Question 7, page 188

To determine the length of ribbon needed, the perimeter of the cushion is needed. Perimeter = sum of side lengths

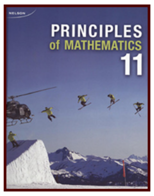
a. $P = \sqrt{63} + \sqrt{50} + \sqrt{72}$
$P = \sqrt{9 \times 7} + \sqrt{25 \times 2} + \sqrt{36 \times 2}$
$P = \sqrt{3^2 \times 7} + \sqrt{5^2 \times 2} + \sqrt{6^2 \times 2}$
$P = \sqrt{3^2} \times \sqrt{7} + \sqrt{5^2} \times \sqrt{2} + \sqrt{6^2} \times \sqrt{2}$
$P = 3\sqrt{7} + 5\sqrt{2} + 6\sqrt{2}$
$P = 3\sqrt{7} + 11\sqrt{2}$

The amount of ribbon needed is $(3\sqrt{7} + 11\sqrt{2})$ cm.

b. To determine the length to the nearest tenth of a centimetre use a calculator.

$P = 7.937\ldots + 15.556\ldots$
$P = 23.493\ldots$
$P = 23.5$
$P = 23.5$ cm

Lesson 1.2: Radicals and Operations



Refer to *Principles of Mathematics 11* pages 191 – 197 and 204 – 210 for more examples.

- Page 198, #1a–d, 5a–f, 8, 13a, 13b, 17a, 19a
- Page 212, #3a–c, 4d, 6d, 10b, 15

Questions 1a to d, page 198

a. $\sqrt{5} \times \sqrt{6}$
$= \sqrt{5 \times 6}$
$= \sqrt{30}$

b. $\sqrt{12} \times \sqrt{20}$
$= \sqrt{4 \times 3} \times \sqrt{4 \times 5}$
$= \sqrt{2^2 \times 3} \times \sqrt{2^2 \times 5}$
$= 2\sqrt{3} \times 2\sqrt{5}$
$= (2 \times 2)\sqrt{3 \times 5}$
$= 4\sqrt{15}$

c. $2\sqrt{3} \times \sqrt{24}$
$= 2\sqrt{3} \times \sqrt{4 \times 6}$
$= 2\sqrt{3} \times \sqrt{2^2 \times 6}$
$= 2\sqrt{3} \times 2\sqrt{6}$
$= (2 \times 2)\sqrt{3 \times 6}$
$= 4\sqrt{18}$
$= 4\sqrt{9 \times 2}$
$= 4\sqrt{3^2 \times 2}$
$= 4 \times 3\sqrt{2}$
$= 12\sqrt{2}$

d. $7\sqrt{32} \times 2\sqrt{48}$
$= (7\sqrt{16 \times 2}) \times (2\sqrt{16 \times 3})$
$= (7\sqrt{4^2 \times 2}) \times (2\sqrt{4^2 \times 3})$
$= (7 \times 4\sqrt{2}) \times (2 \times 4\sqrt{3})$
$= (28\sqrt{2}) \times (8\sqrt{3})$
$= (28 \times 8)\sqrt{2 \times 3}$
$= 224\sqrt{6}$

Questions 5a to 5f, page 198

a. $7(3+\sqrt{12})$
$=(7\times 3)+(7\times\sqrt{12})$
$=21+7\sqrt{12}$
$=21+7\sqrt{4\times 3}$
$=21+7\sqrt{2^2\times 3}$
$=21+7\times 2\sqrt{3}$
$=21+14\sqrt{3}$

b. $\sqrt{5}(4-\sqrt{10})$
$=(\sqrt{5}\times 4)-(\sqrt{5}\times\sqrt{10})$
$=4\sqrt{5}-\sqrt{50}$
$=4\sqrt{5}-\sqrt{25\times 2}$
$=4\sqrt{5}-\sqrt{5^2\times 2}$
$=4\sqrt{5}-5\sqrt{2}$

c. $\sqrt{6}(\sqrt{10}-8\sqrt{3})$
$=(\sqrt{6}\times\sqrt{10})-(\sqrt{6}\times 8\sqrt{3})$
$=\sqrt{60}-8\sqrt{18}$
$=\sqrt{4\times 15}-8\sqrt{9\times 2}$
$=\sqrt{2^2\times 15}-8\sqrt{3^2\times 2}$
$=2\sqrt{15}-(8\times 3)\sqrt{2}$
$=2\sqrt{15}-24\sqrt{2}$

d. $2\sqrt{3}(\sqrt{18}+5\sqrt{2})$
$=(2\sqrt{3}\times\sqrt{18})+(2\sqrt{3}\times 5\sqrt{2})$
$=((2\times 1)\sqrt{3\times 18})+((2\times 5)\sqrt{3\times 2})$
$=2\sqrt{54}+10\sqrt{6}$
$=2\sqrt{9\times 6}+10\sqrt{6}$
$=2\sqrt{3^2\times 6}+10\sqrt{6}$
$=2\times 3\sqrt{6}+10\sqrt{6}$
$=6\sqrt{6}+10\sqrt{6}$
$=16\sqrt{6}$

e. $(6+\sqrt{6})(5+\sqrt{10})$ **[HINT: FOIL]**

$= (6\times 5)+(6\times\sqrt{10})+(\sqrt{6}\times 5)+(\sqrt{6}\times\sqrt{10})$
$= (30)+(6\sqrt{10})+(5\sqrt{6})+(\sqrt{60})$
$= 30 + 6\sqrt{10} + 5\sqrt{6} + (\sqrt{4\times 15})$
$= 30 + 6\sqrt{10} + 5\sqrt{6} + 2\sqrt{15}$

f. $(2\sqrt{3}-5\sqrt{8})^2$ **[Hint: FOIL]**

$= (2\sqrt{3}-5\sqrt{8})(2\sqrt{3}-5\sqrt{8})$
$= (2\sqrt{3}\times 2\sqrt{3})+(2\sqrt{3}\times(-5\sqrt{8}))+((-5\sqrt{8})\times 2\sqrt{3})+((-5\sqrt{8})\times(-5\sqrt{8}))$
$= (2\times 2\sqrt{3\times 3})+(2\times(-5)\sqrt{3\times 8})+((-5)\times 2\sqrt{8\times 3})+((-5)\times(-5)\sqrt{8\times 8})$
$= (4\sqrt{9})+((-10)\sqrt{24})+((-10)\sqrt{24})+(25\sqrt{64})$
$= (4\sqrt{3^2})+(-10\sqrt{24})+(-10\sqrt{24})+(25\sqrt{8^2})$
$= (4\times 3)+(-20\sqrt{24})+(25\times 8)$
$= 12+(-20\sqrt{24})+200$
$= 212+(-20\sqrt{4\times 6})$
$= 212+(-20\sqrt{2^2\times 6})$
$= 212+(-20\times 2\sqrt{6})$
$= 212-40\sqrt{6}$

Question 8, page 198

Area of rectangle = length × width

$A = 4\sqrt{6}\times 3\sqrt{2}$
$A = 4\times 3\sqrt{6\times 2}$
$A = 12\sqrt{12}$
$A = 12\sqrt{4\times 3}$
$A = 12\sqrt{2^2\times 3}$
$A = 12\times 2\sqrt{3}$
$A = 24\sqrt{3}$

Area of triangle $= \frac{1}{2}(\text{base}\times\text{height})$

$A = \frac{1}{2}(4\sqrt{3}\times 2\sqrt{2})$
$A = \frac{1}{2}(4\times 2\sqrt{3\times 2})$
$A = \frac{1}{2}(8\sqrt{6})$
$A = \left(\frac{1}{2}\times 8\right)\sqrt{6}$
$A = 4\sqrt{6}$

Compare $24\sqrt{3}$ and $4\sqrt{6}$. The $\sqrt{3}$ is about 1.7 and $\sqrt{6}$ is about 2.5 using estimation skills. As such, $24\sqrt{3}$ is approximately 40 and $4\sqrt{6}$ is approximately 10.

The area of the rectangle is larger.

Questions 13a and 13b, page 199

a. $\frac{\sqrt{7}}{\sqrt{2}}$

$= \frac{\sqrt{7}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$= \frac{\sqrt{7 \times 2}}{\sqrt{2 \times 2}}$

$= \frac{\sqrt{14}}{\sqrt{4}}$

$= \frac{\sqrt{14}}{2}$

b. $\frac{-1}{4\sqrt{5}}$

$= \frac{-1}{4\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right)$

$= \frac{-1\sqrt{5}}{4\sqrt{5 \times 5}}$

$= \frac{-\sqrt{5}}{4\sqrt{25}}$

$= \frac{-\sqrt{5}}{4 \times 5}$

$= \frac{-\sqrt{5}}{20}$

Question 17a, page 200

a. $$\begin{aligned} S &= \sqrt{\frac{E}{d}} \\ &= \sqrt{\frac{4000}{0.25}} \\ &= \frac{\sqrt{4000}}{\sqrt{0.25}} \\ &= \frac{\sqrt{4000}}{\sqrt{0.25}}\left(\frac{\sqrt{0.25}}{\sqrt{0.25}}\right) \\ &= \frac{\sqrt{4000 \times 0.25}}{\sqrt{0.25 \times 0.25}} \\ &= \frac{\sqrt{1000}}{0.25} \\ &= \frac{\sqrt{100 \times 10}}{0.25} \\ &= \frac{10\sqrt{10}}{0.25} \\ &= \left(\frac{10}{0.25}\right)\sqrt{10} \\ &= 40\sqrt{10} \end{aligned}$$

The speed of sound is $40\sqrt{10}$ m/s.

Question 19a, page 200

$$\begin{aligned}
\text{Area} &= \text{Length} \bullet \text{Width}\\
24\sqrt{3} &= 4\sqrt{6} \bullet w\\
\frac{24\sqrt{3}}{4\sqrt{6}} &= \frac{4\sqrt{6}}{4\sqrt{6}} \bullet w\\
\frac{24\sqrt{3}}{4\sqrt{6}} &= w\\
\left(\frac{24}{4}\right)\frac{\sqrt{3}}{\sqrt{6}} &= w\\
\frac{6\sqrt{3}}{\sqrt{6}} &= w\\
\frac{6\sqrt{3}}{\sqrt{6}}\left(\frac{\sqrt{6}}{\sqrt{6}}\right) &= w\\
\frac{6\sqrt{3 \bullet 6}}{\sqrt{6 \bullet 6}} &= w\\
\frac{6\sqrt{18}}{\sqrt{36}} &= w\\
\frac{6\sqrt{18}}{6} &= w\\
\left(\frac{6}{6}\right)\sqrt{18} &= w\\
\sqrt{9 \bullet 2} &= w\\
\sqrt{3^2 \bullet 2} &= w\\
3\sqrt{2} &= w
\end{aligned}$$

The width is $3\sqrt{2}$ m.

Questions 3a to 3c, page 212

a. $15\sqrt{2x}-7\sqrt{2x}-\sqrt{2x}$

$2x \geq 0$

$\frac{2}{2}x \geq \frac{0}{2}$

$x \geq 0, x \in \text{R}$

Notice that the radicands are all the same. Because they are like radicals, the three terms can be combined by adding and subtracting the coefficients in front of the radicals.

$$15\sqrt{2x}-7\sqrt{2x}-\sqrt{2x}$$
$$=(15-7-1)\sqrt{2x}$$
$$=7\sqrt{2x},\ x \geq 0, x \in \text{R}$$

b. $36\sqrt{3x^3}-10\sqrt{3x^3}+28\sqrt{3x^3}$

$3x^3 \geq 0$

$\frac{3}{3}x^3 \geq \frac{0}{3}$

$x^3 \geq 0$

$\sqrt[3]{x^3} \geq \sqrt[3]{0}$

$x \geq 0, x \in \text{R}$

Combine like radicals.

$$36\sqrt{3x^3}-10\sqrt{3x^3}+28\sqrt{3x^3}$$
$$=(36-10+28)\sqrt{3x^3}$$
$$=54\sqrt{3x^3}$$

Simplify further by looking for perfect square powers of x.

$$=54\sqrt{3x^3}$$
$$=54\sqrt{3x^2 \bullet x}$$
$$=54\sqrt{x^2 \bullet 3x}$$
$$=54\sqrt{x^2}\bullet\sqrt{3x}$$
$$=54x\sqrt{3x},\ x \geq 0, x \in \text{R}$$

c. $(5\sqrt{x})(2\sqrt{x})$
$x \geq 0, x \in \mathrm{R}$

$$\begin{aligned}
&(5\sqrt{x})(2\sqrt{x}) \\
&= (5 \bullet 2)(\sqrt{x} \bullet \sqrt{x}) \\
&= 10(\sqrt{x \bullet x}) \\
&= 10(\sqrt{x^2}) \\
&= 10x, x \geq 0, x \in \mathrm{R}
\end{aligned}$$

Question 4d, page 212

There is an x under a square root sign and an x in the denominator so $x > 0, x \in \mathrm{R}$.

$$\begin{aligned}
&\frac{(3\sqrt{x})(4\sqrt{x^3})}{6\sqrt{x^4}} \\
&= \frac{(3 \bullet 4)(\sqrt{x} \bullet \sqrt{x^3})}{6\sqrt{x^4}} \\
&= \frac{12(\sqrt{x \bullet x^3})}{6\sqrt{x^4}} \\
&= \frac{12\sqrt{x^4}}{6\sqrt{x^4}} \\
&= \left(\frac{12}{6}\right)\left(\sqrt{\frac{x^4}{x^4}}\right) \\
&= 2\sqrt{1} \\
&= 2, x > 0, x \in \mathrm{R}
\end{aligned}$$

Question 6d, page 212

d. $\dfrac{4\sqrt{x^3}}{\sqrt{8x}}$

$x^3 \geq 0$

$\sqrt[3]{x^3} \geq \sqrt[3]{0}$

$x \geq 0, x \in R$

Because there is an x value in the denominator, x cannot be zero either. So, $x \neq 0$. Therefore, the restriction is $x > 0, x \in R$.

$$\frac{4\sqrt{x^3}}{\sqrt{8x}}$$
$$= \frac{4\sqrt{x^2 \bullet x}}{\sqrt{4 \bullet 2x}}$$
$$= \frac{4\sqrt{x^2 \bullet x}}{\sqrt{2^2 \bullet 2x}}$$
$$= \frac{4 \bullet x\sqrt{x}}{2\sqrt{2x}}$$
$$= \left(\frac{4}{2}\right)\left(\frac{x\sqrt{x}}{\sqrt{2} \bullet \sqrt{x}}\right)$$
$$= (2)\left(\frac{x}{\sqrt{2}}\right)\left(\frac{\sqrt{x}}{\sqrt{x}}\right)$$
$$= \left(\frac{2x}{\sqrt{2}}\right)\left(\sqrt{\frac{x}{x}}\right)$$
$$= \left(\frac{2x}{\sqrt{2}}\right)(\sqrt{1})$$
$$= \left(\frac{2x}{\sqrt{2}}\right)$$
$$= \frac{2x}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$ Rationalize the denominator.
$$= \frac{2x \bullet \sqrt{2}}{\sqrt{2} \bullet \sqrt{2}}$$
$$= \frac{2x\sqrt{2}}{\sqrt{4}}$$
$$= \frac{2x\sqrt{2}}{2}$$
$$= x\sqrt{2}, x > 0, x \in R$$

Question 10b, page 213

b. $\dfrac{\sqrt{8x^3}}{\sqrt{2x}}$

$x > 0, x \in \mathrm{R}$

$$
\begin{aligned}
&= \frac{\sqrt{8x^3}}{\sqrt{2x}} \\
&= \sqrt{\left(\frac{8}{2}\right)\left(\frac{x^3}{x}\right)} \\
&= \sqrt{(4)(x^{3-1})} \\
&= \sqrt{4} \bullet \sqrt{x^2} \\
&= 2x,\ x > 0,\ x \in R
\end{aligned}
$$

Question 15, page 213

$\dfrac{(3\sqrt{2} - 2x)(3\sqrt{2} + 2x)}{2\sqrt{x}}$

$x > 0, x \in \mathrm{R}$

$$
\begin{aligned}
&\frac{(3\sqrt{2} - 2x)(3\sqrt{2} + 2x)}{2\sqrt{x}} \\
&= \frac{(3\sqrt{2} \bullet 3\sqrt{2}) + (3\sqrt{2} \bullet 2x) + ((-2x) \bullet 3\sqrt{2}) + ((-2x) \bullet 2x)}{2\sqrt{x}} \\
&= \frac{(9\sqrt{4}) + (6x\sqrt{2}) + (-6x\sqrt{2}) + (-4x^2)}{2\sqrt{x}} \\
&= \frac{(9 \bullet 2) - 4x^2}{2\sqrt{x}} \\
&= \frac{18 - 4x^2}{2\sqrt{x}} \\
&= \left(\frac{18}{2\sqrt{x}}\right) - \left(\frac{4x^2}{2\sqrt{x}}\right) \\
&= \left(\frac{9}{\sqrt{x}}\right) - \left(\frac{2x^2}{\sqrt{x}}\right) \\
&= \left(\frac{9}{\sqrt{x}}\left(\frac{\sqrt{x}}{\sqrt{x}}\right)\right) - \left(\frac{2x^2}{\sqrt{x}}\left(\frac{\sqrt{x}}{\sqrt{x}}\right)\right) \\
&= \left(\frac{9\sqrt{x}}{x}\right) - \left(\frac{2x^2\sqrt{x}}{x}\right) \\
&= \frac{9\sqrt{x} - 2x^2\sqrt{x}}{x},\ x > 0,\ x \in \mathrm{R}
\end{aligned}
$$