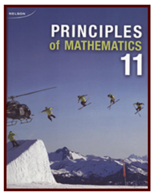
Lesson 2.2: Quadratic Functions and Factors

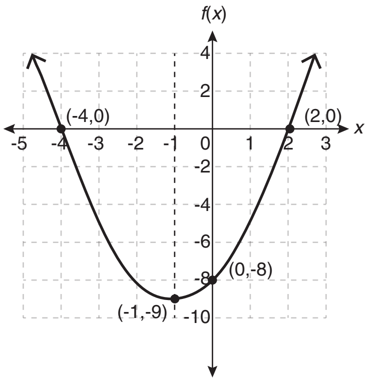


Refer to *Principles of Mathematics 11* for the following practice questions.

- page 346, # 2 and 7
- page 363, # 2, 5, 11a, 11b, and 12
- page 377, # 4

Questions 2, page 346

a. $f(x) = (x+4)(x-2)$

<table>
<tr><td>i) $x + 4 = 0$ $x - 2 = 0$
$x_{intercept} = -4$ $x_{intercept} = 2$</td><td rowspan="4">v) $a > 0$

</td></tr>
<tr><td>ii) $y_{intercept} (x = 0)$
$y = (0+4)(0-2)$
$y = (4)(-2)$
$y_{intercept} = -8$</td></tr>
<tr><td>iii) $x = \frac{-4+2}{2} = \frac{-2}{2}$
$x = -1$</td></tr>
<tr><td>iv) $f(-1) = (-1+4)(-1-2)$
$f(-1) = (3)(-3)$
$f(-1) = -9$
$(-1, -9)$</td></tr>
</table>

b) $g(x) = -2x(x-3)$

i) $-2x = 0$ $\qquad$ $x - 3 = 0$

$\frac{-2}{-2}x = \frac{0}{-2}$

$x = 0$ $\qquad$ $x = 3$

$x_{\text{intercept}} = 0$ $\qquad$ $x_{\text{intercept}} = 3$

ii) $y_{\text{intercept}} (x = 0)$

$y = -2(0)(0-3)$

$y = (0)(-3)$

$y = 0$

$y_{\text{intercept}} = 0$

iii) $x = \frac{0+3}{2} = \frac{3}{2}$

$x = 1.5$

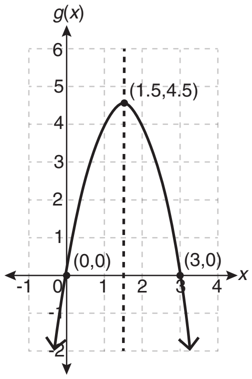
iv) $g(1.5) = -2(1.5)(1.5-3)$

$g(1.5) = (-3)(-1.5)$

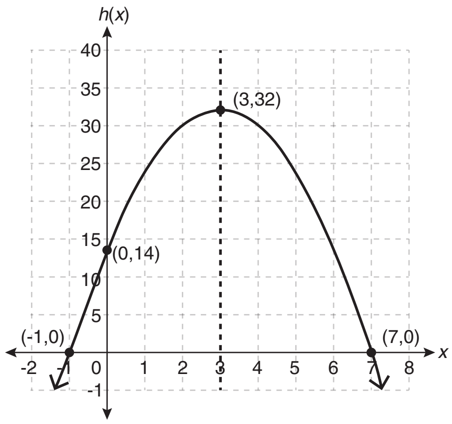
$g(1.5) = 4.5$

$(1.5, 4.5)$

v) $a < 0$



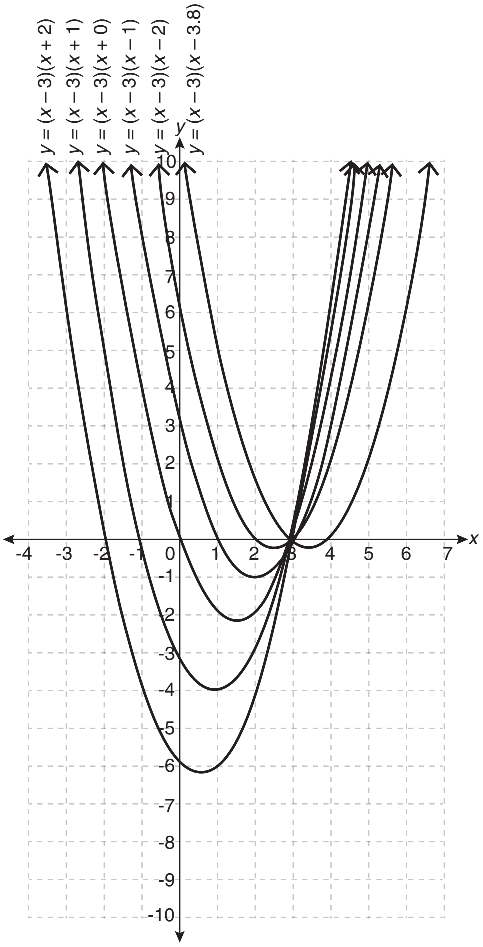
c) $h(x) = -2(x+1)(x-7)$

i) $x + 1 = 0$ $\quad x - 7 = 0$ $x = -1$ $\quad x = 7$ $x_{intercept} = -1$ $\quad x_{intercept} = 7$	v) $a < 0$ 
ii) $y_{intercept}(x = 0)$ $y = -2(0+1)(0-7)$ $y = -2(1)(-7)$ $y = 14$ $y_{intercept} = 14$	
iii) $x = \frac{-1+7}{2} = \frac{6}{2}$ $x = 3$	
iv) $h(3) = -2(3+1)(3-7)$ $h(3) = -2(4)(-4)$ $h(3) = 32$ $(3, 32)$	

Question 7, page 347

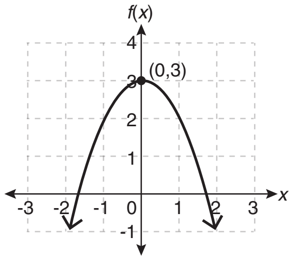
$y = (x - 3)(x + s)$

Changing the *s* value changes only one *x*-intercept value and changes the *y*-intercept.

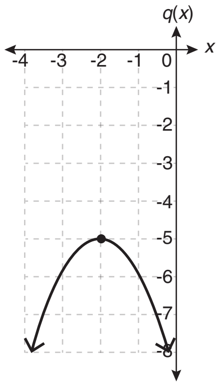


Question 2, page 363

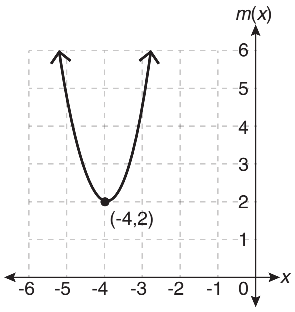
a. Maximum because $a < 0$. The vertex is (0,3) (using (h,k) from the vertex form of the function). Since the vertex has a positive y-value (and thus it lies above the x-axis) and the parabola opens down, the graph of the function will have two x-intercepts.



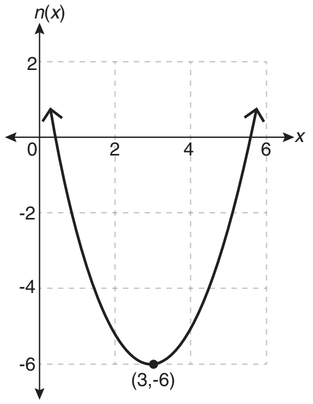
b. Maximum because $a < 0$. The vertex is (–2,–5) (using (h,k) from the vertex form of the function). Since the vertex has a negative y-value (and thus it lies below the x-axis) and the parabola opens down, the graph of the function will have no x-intercepts.



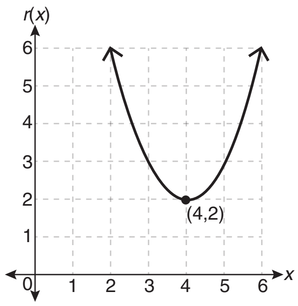
c. Minimum because $a > 0$. The vertex is (–4,2) (using (h,k) from the vertex form of the function). Since the vertex has a positive y-value (and thus it lies above the x-axis) and the parabola opens up, the graph of the function will have no x-intercepts.



d. Minimum because $a > 0$. The vertex is (3,–6) (using (h,k) from the vertex form of the function). Since the vertex has a negative y-value (and thus it lies below the x-axis) and the parabola opens up, the graph of the function will have two x-intercepts.



e. Minimum because $a > 0$. The vertex is (4,2) (using (h,k) from the vertex form of the function). Since the vertex has a positive y-value (and thus it lies above the x-axis) and the parabola opens up, the graph of the function will have no x-intercepts.



Question 5, p. 364

a. Matches iv): The vertex is (3,0) using (h,k) from the function and $a > 0$, so the parabola opens up.

b. Matches iii): The vertex is (–4,–2) using (h,k) from the function and $a < 0$, so the parabola opens down.

c. Matches i): The vertex is (0,–3) using (h,k) from the function and $a < 0$, so the parabola opens down.

d. Matches ii): The vertex is (4,2) using (h,k) from the function and $a > 0$, so the parabola opens up.

Question 11a and b, page 365

a. Use $y = a(x-h)^2 + k$ so, $(h,k) = (0,36)$. Using the vertex and a point on the curve, find the a-value. One of the x-intercepts is $(12,0)$.

$$0 = a(12-0)^2 + 36$$
$$0 = 144a + 36$$
$$-36 = 144a$$
$$\frac{-36}{144} = \frac{144a}{144}$$
$$-\frac{1}{4} = a$$

Substitute a and the coordinates of the vertex into $y = a(x-h)^2 + k$.

$$y = -\frac{1}{4}x^2 + 36$$

b. Use $y = a(x-h)^2 + k$, so $(h,k) = (3,2)$. Using the vertex and a point on the curve, find the a-value. One of the x-intercepts is $(0,0)$.

$$0 = a(0-3)^2 + 2$$
$$0 = 9a + 2$$
$$-2 = 9a$$
$$\frac{-2}{9} = \frac{9a}{9}$$
$$-\frac{2}{9} = a$$

Substitute a and the coordinates of the vertex into $y = a(x-h)^2 + k$.

$$y = -\frac{2}{9}(x-3)^2 + 2$$

Question 12, page 366

a. $(h,k) = (4,-12)$

$$y = a(x-4)^2 - 12$$

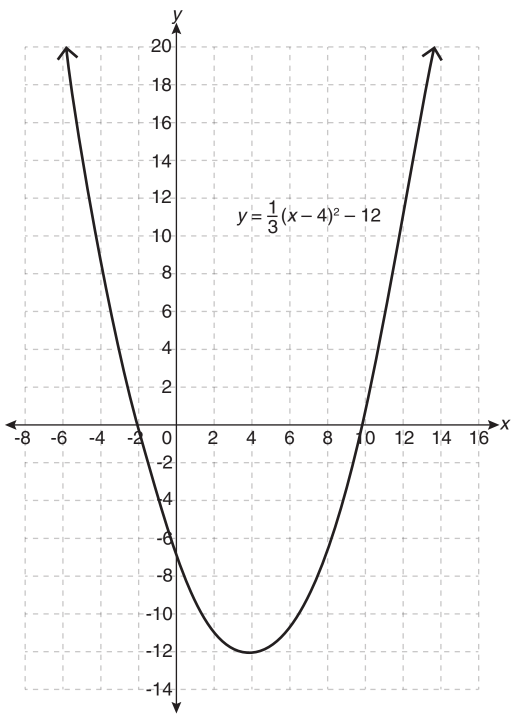
b. $(h,k)=(4,-12)$ and $(x,y)=(13,15)$

$$15 = a(13-4)^2 - 12$$
$$15 = a(9)^2 - 12$$
$$15 = 81a - 12$$
$$27 = 81a$$
$$\frac{27}{81} = \frac{81a}{81}$$
$$\frac{1}{3} = a$$

$$y = \frac{1}{3}(x-4)^2 - 12$$

c. Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \geq -12, y \in R\}$

d.



Question 4, page 377

$f(x) = a(x-3)(x-7)$

Find *a*.	**Find the vertex.**	**Function in vertex form:**
$f(x) = a(x-3)(x-7)$ $(x,y) = (6,3)$ $3 = a(6-3)(6-7)$ $3 = a(3)(-1)$ $3 = -3a$ $\frac{3}{-3} = \frac{-3a}{-3}$ $-1 = a$	The x-intercepts are: $x = 3$ and $x = 7$ The equation of the axis of symmetry is: $x = \frac{3+7}{2} = \frac{10}{2} = 5$ $x = 5$ The coordinates of the vertex are: $f(5) = -1(x-3)(x-7)$ $f(5) = -1(5-3)(5-7)$ $f(5) = -1(2)(-2)$ $f(5) = 4$ $(5,4)$	$a = -1$ $(h,k) = (5,4)$ $y = a(x-h)^2 + k$ $y = -1(x-5)^2 + 4$ Although the value of a was determined from a different form of the function, a in vertex form is the same as the a from the factored form determined in the first column.