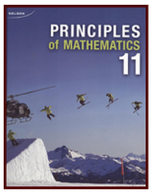
Lesson 2.3: Solving Quadratic Equations



Refer to *Principles of Mathematics 11* for the following practice questions.

- Page 411 # 3, 4, 6, 7, and 9
- Page 420 # 6, 9, and 11

Question 3, page 411

a. $x^2 - 9x - 70 = 0$

Sum of $-9 = 5 + (-14)$ Product of $-70 = (5)(-14)$	LS	RS	LS	RS
$x^2 - 9x - 70 = 0$	$(-5)^2 - 9(-5) - 70$	0	$(14)^2 - 9(14) - 70$	0
$(x + 5)(x - 14) = 0$	$25 + 45 - 70$		$196 - 126 - 70$	
$x + 5 = 0 \quad x - 14 = 0$	$70 - 70$		$70 - 70$	
$x = -5 \quad x = 14$	0		0	
	LS = RS		LS = RS	

b. $x^2 + 19x + 48 = 0$

Sum of $19 = 16 + 3$ Product of $48 = (16)(3)$	LS	RS	LS	RS
$x^2 + 19x + 48 = 0$	$(-16)^2 + 19(-16) + 48$	0	$(-3)^2 + 19(-3) + 48$	0
$(x + 16)(x + 3) = 0$	$256 - 304 + 48$		$9 - 57 + 48$	
$x + 16 = 0 \quad x + 3 = 0$	$-48 + 48$		$-48 + 48$	
$x = -16 \quad x = -3$	0		0	
	LS = RS		LS = RS	

c. $3a^2 + 11a - 4 = 0$

Sum of $11 = 12 + (-1)$ Product of $-12 = (12)(-1)$	LS	RS	LS	RS
$3a^2 + 11a - 4 = 0$	$3\left(\frac{1}{3}\right)^2 + 11\left(\frac{1}{3}\right) - 4$	0	$3(-4)^2 + 11(-4) - 4$	0
$3a^2 - a + 12a - 4 = 0$	$\frac{3}{9} + \left(\frac{11}{3}\right) - 4$		$48 - 44 - 4$	
$(3a^2 - a) + (12a - 4) = 0$	$\frac{1}{3} + \frac{11}{3} - \frac{12}{3}$		$48 - 48$	
$a(3a - 1) + 4(3a - 1) = 0$	0		0	
$(3a - 1)(a + 4) = 0$				
$3a - 1 = 0 \quad a + 4 = 0$				
$a = \frac{1}{3} \quad a = -4$	LS = RS		LS = RS	

d. $6t^2 - 7t - 20 = 0$

Sum of $-7 = -15 + 8$ Product of $-120 = (-15)(8)$	LS	RS	LS	RS
$6t^2 - 7t - 20 = 0$	$6\left(\frac{5}{2}\right)^2 - 7\left(\frac{5}{2}\right) - 20$	0	$6\left(-\frac{4}{3}\right)^2 - 7\left(-\frac{4}{3}\right) - 20$	0
$6t^2 - 15t + 8t - 20 = 0$	$6\left(\frac{25}{4}\right) - \frac{35}{2} - 20$		$6\left(\frac{16}{9}\right) + \frac{28}{3} - 20$	
$(6t^2 - 15t) + (8t - 20) = 0$	$\frac{150}{4} - \frac{70}{4} - 20$		$\frac{96}{9} + \frac{84}{9} - 20$	
$3t(2t - 5) + 4(2t - 5) = 0$	$\frac{80}{4} - 20$		$\frac{180}{9} - 20$	
$(2t - 5)(3t + 4) = 0$	$20 - 20$		$20 - 20$	
$2t - 5 = 0 \quad 3t + 4 = 0$	0		0	
$t = \frac{5}{2} \quad t = -\frac{4}{3}$	LS = RS		LS = RS	

Question 4, page 411

a.

$$12 - 5x = 2x^2$$
$$12 - 12 - 5x + 5x = 2x^2 + 5x - 12$$
$$0 = 2x^2 + 5x - 12$$
$$2x^2 + 5x - 12 = 0$$
$$2x^2 - 3x + 8x - 12 = 0$$
$$x(2x - 3) + 4(2x - 3) = 0$$
$$(2x - 3)(x + 4) = 0$$
$$2x - 3 = 0 \qquad x + 4 = 0$$
$$x = \frac{3}{2} \qquad x = -4$$

b.

$$4x^2 = 9 - 9x$$
$$4x^2 + 9x - 9 = 9 - 9 - 9x + 9x$$
$$4x^2 + 9x - 9 = 0$$
$$4x^2 - 3x + 12x - 9 = 0$$
$$x(4x - 3) + 3(4x - 3) = 0$$
$$(4x - 3)(x + 3) = 0$$
$$4x - 3 = 0 \qquad x + 3 = 0$$
$$x = \frac{3}{4} \qquad x = -3$$

c.

$$49d^2 + 9 = -42d$$
$$49d^2 + 42d + 9 = -42d + 42d$$
$$49d^2 + 42d + 9 = 0$$
$$49d^2 + 21d + 21d + 9 = 0$$
$$7d(7d + 3) + 3(7d + 3) = 0$$
$$(7d + 3)(7d + 3) = 0$$
$$7d + 3 = 0 \qquad 7d + 3 = 0$$
$$d = -\frac{3}{7} \qquad d = -\frac{3}{7}$$

d.

$$169 = 81g^2$$
$$169 - 169 = 81g^2 - 169$$
$$0 = 81g^2 - 169$$
$$81g^2 - 169 = 0$$
$$\sqrt{81g^2} = 9g$$
$$\sqrt{169} = 13$$
$$(9g + 13)(9g - 13) = 0$$
$$9g + 13 = 0 \qquad 9g - 13 = 0$$
$$g = -\frac{13}{9} \qquad g = \frac{13}{9}$$

Question 6, page 411

a. $5u^2 - 10u - 315 = 0$

$\text{GCF} = 5$

$5(u^2 - 2u - 63) = 0$

Sum of $-2 = -9 + 7$
Product of $-63 = (-9)(7)$

$u^2 - 2u - 63 = 0$
$(u - 9)(u + 7) = 0$
$u - 9 = 0 \quad u + 7 = 0$
$u = 9 \qquad u = -7$

b.

$0.25x^2 + 1.5x + 2 = 0$

$\text{GCF} = 0.25$

$\frac{0.25}{0.25}x^2 + \frac{1.5}{0.25}x + \frac{2}{0.25} = 0$

$0.25(x^2 + 6x + 8) = 0$

Sum of $6 = 4 + 2$
Product of $8 = (4)(2)$

$x^2 + 6x + 8 = 0$
$(x + 4)(x + 2) = 0$
$x + 4 = 0 \quad x + 2 = 0$
$x = -4 \qquad x = -2$

c. $1.4y^2 + 5.6y - 16.8 = 0$

$\text{GCF} = 1.4$

$\frac{1.4}{1.4}y^2 + \frac{5.6}{1.4}y - \frac{16.8}{1.4} = 0$

$1.4(y^2 + 4y - 12) = 0$

Sum of $4 = 6 + (-2)$
Product of $-12 = (6)(-2)$

$y^2 + 4y - 12 = 0$
$(y + 6)(y - 2) = 0$
$y + 6 = 0 \quad y - 2 = 0$
$y = -6 \qquad y = 2$

d. $\frac{1}{2}k^2 + 5k + 12.5 = 0$

$\text{GCF} = \frac{1}{2}$

$\frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)}k^2 + \frac{5}{\left(\frac{1}{2}\right)}k + \frac{12.5}{\left(\frac{1}{2}\right)} = 0$

$\frac{1}{2}(k^2 + 10k + 25) = 0$

Sum of $10 = 5 + 5$
Product of $25 = (5)(5)$

$k^2 + 10k + 25 = 0$
$(k + 5)(k + 5) = 0$
$k + 5 = 0 \quad k + 5 = 0$
$k = -5 \quad k = -5$

Question 7, page 412

$x_{\text{int}} = -5 \quad x_{\text{int}} = -12$
factors: $(x + 5)(x + 12)$
$y = (x + 5)(x + 12)$
$y = x^2 + 17x + 60$

The function could be $y = x^2 + 17x + 60$ and therefore the related quadratic equation could be $0 = x^2 + 17x + 60$.

Question 9, page 412

$$\begin{aligned} 5x - 8 &= 20x^2 - 32x \\ 5x - 5x - 8 + 8 &= 20x^2 - 32x - 5x + 8 \\ 0 &= 20x^2 - 37x + 8 \\ 20x^2 - 37x + 8 &= 0 \\ 20x^2 - 32x - 5x + 8 &= 0 \\ 4x(5x - 8) - 1(5x - 8) &= 0 \\ (5x - 8)(4x - 1) &= 0 \end{aligned}$$

$5x - 8 = 0 \quad 4x - 1 = 0$
$x = \frac{8}{5} \quad x = \frac{1}{4}$

Left Side	Right Side
$5x-8$	$20x^2-32x$
$5\left(\frac{8}{5}\right)-8$	$20\left(\frac{8}{5}\right)^2-32\left(\frac{8}{5}\right)$
$8-8$	$20\left(\frac{64}{25}\right)-\frac{256}{5}$
0	$\frac{1280}{25}-\frac{256}{5}$
	$\frac{1280}{25}-\frac{1280}{25}$
	0
LS = RS	

Left Side	Right Side
$5x-8$	$20x^2-32x$
$5\left(\frac{1}{4}\right)-8$	$20\left(\frac{1}{4}\right)^2-32\left(\frac{1}{4}\right)$
$\frac{5}{4}-8$	$20\left(\frac{1}{16}\right)-\frac{32}{4}$
$\frac{5}{4}-\frac{32}{4}$	$\frac{20}{16}-\frac{32}{4}$
$-\frac{27}{4}$	$\frac{20}{16}-\frac{128}{16}$
	$-\frac{108}{16}$
	$-\frac{27}{4}$
LS = RS	

Question 6, page 420

a. $3x^2-6x-1=0$

$a=3, b=-6, c=-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(3)(-1)}}{2(3)}$$

$$x=\frac{6\pm\sqrt{36+12}}{6}$$

$$x=\frac{6\pm\sqrt{48}}{6}$$

$$x=\frac{6\pm\sqrt{16\cdot 3}}{6}$$

$$x=\frac{6\pm\sqrt{4^2\cdot 3}}{6}$$

$$x=\frac{6\pm 4\sqrt{3}}{6}$$

$$x=\frac{3\pm 2\sqrt{3}}{3}$$

$$x=\frac{3+2\sqrt{3}}{3} \qquad x=\frac{3-2\sqrt{3}}{3}$$

b. $x^2+8x+3=0$

$a=1, b=8, c=3$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-8\pm\sqrt{(8)^2-4(1)3}}{2(1)}$$

$$x=\frac{-8\pm\sqrt{64-12}}{2}$$

$$x=\frac{-8\pm\sqrt{52}}{2}$$

$$x=\frac{-8\pm\sqrt{4\cdot 13}}{2}$$

$$x=\frac{-8\pm\sqrt{2^2\cdot 13}}{2}$$

$$x=\frac{-8\pm 2\sqrt{13}}{2}$$

$$x=-4\pm\sqrt{13}$$

$$x=-4+\sqrt{13} \qquad x=-4-\sqrt{13}$$

c. $8x^2 + 8x - 1 = 0$

$a = 8, b = 8, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(8)(-1)}}{2(8)}$$

$$x = \frac{-8 \pm \sqrt{64 + 32}}{16}$$

$$x = \frac{-8 \pm \sqrt{96}}{16}$$

$$x = \frac{-8 \pm \sqrt{16 \cdot 6}}{16}$$

$$x = \frac{-8 \pm \sqrt{4^2 \cdot 6}}{16}$$

$$x = \frac{-8 \pm 4\sqrt{6}}{16}$$

$$x = \frac{-2 \pm \sqrt{6}}{4}$$

$$x = \frac{-2 + \sqrt{6}}{4} \qquad x = \frac{-2 - \sqrt{6}}{4}$$

d. $9x^2 - 12x - 1 = 0$

$a = 9, b = -12, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(9)(-1)}}{2(9)}$$

$$x = \frac{12 \pm \sqrt{144 + 36}}{18}$$

$$x = \frac{12 \pm \sqrt{180}}{18}$$

$$x = \frac{12 \pm \sqrt{36 \cdot 5}}{18}$$

$$x = \frac{12 \pm \sqrt{6^2 \cdot 5}}{18}$$

$$x = \frac{12 \pm 6\sqrt{5}}{18}$$

$$x = \frac{2 \pm \sqrt{5}}{3}$$

$$x = \frac{2 + \sqrt{5}}{3} \qquad x = \frac{2 - \sqrt{5}}{3}$$

Question 9, 420

a. Factoring is possible, however the factors will not involve integers.

b. $$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{-174.96 \pm \sqrt{174.96^2 - 4(116.64)(65.61)}}{2(116.64)}$$

$$z = -0.75$$

c. Choices and explanations will vary. The quadratic formula and solving graphically with technology are two choices.

Question 11, page 421

Let B represent the width of the border. The length of the garden and border is $6.25 + 2B$ and the width of the garden and border is $5.00 + 2B$. The area of the garden and border is $(6.25 + 2B)(5 + 2B)$. Subtracting the area of just the garden gives $(6.25 + 2B)(5.00 + 2B) - (6.25)(5.00)$, which represents the area of the border.

$$A = (6.25 + 2B)(5.00 + 2B) - (6.25)(5.00)$$
$$6.0 = 31.25 + 12.5B + 10B + 4B^2 - 31.25$$
$$0 = 4B^2 + 22.5B - 6$$

$$B = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$B = \frac{-22.5 \pm \sqrt{22.5^2 - 4(4)(-6)}}{2(4)}$$
$$B = \frac{-22.5 \pm \sqrt{602.25}}{8}$$
$$B = -5.880, \quad 0.255$$

The width of the border is positive and therefore the width is 0.255 m.