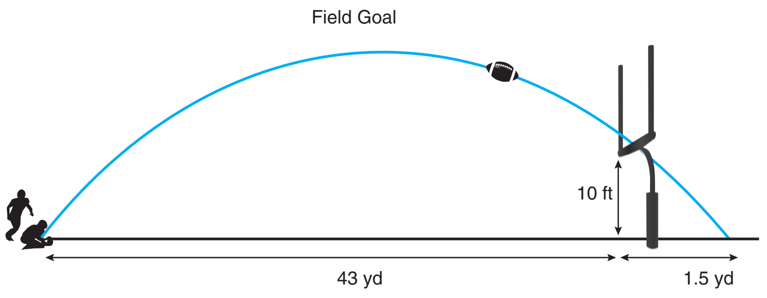
Practice Run

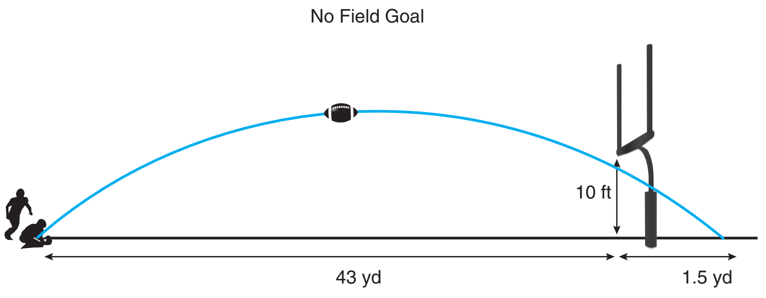
To score a 3-point field goal in the game of football, the place kicker must kick the ball between the uprights and above the 10-foot high cross-bar, which are located in the end zone of the football field.

A ball is positioned 43 yards from the end zone, where the place kicker will attempt to kick a field goal.

The flight of the ball is parabolic, the maximum height reached by the ball is 23 yards above the turf, and the ball lands 1.5 yards behind the uprights.

Will the football clear the cross-bar and be a successful field goal?





Compare your answers.

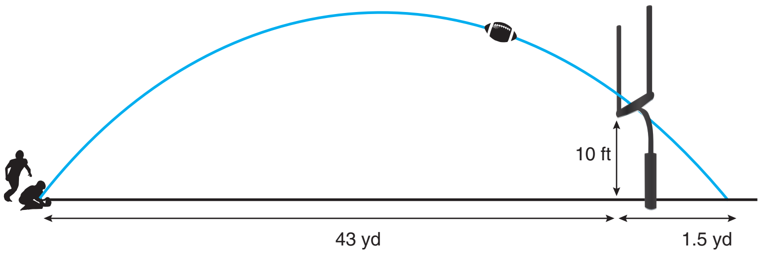
To score a 3-point field goal in the game of football, the place kicker must kick the ball between the uprights and above the 10-foot high cross-bar, which are located in the end zone of the football field.

A ball is positioned 43 yards from the end zone, where the place kicker will attempt to kick a field goal.

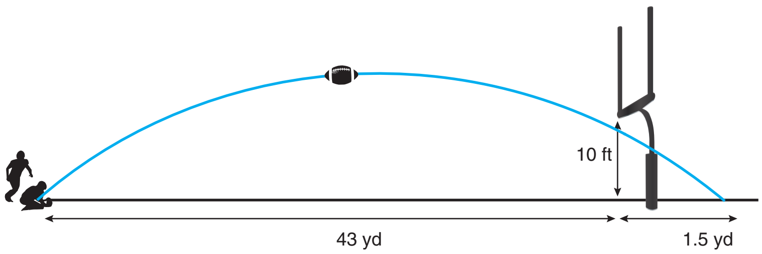
The flight of the ball is parabolic, the maximum height reached by the ball is 23 yards above the turf, and the ball lands 1.5 yards behind the uprights.

Will the football clear the cross-bar and be a successful field goal?

Field Goal



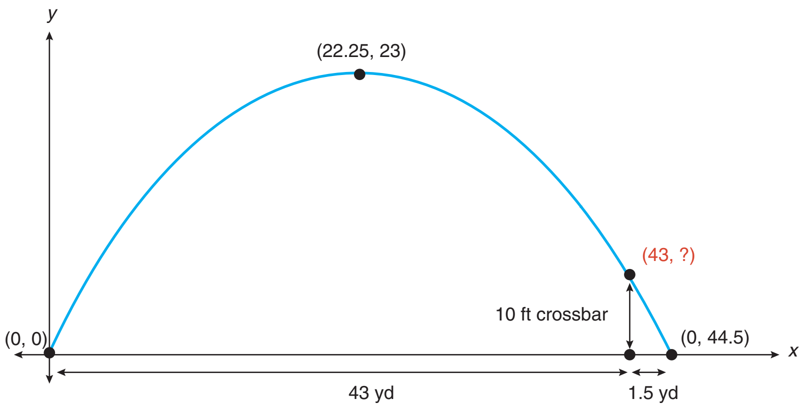
No Field Goal



Step 1: Organize the information that is given.

Mark the point where the football starts as (0,0) and the point where the football lands as (44.5, 0).

The vertex of the quadratic function is (22.25, 23) because the maximum occurs at the half-way mark.



Step 2: Select a form that works with the given information.

Because the vertex is known, the vertex form of a quadratic function would be a good form to work with.

$h = 22.25$ and $k = 23$

$$f(x) = a(x - h)^2 + k$$
$$f(x) = a(x - 22.25)^2 + 23$$

Step 3: Determine the value of a.

Substitute a known point (other than the vertex, since you used it already) into the equation of the function and solve for a. The point (0, 0) can be used.

$$\begin{aligned} f(x) &= a(x - 22.25)^2 + 23 \\ 0 &= a(0 - 22.25)^2 + 23 \\ -23 &= a(-22.25)^2 + 23 - 23 \\ -23 &= 495.0625a \\ -23 \div 495.0625 &= 495.0625a \div 495.0625 \\ -0.0464587805... &= a \\ -0.046 &\doteq a \end{aligned}$$

Step 4: Write the equation of the quadratic function in vertex form.

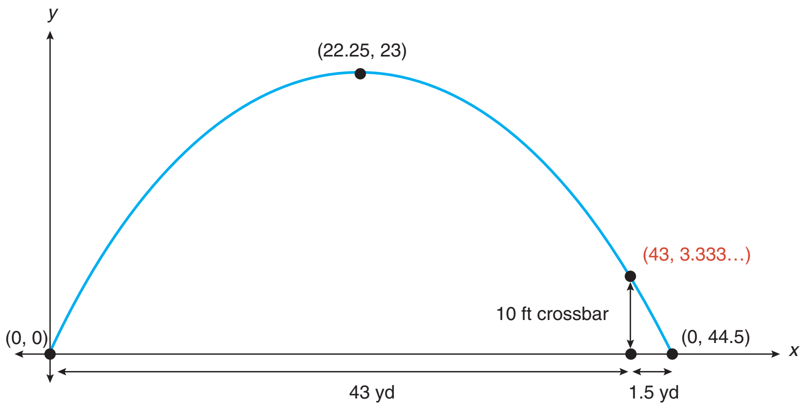
$$f(x) \doteq -0.046(x - 22.25)^2 + 23$$

Step 5: Use this function to solve the problem.

The height of the cross-bar between the uprights is 10 ft. So, 43 yds from the point labelled (0,0), the football must be at least 10 ft in the air in order to be deemed a field goal.

Note: in order to work with this function $f(x) \doteq -0.046(x-22.25)^2+23$, all measurements must be in yards (since the vertex used to the define the function corresponds to a length and height in yards). As such, we must convert the height of the 10 foot-high cross-bar to yards.

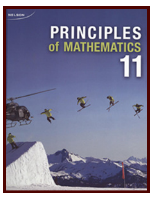
$$\frac{3\text{ ft}}{1\text{ yd}} = \frac{10\text{ ft}}{x}$$
$$3\text{ ft} \cdot x = 10\text{ ft} \cdot 1\text{ yd}$$
$$x = \frac{10\text{ ft} \cdot 1\text{ yd}}{3\text{ ft}}$$
$$x = 3.33\ldots\text{yards}$$



Determine the value of the function (height of the ball) when $x = 43$ yds.

$$f(x) \doteq -0.046(x-22.25)^2+23$$
$$f(43) \doteq -0.046(43-22.25)^2+23$$
$$f(43) \doteq -0.046(20.75)^2+23$$
$$f(43) \doteq -0.046(430.5625)+23$$
$$f(43) \doteq -19.81+23$$
$$f(43) \doteq 3.19\text{ yds}$$

Step 6: In conclusion, the football player was unsuccessful in scoring the 3 points. The vertical height of the ball at $x = 43$ yds is 3.19 yds, which is not over the required 3.33 yards.



Please refer to page 343, Example 4, and page 357, Example 2, of *Principles of Mathematics 11* for more examples of applications of quadratic functions.