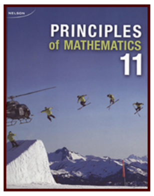
Strengthening and Conditioning

Lesson 1.1: Introduction to Radicals



Refer to *Principles of Mathematics 11* pages 176 – 181 and 184 – 187 for more examples.

- Page 182, #1a, b, d, and e, 5a, and 5b, 9, 10b, 11a, 13, 20
- Page 188, #2a, and 2b, 4a, 4b, 5b, 6c, 7

Questions 1a, b, d, and e, page 182

a. Given statement: the principal square root of 25 is –5.

The statement is false. Both $(+5) \times (+5) = 25$ and $(-5) \times (-5) = 25$. However, by definition, the principal square root is the positive possibility.

b. Given statement: $\sqrt{16} = \pm 4$

The statement is false. The square root sign alone indicates that the principal square root should be taken. It must be specified if the negative solution is to be considered. For example, $\pm\sqrt{16} = \pm 4$ is correct because it indicates that both the positive and negative values are required.

d. Given statement: $\sqrt{-4} = -2$
The statement is false. The square root of a negative number does not exist in the Real Number system.

e. Given statement: $-\sqrt{4} = -2$
The statement is true. To have a negative sign in front of the square root sign means to multiply the square root by –1.
$-\sqrt{4} = (-1) \times \sqrt{4} = (-1) \times 2 = -2$

Questions 5a, and 5b page 182

a. $\sqrt{4000}$
$= \sqrt{400 \times 10}$
$= \sqrt{20^2 \times 10}$
$= \sqrt{20^2} \times \sqrt{10}$
$= 20\sqrt{10}$

b. $-\sqrt{2835}$
$=-\sqrt{81 \times 35}$
$=-\sqrt{9^2 \times 35}$
$=-\sqrt{9^2} \times \sqrt{35}$
$=-9\sqrt{35}$

Question 9, page 182

The mistake starts at the step when he states $\sqrt{(-4)(-4)}$. It is true that (–4)(–4) = 16, but the negative is 'lost' in the process of squaring. Here is the correct process:

$-\sqrt{16}$
$= (-1)(\sqrt{16})$
$= (-1)(\sqrt{4^2})$
$= (-1)(4)$
$=-4$

Question 10b, page 182

b. The radical $\sqrt{41000}$ is an Irrational Number. To estimate $\sqrt{41000}$, consider a value close to 41000 that is a perfect square (has a Whole Number square root).

$\sqrt{40000} = 200$

Next, determine sequentially the square(s) of Whole Numbers larger than 200.

$201^2 = 40\,401$
$202^2 = 40\,804$
$203^2 = 41\,209$

Because 41 000 is between 40 804 and 41 209, $\sqrt{41000}$ must be between 202 and 203.

Now using some logical thinking, 41 000 is about half way between 40 804 and 41 209, so 41 000 ≐ 202.5.

Using a calculator, the approximate value, to the nearest hundredth is $\sqrt{41000} = 202.48$.

Question 11a, page 183

a. $6\sqrt{5}$
$= \sqrt{6^2} \times \sqrt{5}$
$= \sqrt{36} \times \sqrt{5}$
$= \sqrt{36 \times 5}$
$= \sqrt{180}$

Question 13, page 183

$2\sqrt{4}$ and $5\sqrt[3]{4}$ **cannot** be added and then simplified in radical form because they are not like radicals. Two criteria that must be met to be like radicals. First, the index must be the same. The index of $2\sqrt{4}$ is 2 (square root) and the index of $5\sqrt[3]{4}$ is 3 (cube root). The second criteria is that the radicand must be the same. In this case, the radicand is the same as it is 4 in both radicals. However, because the first criteria is not met, the two radicals are not like radicals.

Question 20, page 183

a. The statement $(\sqrt{x})^4 = x^2$ is true.

Examine the left side of the equation separately. Expand the expression $(\sqrt{x})^4$.

Left Side	Right Side
$(\sqrt{x})^4$	x^2
$(\sqrt{x})(\sqrt{x})(\sqrt{x})(\sqrt{x})$	
$(\sqrt{x})^2(\sqrt{x})^2$	
$(x)(x)$	
x^2	x^2

Left side = Right Side

It has been shown that $(\sqrt{x})^4 = x^2$. However, $x \geq 0, x \in \mathrm{R}$.

b. The statement $(\sqrt{x})^3 = x\sqrt{x}$ is true.

Examine the left side of the equation separately. Expand the expression $(\sqrt{x})^3$.

Left Side	Right Side
$(\sqrt{x})^3$	$x\sqrt{x}$
$(\sqrt{x})(\sqrt{x})(\sqrt{x})$	
$(\sqrt{x})^2(\sqrt{x})$	
$x\sqrt{x}$	$x\sqrt{x}$

Left side = Right Side

It has been shown that $(\sqrt{x})^3 = x\sqrt{x}$. However, $x \geq 0, x \in R$.

Questions 2a and 2b, page 188

a. $4\sqrt{6} + 3\sqrt{6} + 2\sqrt{6}$
$= (4 + 3 + 2)\sqrt{6}$
$= 9\sqrt{6}$

b. $15\sqrt{3} - 3\sqrt{3} - 8\sqrt{3}$
$= (15 - 3 - 8)\sqrt{3}$
$= 4\sqrt{3}$

Questions 4a and 4b, page 188

a. $\sqrt{8} - \sqrt{32} + \sqrt{512}$
$= \sqrt{4 \times 2} - \sqrt{16 \times 2} + \sqrt{256 \times 2}$
$= \sqrt{2^2 \times 2} - \sqrt{4^2 \times 2} + \sqrt{16^2 \times 2}$
$= \sqrt{2^2} \times \sqrt{2} - \sqrt{4^2} \times \sqrt{2} + \sqrt{16^2} \times \sqrt{2}$
$= 2\sqrt{2} - 4\sqrt{2} + 16\sqrt{2}$
$= (2 - 4 + 16)\sqrt{2}$
$= 14\sqrt{2}$

b. $-\sqrt{27}+\sqrt{75}-\sqrt{12}$
$=-\sqrt{9\times 3}+\sqrt{25\times 3}-\sqrt{4\times 3}$
$=-\sqrt{3^2\times 3}+\sqrt{5^2\times 3}-\sqrt{2^2\times 3}$
$=-\sqrt{3^2}\times\sqrt{3}+\sqrt{5^2}\times\sqrt{3}-\sqrt{2^2}\times\sqrt{3}$
$=-3\sqrt{3}+5\sqrt{3}-2\sqrt{3}$
$=(-3+5-2)\sqrt{3}$
$=0\sqrt{3}$
$=0$

Question 5b, page 188

b. $7\sqrt{3}+2\sqrt{45}+\sqrt{108}$
$=7\sqrt{3}+2\sqrt{9\times 5}+\sqrt{36\times 3}$
$=7\sqrt{3}+2\sqrt{3^2\times 5}+\sqrt{6^2\times 3}$
$=7\sqrt{3}+(2\times\sqrt{3^2}\times\sqrt{5})+\sqrt{6^2}\times\sqrt{3}$
$=7\sqrt{3}+(2\times 3\times\sqrt{5})+6\sqrt{3}$
$=7\sqrt{3}+6\sqrt{5}+6\sqrt{3}$
$=7\sqrt{3}+6\sqrt{3}+6\sqrt{5}$
$=13\sqrt{3}+6\sqrt{5}$

Question 6c, page 188

c. $5\sqrt{32}-7\sqrt{2}-\sqrt{484}$
$=5\sqrt{16\times 2}-7\sqrt{2}-\sqrt{22^2}$
$=5\sqrt{4^2\times 2}-7\sqrt{2}-22$
$=5\times\sqrt{4^2}\times\sqrt{2}-7\sqrt{2}-22$
$=5\times 4\sqrt{2}-7\sqrt{2}-22$
$=20\sqrt{2}-7\sqrt{2}-22$
$=(20-7)\sqrt{2}-22$
$=13\sqrt{2}-22$

Question 7, page 188

To determine the length of ribbon needed, the perimeter of the cushion is needed. Perimeter = sum of side lengths

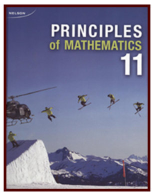
a. $P = \sqrt{63} + \sqrt{50} + \sqrt{72}$
$P = \sqrt{9 \times 7} + \sqrt{25 \times 2} + \sqrt{36 \times 2}$
$P = \sqrt{3^2 \times 7} + \sqrt{5^2 \times 2} + \sqrt{6^2 \times 2}$
$P = \sqrt{3^2} \times \sqrt{7} + \sqrt{5^2} \times \sqrt{2} + \sqrt{6^2} \times \sqrt{2}$
$P = 3\sqrt{7} + 5\sqrt{2} + 6\sqrt{2}$
$P = 3\sqrt{7} + 11\sqrt{2}$

The amount of ribbon needed is $(3\sqrt{7} + 11\sqrt{2})$ cm.

b. To determine the length to the nearest tenth of a centimetre use a calculator.

$P = 7.937... + 15.556...$
$P = 23.493...$
$P = 23.5$
$P = 23.5$ cm

Lesson 1.2: Radicals and Operations



Refer to *Principles of Mathematics 11* pages 191 – 197 and 204 – 210 for more examples.

- Page 198, #1a–d, 5a–f, 8, 13a, 13b, 17a, 19a
- Page 212, #3a–c, 4d, 6d, 10b, 15

Questions 1a to d, page 198

a. $\sqrt{5} \times \sqrt{6}$
$= \sqrt{5 \times 6}$
$= \sqrt{30}$