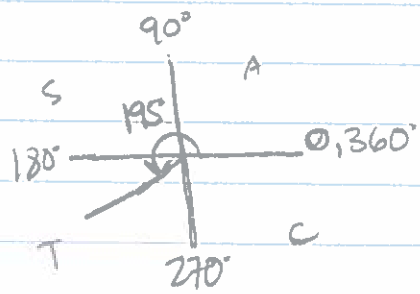
Find θ_R; $\cos\theta_R = \frac{2}{3}$

$$\theta_R = \cos^{-1}\left(\frac{2}{3}\right)$$

$$\theta_1 = \theta_R = 66^\circ$$

$$\theta_2 = 360^\circ - 66^\circ = 294^\circ$$

15.



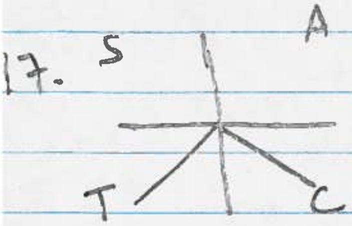
The terminal arm is in quadrant III, and sin is negative here. Sin is also negative in quadrant IV, therefore the terminal arm of the second angle is in quadrant 4.

$$\theta_R = 195^\circ - 180^\circ = 15^\circ$$

$$\theta_2 = 360^\circ - 15^\circ = 345^\circ$$

16. OMITED

17.



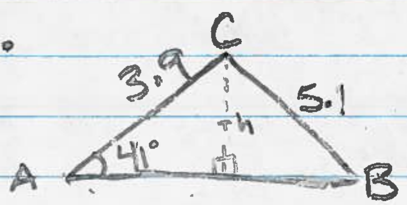
Sin is negative in quadrant III & IV

$\theta_R = \sin^{-1}\left(\frac{-4}{7}\right) = 35°$

$\theta_{III} = 180° + 35° = 215°$

$\theta_{IV} = 360° - 35° = 325°$

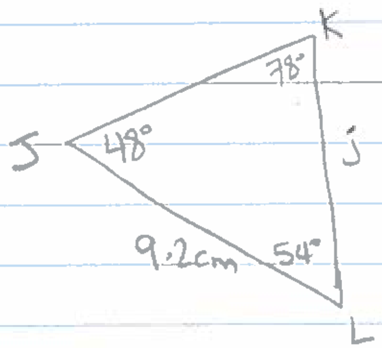
18.



See page 107 of the textbook; Key Ideas

Since $a \geq b$, there is one solution

19.



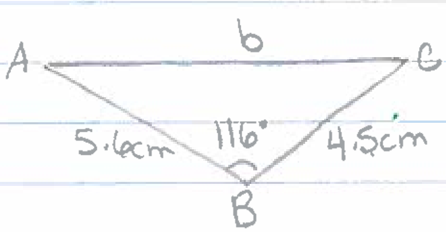
$\angle K = 180° - 48° - 54° = 78°$

$$\frac{j}{\sin 48°} = \frac{9.2}{\sin 78°}$$

$$j = \frac{9.2(\sin 48°)}{\sin 78°}$$

$j = 6.99$ cm

20.



$b^2 = a^2 + c^2 - 2ac\cos B$

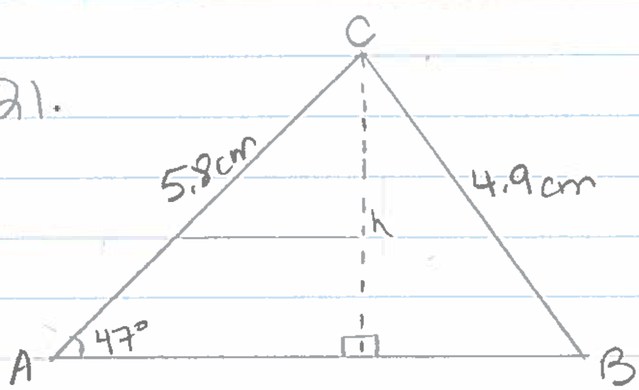
$= (4.5)^2 + (5.6)^2 - 2(4.5)(5.6)\cos 116°$

$= 20.25 + 31.36 - (-22.093\ldots)$

$\sqrt{b^2} = \sqrt{73.7039\ldots}$

$b = 8.6$ cm.

21.



Note the ambiguous case of the sine law exists.

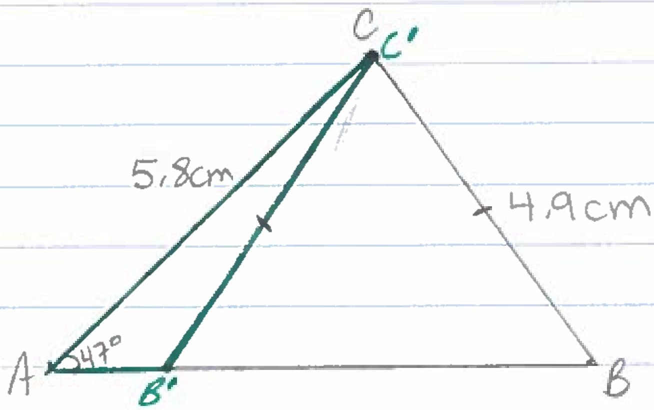
$$\sin 47° = \frac{h}{5.8}$$

$h = 4.24\ldots$

since $h < a < b$

$4.24\ldots < 4.9 < 5.8$

There are two possible triangles: $\triangle ABC$ + $\triangle AB'C'$



In triangle ABC, solve ∠B:

$$\frac{\sin 47^\circ}{4.9} = \frac{\sin B}{5.8}$$

$$\sin B = \frac{(\sin 47^\circ)(5.8)}{4.9}$$

$$\angle B = 59.9608\ldots^\circ \doteq 60^\circ$$

$$\therefore \angle C = 180^\circ - 47^\circ - 59.9608\ldots = 73.0391\ldots^\circ$$

$$\angle C \doteq 73^\circ$$

Now determine side c

$$\frac{c}{\sin 73.0391\ldots} = \frac{4.9}{\sin 47}$$

$$c = \frac{4.9(\sin 73.0391)}{\sin 47}$$

$$c = 6.4084\ldots\text{ cm} \doteq 6.4\text{ cm.}$$

Therefore in triangle ABC, $\angle B = 60^\circ$, $\angle C = 73^\circ$, $c = 6.4$ cm.

Next solve $\triangle AB'C'$

① Use $\angle B$ to determine $\angle B'$

$\angle B' = 180° - 59.9608... = 120.0391...° = 120°$

② Determine $\angle C'$

$\angle C' = 180° - \angle A - \angle B'$

$= 180° - 47° - 120.0391...°$

$= 12.9608...°$

$= 13°$

③ Determine side c'

$$\frac{c'}{\sin 12.9608...°} = \frac{4.9}{\sin 47°}$$

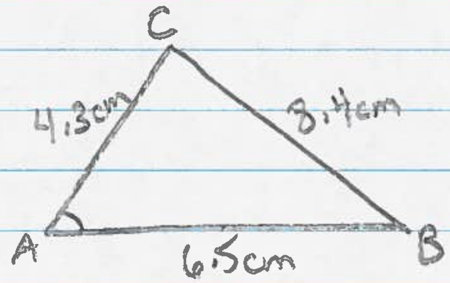
$$c = \frac{4.9(\sin 12.9608...°)}{\sin 47°}$$

$c' = 1.5026...°$

$c' = 1.5$ cm

Therefore in triangle $AB'C'$ $\angle B' = 120°$ $\angle C' = 13°$, $c = 1.5$ cm

22.



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{(4.3)^2 + (6.5)^2 - (8.4)^2}{(2)(4.3)(6.5)}$$

$$\cos A = (^-0.17567...)$$

$$A = \cos^{-1}(^-0.17567...)$$

$$A = 100°$$

Ch 4. Factoring and Radical Equations

1. $x^2+5x-24$ $\quad\quad$ $(8)(-3)=-24$

$(x+8)(x-3)$ $\quad\quad$ $8+(-3)=5$

2. $16f^2-81$ $\quad\quad$ Difference of squares

$(4f-9)(4f+9)$

3. $9x^2-30x+25$ $\quad\quad$ $ac=225$ $\quad\quad$ $(-15)(-15)=225$

$9x^2-15x \mid -15x+25$ $\quad\quad$ $b=-30$ $\quad\quad$ $(-15)+(-15)=30$

$3x(3x-5)-5x(3x-5)$

$=(3x-5)(3x-5)$

$=(3x-5)^2$

4. $36(2x+3)^2-49(y-5)^2$ $\quad\quad$ $a=(2x+3)$ $\quad\quad$ $b=(y-5)$

$36a^2-49b^2$

$(6a-7b)(6a+7b)$

$(6(2x+3)-7(y-5))(6(2x+3)+7(y-5))$

$(12x+18-7y+35)(2x+18+7y-35)$

$(12x-7y+53)(2x+7y-17)$

5. $(x)^2 = \left(\sqrt{13x-36}\right)^2$

$x^2 = 13x - 36$

$x^2 - 13x + 36 = 0$ $\qquad (-4)(-9) = 36$

$(x-4)(x-9) = 0$ $\qquad (-4) + (-9) = -13$

$x = 4 \qquad x = 9$

6. $\sqrt{2x-7} = 1$

$2x - 7 \geq 0$

$\frac{2x}{2} \geq \frac{7}{2}$

$x \geq \frac{7}{2}$

7. $-5x^2 + 45x + 180$

$-5(x^2 - 9x - 36)$ $\qquad (-12)(3) = -36$

$-5(x-12)(x+3)$ $\qquad (-12) + 3 = -9$

8. $64x^2 - 144y^2$

$16(4x^2 - 9y^2)$

$16(2x - 3y)(2x + 3y)$

9. $16(x+3)^2 - 25(x-6)^2$ $\qquad a = x+3$

$16a^2 - 25b^2$ $\qquad b = x-6$

$(4a - 5b)(4a + 5b)$

$(4(x+3) - 5(x-6))(4(x+3) + 5(x-6))$

$= (4x + 12 - 5x + 30)(4x + 12 + 5x - 30)$

$= (-x + 42)(9x - 18)$

10. $4(x-6)^2 + 40(x-6) - 156$ $\qquad a = (x-6)$

$4a^2 + 40a - 156$

$4(a^2 + 10a - 39)$ $\qquad (13)(-3) = -39$

$4(a+13)(a-3)$ $\qquad (13) + (-3) = 10$

$4((x-6)+13)((x-6)-3)$

$4(x-6+13)(x-6-3)$

$4(x+7)(x-9)$

1. $4x^4 - 31x^2 - 45 = 0$

$4(x^2)^2 - 31x^2 - 45$ $\qquad a = x^2$

$4a^2 - 31a - 45$ $\qquad (-36)(5) = -180$

$4a^2 - 36a \mid + 5a - 45$ $\qquad (-36) + (5) = -31$

$4a(a-9) + 5(a-9)$

$(4a+5)(a-9)$

$(4x^2+5)(x^2-9)$

$(4x^2+5)(x-3)(x+3)$

$4x^2 + 5 = 0 \qquad x - 3 = 0 \qquad x + 3 = 0$

$4x^2 = -5$

$\sqrt{x^2} = \sqrt{-\frac{5}{4}} \qquad x = 3 \qquad x = -3$

undefined

12. $6+\sqrt{8+x^2} = x$

$(\sqrt{8+x^2})^2 = (x-6)^2$

$8+x^2 = (x-6)(x-6)$

$8+x^2 = x^2-6x-6x+36$

$8+\cancel{x^2} = \cancel{x^2}-12x+36$

$\frac{12x}{12} = \frac{28}{12}$

$x = 7/3$

Verify: $x = 7/3$

$6+\sqrt{8+(7/3)^2} = \frac{7}{3}$

$6+\sqrt{8+49/9} = 7/3$

$6+\sqrt{121/9} = 7/3$

$6+11/3 = 7/3$

$29/3 \neq 7/3$

No Solution or the solution is extraneous

13. $-3\sqrt{x} - 27 = -15\sqrt{x}+5$

$\frac{12\sqrt{x}}{12} = \frac{32}{12}$

$(\sqrt{x})^2 = \left(\frac{8}{3}\right)^2$

$x = \frac{64}{9}$.

Verify $x = 64/9$

$-3\sqrt{64/9} - 27 = -15\sqrt{64/9}+5$

$-3(8/3) - 27 = -15(8/3)+5$

$-8-27 = -40+5$

$-35 = -35$

14. $(\sqrt{x+4})^2 = (\sqrt{7x-6})^2$

$$x+4 = 7x-6$$
$$\frac{10}{6} = \frac{6x}{6}$$
$$x = \frac{10}{6} = \frac{5}{3}$$

Verify $x = 5/3$

$$\sqrt{(5/3)+4} = \sqrt{7(5/3)-6}$$
$$2.3804.... = 2.3804....$$

15. $(\sqrt{x+9})^2 = (\sqrt{2-x}+1)^2$

$$x+9 = (\sqrt{2-x}+1)(\sqrt{2-x}+1)$$
$$x+9 = (2-x+2\sqrt{2-x}+1)$$
$$\frac{2x+6}{2} = \frac{2\sqrt{2-x}}{2}$$
$$(x+3)^2 = (\sqrt{2-x})^2$$
$$(x+3)(x+3) = 2-x$$
$$x^2+6x+9 = 2-x$$
$$x^2+7x+7 = 0$$

$a=1 \quad b=7 \quad c=7$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$= \frac{-7 \pm \sqrt{7^2-4(1)(7)}}{2(1)}$$
$$x = \frac{-7 \pm \sqrt{21}}{2}$$

Verify $x = \frac{-7 \pm \sqrt{21}}{2}$

$$\sqrt{\left(\frac{-7+\sqrt{21}}{2}\right)+9} = \sqrt{2-\left(\frac{-7+\sqrt{21}}{2}\right)}+1$$
$$2.7912... = 2.7912...$$

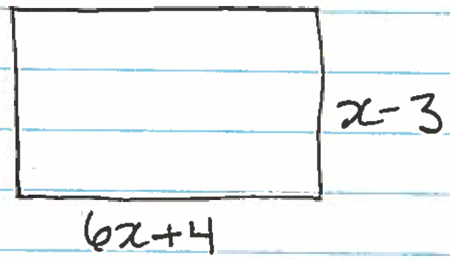
$\therefore x = \frac{-7+\sqrt{21}}{2}$ is a solution

when $x = \frac{-7-\sqrt{21}}{2}$

$$\sqrt{\left(\frac{-7-\sqrt{21}}{2}\right)+9} = \sqrt{2-\left(\frac{-7-\sqrt{21}}{2}\right)}+1$$
$$1.7912... \neq 3.7912...$$

$\therefore x = \frac{-7-\sqrt{21}}{2}$ is extraneous

16.



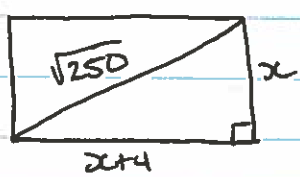
$A = 28cm^2$
$x = ?$

$$A = lw$$
$$28 = (6x+4)(x-3)$$
$$28 = 6x^2 - 18x + 4x - 12$$
$$0 = 6x^2 - 14x - 40$$
$$0 = 3x^2 - 7x - 20$$
$$0 = 3x^2 - 12x + 5x - 20$$
$$0 = 3x(x-4) + 5(x-4)$$
$$0 = (3x+5)(x-4)$$

$(-12)(5) = -60$
$(-12) + (5) = -7$

$3x + 5 = 0 \qquad x - 4 = 0$
$x = -\frac{5}{3} \qquad x = 4.$

17



$$a^2 + b^2 = c^2$$
$$(x+4)^2 + (x^2) = (\sqrt{250})^2$$
$$(x+4)(x+4) + x^2 = 250$$
$$x^2 + 8x + 16 + x^2 = 250$$
$$2x^2 + 8x - 234 = 0$$
$$x^2 + 4x - 117 = 0$$
$$x^2 + 13x - 9x - 117 = 0$$
$$x(x+13) - 9(x+13) = 0$$
$$(x-9)(x+13) = 0$$

$(13)(-9) = -117$
$13 + (-9) = 4$

$x = 9 \qquad x \neq -13$ ← cannot be negative

The dimensions are 9 cm by 13 cm.

18. $t = \sqrt{\frac{h}{4.9}}$

$$(4)^2 = \left(\sqrt{\frac{h}{4.9}}\right)^2$$

$$16 = \frac{h}{4.9}$$

It was dropped prom a height of 78.4 m.

$$h = 78.4 \text{ m}$$

Verify $h = 78.4$

$$4 = \sqrt{\frac{78.4}{4.9}}$$

$$4 = \sqrt{16}$$

$$4 = 4 \checkmark$$

Ch 5 Quadratic Functions & Equations.

1. The x-intercepts (roots)

2. $y = -7(x-5)^2 + 8$

$y = -7(0-5)^2 + 8$

$y = -167$

3. $x = -0.41 \quad x = 4.91$

4. $A = -x^2 + 40x$. complete the square to find the maximum.

$A = -1(x^2 - 40x)$

$A = -1(x^2 - 40x + 400 - 400)$

$A = -1(x^2 - 40x + 400) + 400$

$A = -1(x - 20)^2 + 400$

$\therefore$ vertex (max) at $(20, 400)$.

The width that gives the maximum area 20m.

5. The discriminat must be equal to 0.

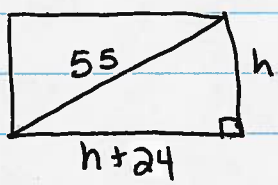
6. $y = x^2 \rightarrow y = x^2 + 5$ translation 5 units up

7. $y = x^2 \rightarrow y = (x-15)^2$ translation 15 units right.

8. False: C, the vertex of $y = x^2$ is at the origin.

9.
$$y = 4(x-9)^2 - 2$$
$$y = 4(x-9)(x-9) - 2$$
$$y = 4(x^2 - 18x + 81) - 2$$
$$y = 4x^2 - 72x + 324 - 2$$
$$y = 4x^2 - 72x + 322.$$

10. let h = height, then $h+24$ is the width



$$a^2 + b^2 = c^2$$
$$(h+24)^2 + h^2 = 55^2$$
$$(h+24)(h+24) + h^2 = 3025$$
$$h^2 + 48h + 576 + h^2 = 3025$$
$$2h^2 + 48h - 2449 = 0$$

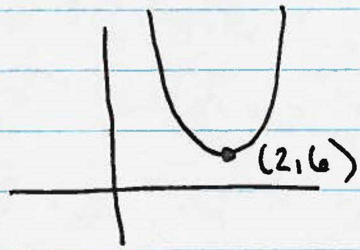
11. $h(t) = 225t - 7t^2$ Complete the square to find the maximum.
$$h(t) = -7t^2 + 225t$$
$$h(t) = -7\left[t^2 - \frac{225}{7}t + \left(\frac{225}{14}\right)^2 - \left(\frac{225}{14}\right)^2\right]$$

$$h(t) = -7\left(t^2 - \frac{225}{14}\right)^2 + 1808.035\ldots$$

vertex is $(16.071\ldots, 1808.035\ldots)$.

$\therefore$ The height is 1808.04 m

12. The vertex is at $(2,6)$ and the parabola opens up.



Domain: $x \in R$
Range: $y \geq 6$.

13. $\frac{2(x-5)^2}{2} = \frac{56}{2}$

$\sqrt{(x-5)^2} = \sqrt{28}$

$x - \cancel{5} = \pm\sqrt{28} + 5$
$\cancel{+5}$

$x = \pm\sqrt{28} + 5$
or $x = 5 \pm 2\sqrt{7}$.

14. $y = -3x^2 + 18x + 22$
$y = -3(x^2 - 6x + 9 - 9) + 22$
$y = -3(x^2 - 6x + 9) + 27 + 22$
$y = -3(x-3)^2 + 49$

15 a) $x = 2$ and $x = 6$
b) $(4,7)$
c) $x = 4$
d) $x \in R$
e) $y \leq 7$.

16. $y = x^2$ becomes $y = -2(x+4)^2 + 7$

Given $y = a(x-p)^2 + q$

a: if $|a| > 1$, the graph becomes narrower
if $0 < |a| < 1$, the graph becomes wider
$\Big\}$ a affects the y-coordinate of a point so that $(a)y$
if a is negative, the graph is reflected accross the x-axis.

p: if $p > 0$, the graph shifts to the right
if $p < 0$, the graph shifts to the left.
$\Big\}$ p will affect the x-coordinate

q: if $q > 0$, the graph shifts up
if $q < 0$, the graph shifts down
$\Big\}$ q will affect the y-coordinate.

1. Value of $a = -2$

$(-2, 4) \rightarrow [-2, 4(-2)] \rightarrow (-2, -8)$

2. value of $p = -4$

$(-2, -8) \rightarrow (-2-4, -8) \rightarrow (-6, -8)$

3. value of $q = 7$

$(-6, -8) \rightarrow (-6, -8+7) \rightarrow (-6, -1)$

The point is $(-6, -1)$.

17. The vertex is $(\overset{p}{-3},\overset{q}{1})$ and $(\overset{x}{-1},\overset{y}{3})$ is a point on the graph.

$$y = a(x-p)^2 + q$$
$$3 = a(-1-(-3))^2 + 1$$
$$2 = a(2^2)$$
$$\frac{2}{4} = \frac{4a}{4}$$
$$a = \frac{1}{2}$$

$$\therefore \quad y = \tfrac{1}{2}(x-(-3))^2 + 1$$

$$y = \tfrac{1}{2}(x+3)^2 + 1$$

18. $h = 200 + 76t - 16t^2$

a) find the vertex:

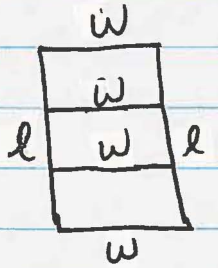
$$h = -16t^2 + 76t + 200$$
$$h = -16\left(t^2 - \frac{19}{4}t + \left(\frac{19}{8}\right)^2 - \left(\frac{19}{8}\right)^2\right) + 200$$
$$h = -16\left(t^2 - \frac{19}{4}t + \frac{361}{64}\right) + \frac{361}{4} + 200$$
$$h = -16\left(t - \frac{19}{8}\right)^2 + \frac{1161}{4}$$
$$h = -16(t - 2.375)^2 + 290.25$$

vertex is $(2.375, 290.25)$

Then it take 2.375 sec to reach the maximum height.

b. The maximum height is 290.25m

19.



$$2l + 4w = 1200$$
$$2l = 1200 - 4w$$
$$l = 600 - 2w$$

$$A = lw$$
$$A = (600 - 2w)(w)$$
$$A = 600w - 2w^2$$

Find vertex:

$$A = -2w^2 + 600w$$
$$A = -2(w^2 - 300w + 150^2 - 150^2)$$
$$A = -2(w^2 - 300w + 150^2) + 45000$$
$$A = -2(w - 150)^2 + 45000$$

vertex: (150, 45000)

$\therefore$ the width is 150m Since $l = 600 - 2w$, the length is 300m.

Let x = price increases

20. Revenue = (Price of burger)(# sold)

$$y = (1.49 + 0.05x)(3000 - 50x)$$

Graph the function using window settings:

$x: [-50, 60, 10]$
$y: [-1, 6000, 1000]$

The vertex occurs at $(15, 5040)$, so the maximum revenue occurs when there are 15 price increases

$\therefore\ 1.49 + 0.05(15) = \$2.24$

The price of the cheeseburger that will maximize revenue is \$2.24.

21. Two different roots occur when the discriminant $b^2 - 4ac > 0$

given $x^2 + 23x + k$ $\quad a = 1,\ b = 23,\ c = k$

$$b^2 - 4ac > 0$$
$$23^2 - 4(1)(k) > 0$$
$$529 - 4k > 0$$
$$-4k \cancel{>} -529$$
$$\frac{-4k}{-4} < \frac{-529}{-4}$$

$$k < 132.25$$

22. $y = 2x^2 + 63x - 300$

$a = 2 \quad b = 63 \quad c = -300$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-63 \pm \sqrt{63^2 - 4(2)(-300)}}{2(2)}$$

$$= \frac{-63 \pm \sqrt{6369}}{4}$$

23. The error occurs in second row. $9x$ should be $6x$.

$$y = 3x^2 + 18x - 7$$
$$y = 3(x^2 + 6x + 9 - 9) - 7$$
$$y = 3(x^2 + 6x + 9) - 27 - 7$$
$$y = 3(x + 3)^2 - 34.$$

Ch6 Rational Expressions and Equations

1. $\dfrac{x^2-2x+35}{3x^2-11x-20}$

$$3x^2-11x-20 \neq 0$$
$$3x^2-15x+4x-20 \neq 0$$
$$3x(x-5)+4(x-5) \neq 0$$
$$(3x+4)(x-5) \neq 0$$
$$3x+4 \neq 0 \qquad x-5 \neq 0$$
$$x \neq -\frac{4}{3} \qquad x \neq 5$$

2. $\dfrac{2x^2+5x-12}{x^2-16} = \dfrac{(2x-3)\cancel{(x+4)}}{\cancel{(x+4)}(x-4)}$ NPV $= x \neq \pm 4$

$$= \frac{2x-3}{x-4}$$

3. $\dfrac{4x}{12x^2y} \div \dfrac{8x^2}{3x} \times \dfrac{15y}{6xy} = \dfrac{\cancel{4}x}{\cancel{12}x^2y} \cdot \dfrac{\cancel{3}x}{8x^2} \cdot \dfrac{15y}{6xy}$

$$= \frac{15}{48x^3y}$$

$$= \frac{5}{16x^3y}$$

4. $$\frac{b^2+2b-8}{b^2-4} \times \frac{4b^2+3b-10}{8b^2+35b+12}$$

$$= \frac{(\cancel{b+4})(\cancel{b-2})}{(\cancel{b+2})(\cancel{b-2})} \times \frac{(4b-5)(\cancel{b+2})}{(8b+3)(\cancel{b+4})}$$

$$= \frac{4b-5}{8b+3}$$

5. $$\frac{5m}{2m-4} + 1 - \frac{3}{3m-6}$$

$$= \frac{5m}{2(m-2)} + 1 - \frac{\cancel{3}}{\cancel{3}(m-2)} \qquad LCD = 2(m-2)$$

$$= \frac{5m}{2(m-2)} + \frac{1(2)(m-2)}{(2)(m-2)} - \frac{1(2)}{(2)(m-2)}$$

$$= \frac{5m+2m-4-2}{2(m-2)}$$

$$= \frac{7m-6}{2(m-2)}$$

6. $\dfrac{5y^2+2}{x^2y^2} - \dfrac{7}{x}$ LCD: x^2y^2

$= \dfrac{5y^2+2}{x^2y^2} - \dfrac{7}{x}\cdot\dfrac{(xy^2)}{(xy^2)}$

$= \dfrac{5y^2+2-7xy^2}{x^2y^2} = \dfrac{5y^2-7xy^2+2}{x^2y^2}$

7. $\dfrac{x}{3x+4} + \dfrac{4}{2x-1}$ LCD: $(3x+4)(2x-1)$

$= \dfrac{x}{(3x+4)}\cdot\dfrac{(2x-1)}{(2x-1)} + \dfrac{4}{(2x-1)}\cdot\dfrac{(3x+4)}{(3x+4)}$

$= \dfrac{2x^2-1x+12x+16}{(3x+4)(2x-1)}$

$= \dfrac{2x^2+11x+16}{(2x-1)(3x+4)}$

8. $$\frac{\frac{t}{2}+1}{\frac{1}{4}+\frac{t+1}{t^2}} = \left(\frac{t}{2}+1\right) \div \left(\frac{1}{4}+\frac{t+1}{t^2}\right)$$

$$= \left(\frac{t}{2}+\frac{2}{2}\right) \div \left(\frac{1}{4}\cdot\frac{t^2}{t^2} + \frac{4}{4}\cdot\frac{(t+1)}{(t^2)}\right)$$

$$= \frac{t+2}{2} \div \frac{t^2+4t+4}{4t^2}$$

$$= \frac{t+2}{2} \times \frac{4t^2}{t^2+4t+4}$$

$$= \frac{4t^3+8t^2}{2(t+2)(t+2)}$$

$$= \frac{4t^2\cancel{(t+2)}}{2\cancel{(t+2)}(t+2)}$$

$$= \frac{2t^2}{t+2}$$

9. $\dfrac{3}{x+1} = \dfrac{5}{3x-1}$ NPV: $x+1 \neq 0$; $x \neq -1$
$3x-1 \neq 0$; $x \neq \dfrac{1}{3}$

$(3x-1)(3) = (5)(x+1)$
$9x-3 = 5x+5$
$\dfrac{4x}{4} = \dfrac{8}{4}$
$x = 2$

10. $\dfrac{3x}{x+1} - \dfrac{x}{x-1} = \dfrac{2x+3}{x+1}$ NPV: $x+1 \neq 0$; $x \neq -1$
$x-1 \neq 0$; $x \neq 1$

$\cancel{(x+1)}(x-1) \cdot \dfrac{3x}{\cancel{x+1}} - \dfrac{x(x+1)\cancel{(x-1)}}{\cancel{x-1}} = \dfrac{(2x+3)\cancel{(x+1)}(x-1)}{\cancel{x+1}}$ LCD: $(x+1)(x-1)$

$(x-1)(3x) - x(x+1) = (2x+3)(x-1)$
$3x^2 - 3x - x^2 - x = 2x^2 - 2x + 3x - 3$
$\cancel{2x^2} - 4x = \cancel{2x^2} + x - 3$
$\dfrac{-5x}{-5} = \dfrac{-3}{-5}$
$x = \dfrac{3}{5}$

11. $\dfrac{2}{n^2-16} + \dfrac{1}{n+1} = \dfrac{3}{n-4}$

$= \dfrac{2}{(n-4)(n+4)} + \dfrac{1}{n+1} = \dfrac{3}{n-4}$ NPVS: $n \neq 4, -4, -1$

LCD: $(n-4)(n+4)(n+1)$

$\cancel{(n-4)}\cancel{(n+4)}(n+1) \cdot \dfrac{2}{\cancel{(n-4)}\cancel{(n+4)}} + \dfrac{1}{\cancel{n+1}}(n-4)(n+4)\cancel{(n+1)} = \dfrac{3}{\cancel{n-4}}\cancel{(n-4)}(n+4)(n+1)$

$= 2(n+1) - 1(n-4)(n+4) = 3(n+4)(n+1)$

$2n + 2 - (n^2 - 4n + 4n - 16) = (3n+12)(n+1)$

$2n + 2 - n^2 + 16 = 3n^2 + 3n + 12n + 12$

$-n^2 + 2n + 18 = 3n^2 + 15n + 12$

$+n^2 - 2n - 18 = +n^2 - 2n - 18$

$0 = 4n^2 + 13n - 6$

$a = 4 \quad b = 13 \quad c = -6$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-13 \pm \sqrt{13^2 - 4(4)(-6)}}{2(4)}$

$= \dfrac{-13 \pm \sqrt{265}}{8}$

12. Organize the info in a distance - speed - time table.

13. Remember that time = $\frac{\text{distance}}{\text{speed}}$. Let x = Heidi's speed.

	Distance	÷	Speed	=	Time
Melanie	404	÷	$x+10$	=	$404/x+10$
Heidi	364	÷	x	=	$364/x$

Since it took the same amount of time to travel then

Melanie's time = Heidi's time

$$\frac{404}{x+10} = \frac{364}{x}$$

Solve for x, LCD = $x(x+10)$

$$(x)\cancel{(x+10)}\frac{404}{\cancel{x+10}} = \frac{364}{\cancel{x}}(\cancel{x})(x+10)$$

$$404x = 364x + 3640$$

$$\frac{40x}{40} = \frac{3640}{40}$$

$$x = 91.$$

Heidi's speed was 91 km/h.

13. Let x = Morgan's time, then $2x$ is Bayley's time

$$\frac{1}{\text{time taken by A}} + \frac{1}{\text{time taken by B}} = \frac{1}{\text{time taken together}}$$

$$8x\left(\frac{1}{x} + \frac{4 \cdot 1}{2x}\right) = \left(\frac{1}{\cancel{8}}\right)\cancel{8x} \qquad LCD = 8x$$

$$8 + 4 = x$$
$$12 = x$$

Then it would take Bailey 12 hrs to paint the room alone, and it would take Morgan 24 hrs to paint the room alone.

Chapter 7 Absolute Value and Reciprocal Functions

1. See Key

2. $-2x+8 \geq 0$

$$\begin{array}{r} -2x+8 \geq 0 \\ \underline{-8 \quad -8} \\ \frac{-2x}{-2} \geq \frac{-8}{-2} \\ x \leq 4 \end{array}$$

① $-2x+8, \; x \leq 4.$

② $-(-2x+8), \; x > 4$

$2x-8, \; x > 4.$

3. Find the equation of the positive part of the parabola between the x-intercepts.

vertex at $\underset{p}{(2}, \underset{q}{12)}$ point: $\underset{x}{(0}, \underset{y}{4)}$

$$y = a(x-p)^2 + q$$
$$4 = a(0-2)^2 + 12$$
$$\frac{-8}{4} = \frac{4a}{4}$$
$$a = -2$$

$y = -2(x-2)^2 + 12$ is the equation of $f(x)$.

$y = |-2(x-2)^2 + 12|$ is the absolute value equation of $f(x)$

4. Please call if you need help solving with the calculator graphically.

Algebraically:

Case 1: $-2x^2 + \cancel{5} = 6 - 5$

$$\frac{-\cancel{2}x^2}{\cancel{-2}} = \frac{1}{-2}$$

$$\sqrt{x^2} = \sqrt{-\frac{1}{2}}$$

undefined

Case 2: $-(-2x^2+5) = 6$

$$2x^2 - \cancel{5} = 6 + 5$$

$$\frac{\cancel{2}x^2}{\cancel{2}} = \frac{11}{2}$$

$$\sqrt{x^2} = \sqrt{5.5}$$

$$\boxed{x = \pm\ 2.35}$$

The graph of $y = |-2x^2+5|$ and $y=6$ intersect at $(-2.35, 6)$ and $(2.35, 6)$

5. $|3x^2 - 8| = 3$

Case 1: $3x^2 - \cancel{8} = 3 + 8$

$$\frac{\cancel{3}x^2}{3} = \frac{11}{3}$$

$$\sqrt{x^2} = \sqrt{\frac{11}{3}}$$

$$x = \pm\sqrt{\frac{11}{3}}$$

Case 2: $-(3x^2 - 8) = 3$

$$-3x^2 + \cancel{8} = 3 - 8$$

$$-3x^2 = {}^{-}5$$

$$\sqrt{x^2} = \sqrt{\frac{5}{3}}$$

$$x = \pm\sqrt{\frac{5}{3}}$$

Checking graphically, all 4 solutions are valid.

6. $y = \dfrac{1}{-7x+6}$

7. See answer key.

8. The zero of the graph occurs at $x=3$, then $x=3$ is the equation of the vertical asymptotes of the graph of the reciprocal function

9. $x-6 \neq 0$ $\therefore$ Domain: $x \neq 6$, $x \in R$
$x \neq 6$ Range: $y \neq 0$, $y \in R$.

10. The zero's of the graph occur at $x=-5$ and $x=4$, then the vertical asymptotes of the reciprocal function are at $x=-5$ and $x=4$.

11. See answer key

12. $|5^2 - 18| + |4^3 \times 6|$
$= |25 - 18| + |64 \times 6|$
$= |7| + |384|$
$= 391$

13. Invariat points occur at $y=1$ and -1.
Substitute $y=1$ and -1 into the function and solve for x.

$$1 = \frac{1}{2x-8}$$

$$2x - \cancel{8} = 1 + 8$$

$$\frac{2x}{2} = \frac{9}{2}$$

$$x = \frac{9}{2} = 4.5$$

$(4.5, 1)$

$$-1 = \frac{1}{2x-8}$$

$$-1(2x-8) = 1$$

$$-2x + \cancel{8} = 1 - 8$$

$$-2x = -7$$

$$x = \frac{7}{2} = 3.5$$

$(3.5, -1)$.

14. a) see key

b) Domain: $x \in R$

Range: $y \geq 0$, $y \in R$.

c) Determine the equation of $y = f(x)$. A point on the line is $(-4, 0)$.

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{6}{2} = 3$$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 3(x - (-4))$$

$y = 3x + 12$ then the equation of $y = |f(x)|$ is $y = |3x + 12|$

Piecewise:

① $y = 3x + 12$ $\quad x \geq -4$.

② $y = -(3x + 12)$ $\quad x < 4$

$y = -3x - 12$ $\quad x < 4$.

15. $|3x+5|-2=7 \ +2$

$|3x+5|=9$

Case 1: $3x+5=9 -5$

$\frac{3x}{3}=\frac{4}{3}$

$x = 4/3$ or 1.5

Case 2: $-(3x+5)=9$

$-3x-\cancel{5}=9+5$

$\frac{-3x}{-3}=\frac{14}{-3}$

$x=-14/3$ or $-4.\overline{6}$

Checking both solutions, they are both valid.

16. Case 1: $6x-5=-2x+3$

$8x=8$

$x=1.$

Case 2: $-(6x-5)=-2x+3$

$-6x+5=-2x+3$

$-4x=-2$

$x=\frac{1}{2}$

check both solutions, they are both valid.

17. $|2x^2-6|=12$

Case 1: $2x^2-6=12$

$2x^2=18$

$x^2=9$

$x=\pm3$

Case 2: $-(2x^2-6)=12$

$-2x^2+6=12$

$-2x^2=6$

$\sqrt{x^2}=\sqrt{-3}$

undefined

Check both solutions algebraically or graphically.. Both $x=3$ and $x=-3$ are valid solutions.

18. Case 1: $x^2+6x+8=4x+15$

$$x^2+2x-7=0$$

$a=1 \quad b=2 \quad c=-7$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-2\pm\sqrt{2^2-4(1)(-7)}}{2(1)}$$

$$=\frac{-2\pm\sqrt{32}}{2}$$

$$=\frac{-2\pm4\sqrt{2}}{2}$$

$$=-1\pm2\sqrt{2}$$

Check: $-1+2\sqrt{2}$

$$|x^2+6x+8|=4x+15$$

$$(-1+2\sqrt{2})^2+(6)(-1+2\sqrt{2})+8=4(-1+2\sqrt{2})+15$$

$22.3137\ldots \quad 22.3137\ldots$

LS = RS so $x=-1+2\sqrt{2}$ is a solution

Check: $-1-2\sqrt{2}$

$$|(-1-2\sqrt{2})^2+6(-1-2\sqrt{2})+8|=4(-1-2\sqrt{2})+15$$

$$|-0.313708\ldots|=-0.3137$$

$$=0.31370\ldots\neq-0.313$$

LS $\neq$ RS so $x=-1-2\sqrt{2}$ is NOT a solution

Case 2: $-(x^2+6x+8) = 4x+15$

$$-x^2-6x-8 = 4x+15$$

$$0 = x^2+10x+23$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{10^2-4(1)(23)}}{2(1)}$$

$$= \frac{-10 \pm \sqrt{8}}{2}$$

$$= \frac{-10 \pm 2\sqrt{2}}{2}$$

$$= -5 \pm \sqrt{2}$$

Check: $-5+\sqrt{2}$

$$|x^2+6x+8| = 4x+15$$

$$|(-5+\sqrt{2})^2+6(-5+\sqrt{2})+8| = 4(-5+\sqrt{2})+15$$

$$|-0.6568\ldots| = 0.6568\ldots$$

$$0.6568\ldots = 0.6568\ldots$$

LS = RS so $x = -5+\sqrt{2}$ is a solution

Check: $-5-\sqrt{2}$

$|x^2+6x+8| = 4x+15$

$|(-5-\sqrt{2})^2+6(-5-\sqrt{2})+8| = 4(-5-\sqrt{2})+15$

$|10.6568...| = -10.6568...$

$10.6568... \neq -10.6568.$

The LS $\neq$ RS so $x=-5-\sqrt{2}$ is not a solution.

$\therefore$ $x=-1+2\sqrt{2}$ and $x=-5+\sqrt{2}$

19. See key.

Ch 8 Linear and Quadratic Systems of Equations and Inequalities

1. The graph is above the x-axis between $x = -2$ and $x = 4$
$\therefore \quad -2 \le x \le 4.$

2. $5x + 3y > 7$

A: $(-2, -2)$ NO

$5(-2) + 3(-2) > 7$
$-16 \not> 7$ False statement.

B. $(1, -4)$ NO
$5(1) + 3(-4) > 7$
$-7 \not> 7$ False statement

C. $(2, 4)$ YES
$5(2) + 3(4) > 7$
$17 > 7$ True Statement.

D. $(2, -6)$ NO
$5(2) + 3(-6) > 7$
$-8 \not> 7$ False statement

3. $30x + 60y \le 40(60)$

$30x + 60y \le 2400$

4. $m = \frac{\text{rise}}{\text{run}} = \frac{2}{5}$ point$(2,0)$ Find the equation of the line.

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{2}{5}(x - 2)$

$y = \frac{2x}{5} - \frac{4}{5}$.

Then the equation of the inequality is.

$(5)y \ge \frac{2x-4}{\cancel{5}}(\cancel{5})$

$5y \ge 2x - 4$.

5. They slope intercept form of the line is

$\frac{4x + 3y}{3} \le \frac{12 - 4x}{3}$

$y \le 4 - \frac{4}{3}x$ $\therefore$ y-intercept is at 4 and the line has a negative slope of $-\frac{4}{3}$. The line and the area below the line is included.

$\therefore$ B.

6. $y > x^2 - 4$

A. $2 > (-3)^2 - 4$
$2 \ngtr 5$ False statement.

B. $-4 > (2)^2 - 4$
$-4 \ngtr 0$ False statement

C. $5 > (-4)^2 - 4$
$5 \ngtr 12$ False statement

D. $5 > (1)^2 - 4$ True $\therefore$ $(1, 5)$
$5 > -3$

7. From the inequality $y > -0.5(x+1)^2 - 2$, the vertex is at $(-1, -2)$, and the area above the graph is included $\therefore$ B.

8. $y_1 = 2x^2 - 8$ $(-0.91, -6.33)$
$y_2 = \dfrac{(-9+4x)}{2}$ $(1.91, -0.67)$

9. $50x + 25y \geq 900$

10. $\leq$ or $\geq$

11. $5x^2 + 18x - 8 = 0$

$(5x-2)(x+4) = 0$

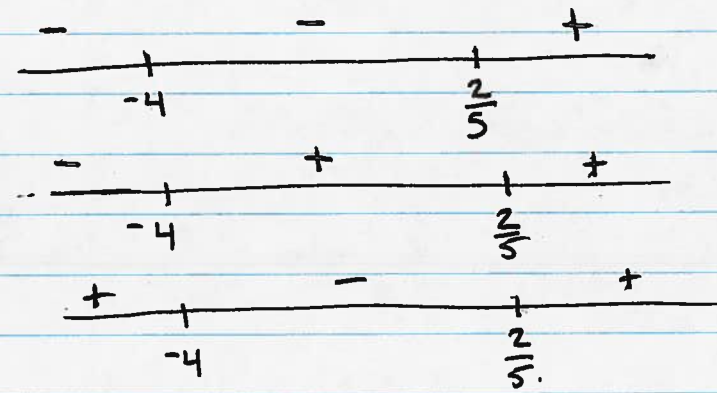
$5x - 2 = 0 \qquad x + 4 = 0$

$x = \frac{2}{5} \qquad x = -4.$

$(5x-2)$

$(x+4)$

$(5x-2)(x+4)$



$(5x-2)(x+4) > 0$ in the intervals $x < -4$ and $x > \frac{2}{5}$

12. $4x + 8y < 24$

$4(p) + 8(-4) < 24$

$4p - 32 < 24$

$4p < 56$

$p < 14.$

13. $y < (x-3)^2 - 2$

vertex: $(3, -2)$, opens up.

x-int: $0 = (x-3)^2 - 2$

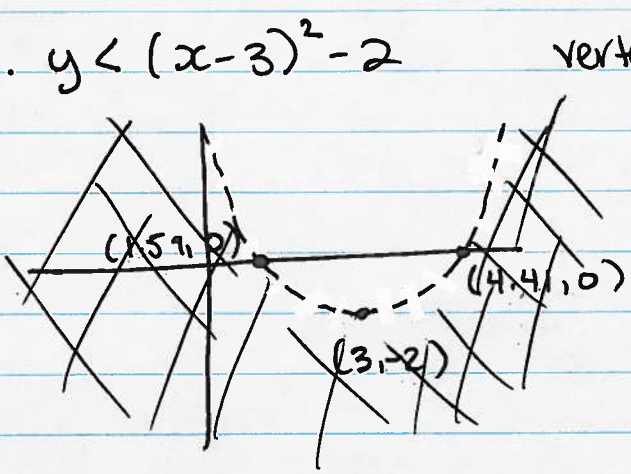
$\sqrt{2} = \sqrt{(x-3)^2}$

$\pm\sqrt{2} = x - 3$

$\pm\sqrt{2} + 3 = 0$

zeros @ $x = 4.41$ and 1.59.

shaded below, dotted line



b) any point below the parabola will satisfy the inequality ie) (0,0), (-2,-5) ...

14. Dertermine the equation of the parabola:
vetex(1,3) point. (0,4)

$y = a(x-p)^2 + q$

$4 = a(0-1)^2 + 3$

$1 = 1a$

$a = 1$

$\therefore\ y = (x-1)^2 + 3$ is the equation of the parabola

Since its shaded above a dotted curve then

$y > (x-1)^2 + 3$.

15. $2x + 7 = x^2 + 3x - 49$

$0 = x^2 + x - 56$

$0 = (x-7)(x+8)$

$x = 7$ or $x = 8$

when $x = 7$

$y = 2(7) + 7$

$y = 21$

$\therefore\ (7,21)$

when $x = -8$

$y = 2(-8) + 7$

$y = -9$

$\therefore\ (-8,-9)$

16. $4x-(-4x^2-4x+20)=12$

$4x+4x^2-4x+20=12$

$4x^2+8x-32=0$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x=^-4 \qquad x=2$

when $x=-4$

$4(-4)-y=12$

$y=^-28$

when $x=2$

$4(2)-y=12$

$y=-4$

$\therefore \quad (-4,^-28) \qquad (2,-4)$

17. $3x^2-5=-4x^2+1$

$7x^2=6$

$\sqrt{x^2}=\sqrt{\frac{6}{7}}$

$x=\pm\sqrt{\frac{6}{7}}$

$\doteq 0.925.$

when $x=\pm\sqrt{\frac{6}{7}}$

$y=3\left(\pm\sqrt{\frac{6}{7}}\right)^2-5$

$y=3\left(\frac{6}{7}\right)-5$

$y=\frac{-17}{7}\doteq-2.428$

$\therefore \quad (-0.925\ldots,-2.428.)$

$(0.925\ldots,-2.428\ldots)$

18. $(x-3)^2 = -2x^2 - 15x + 39$

$x^2 - 6x + 9 = -2x^2 - 15x + 39$

$3x^2 + 9x - 30 = 0$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x = -5 \qquad x = 2$

when $x = -5$

$y = (-5-3)^2$

$y = 64$

when $x = 2$

$y = (2-3)^2$

$y = 1$

$\therefore (-5, 64) \qquad (2, 1).$

19. $-16t^2 + 177t + 4 = 80t + 150$

$0 = 16t^2 - 97t + 146$

$$x = \frac{97 \pm \sqrt{(-97)^2 - 4(16)(146)}}{2(16)}$$

$x = 3.2831\ldots$ or $2.779\ldots$

It first hits the balloon at $t = 2.78s$.

$h = 80(2.779\ldots) + 150$

$h = 372.3\,m$

$\therefore (2.78, 372.3).$

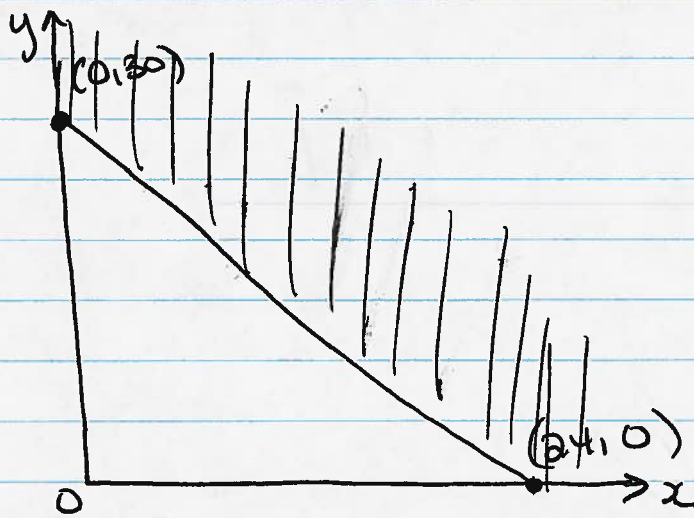
a) 2.78 seconds

b) 372.3 metres

20. Let x = number of shoes sold
y = number of ties sold

then $25x + 20y \geq 600$

b)



$x = 0$
$\cancel{25(0)} + 20y = 600$
$y = 30$
$(0,30)$

$y = 0$
$25x + \cancel{20(0)} = 600$
$x = 24.$

c). let z = # of shoes + z = # of ties

$25z + 20z \geq 600$
$45z \geq 600$
$z \geq 13.3\ldots$

They would need to sell at least 14 shoes and 14 ties.