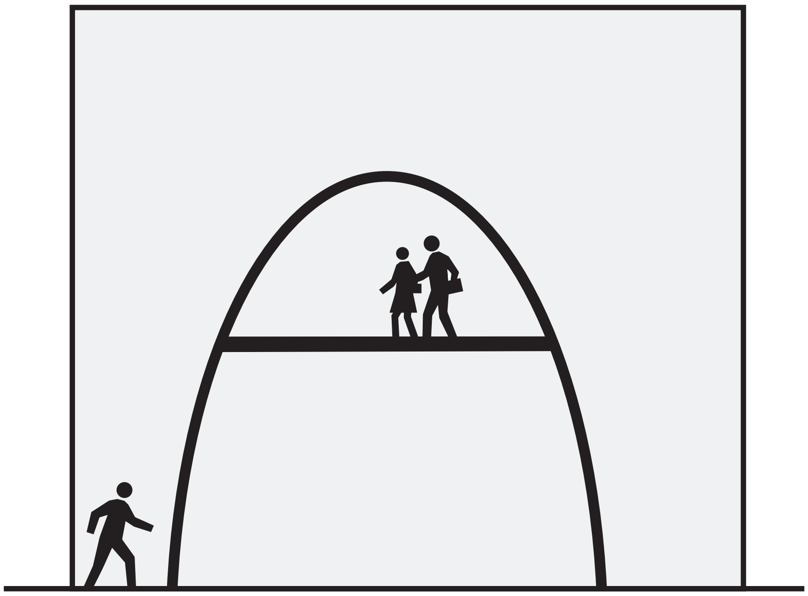
Practice Run

Solve the following problems. Be sure to show all of your steps.

1. An architect has designed an archway that is in the entrance of a new computer engineering office. In the design, the highest point of the archway is in the middle. The function that defines the shape of the archway is $h(x) = -\frac{5}{9}x(x-18)-25$. To grant access from one side of the building's second floor to the other side, there will be a bridge that passes under the archway, 12 feet above the floor.



a. Sketch a diagram to represent the information given.

b. The engineers plan to move a 6 foot by 8 foot painting into the new office. The cart they have to roll the painting on is 8 inches tall. Will there be enough room to roll the painting across the bridge under the arch? Note: 1 foot = 12 inches.

c. State the domain and range of the function.

2. Carter and Jordan are warming up for their ball game by playing catch with a softball. They are standing 12 metres from each other and they each catch the softball in their gloves, 1.5 metres above the ground. When Carter throws the ball to Jordan, it reaches a maximum height of 2.5 metres.

   - Let *x* be the horizontal distance, in metres, between Carter and the softball.
   - Let *f*(*x*) be the height, in metres, of the softball.

   a. Sketch the graph that models the flight path of the softball thrown by Carter.

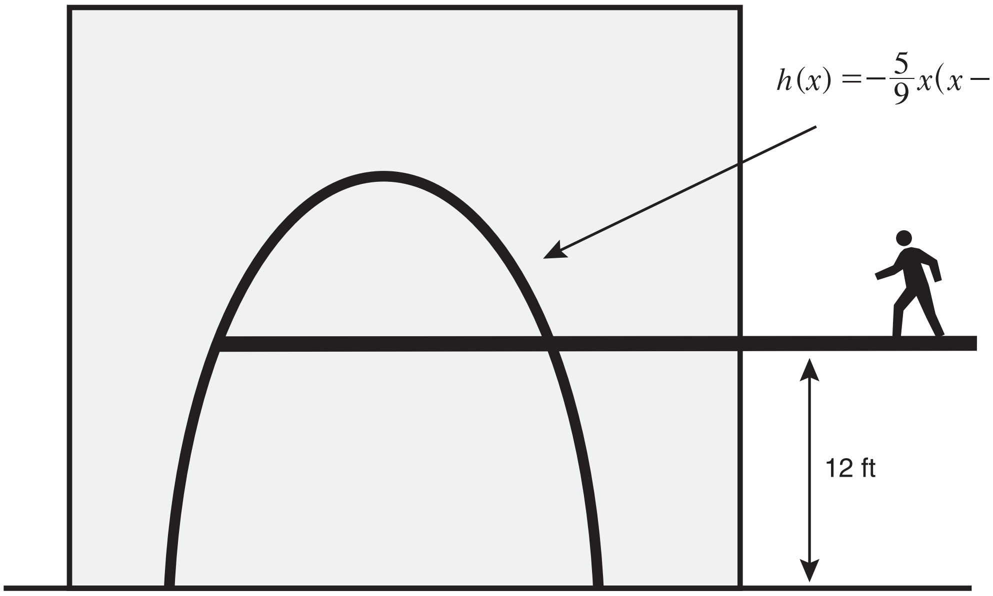
   b. Determine a quadratic function that models the softball's flight path.

   c. State the domain and range of the function.

Compare your answers.

1. An architect has designed an archway that is in the entrance of a new computer engineering office. In the design, the highest point of the archway is in the middle. The function that defines the shape of the archway is $h(x) = -\frac{5}{9}x(x-18)-25$. To grant access from one side of the building's second floor to the other side, there will be a bridge that passes under the archway, 12 feet above the floor.

   a. Sketch a diagram to represent the information given.



b. The engineers plan to move a 6×8 foot painting into the new office. The cart they have to roll the painting on is 8 inches tall. Will there be enough room to roll the painting across the bridge under the arch?
Note: 1 foot = 12 inches.

To find the maximum height, find the vertex.
Begin by finding the zeros of the function.

$h(x) = -\frac{5}{9}x(x-18)-25$

Using technology, the zeros of the function are $x = 3$ and $x = 15$.

Find the equation of the axis of symmetry for the graph of the function.

$x = \frac{3+15}{2}$
$x = 9$

Determine the y-coordinate of the vertex.

$h(9) = -\frac{5}{9}(9)((9)-18)-25$
$h(9) = -5(-9)-25$
$h(9) = 45-25$
$h(9) = 20$
Vertex = (9, 20)

The maximum height is 20 ft. Since the bridge is 12 feet above the ground, there is an 8 foot clearance from the bridge to the very top of the archway. With its long edge on the cart, the painting and cart will be 6 ft 8 in tall and will pass under the arch.

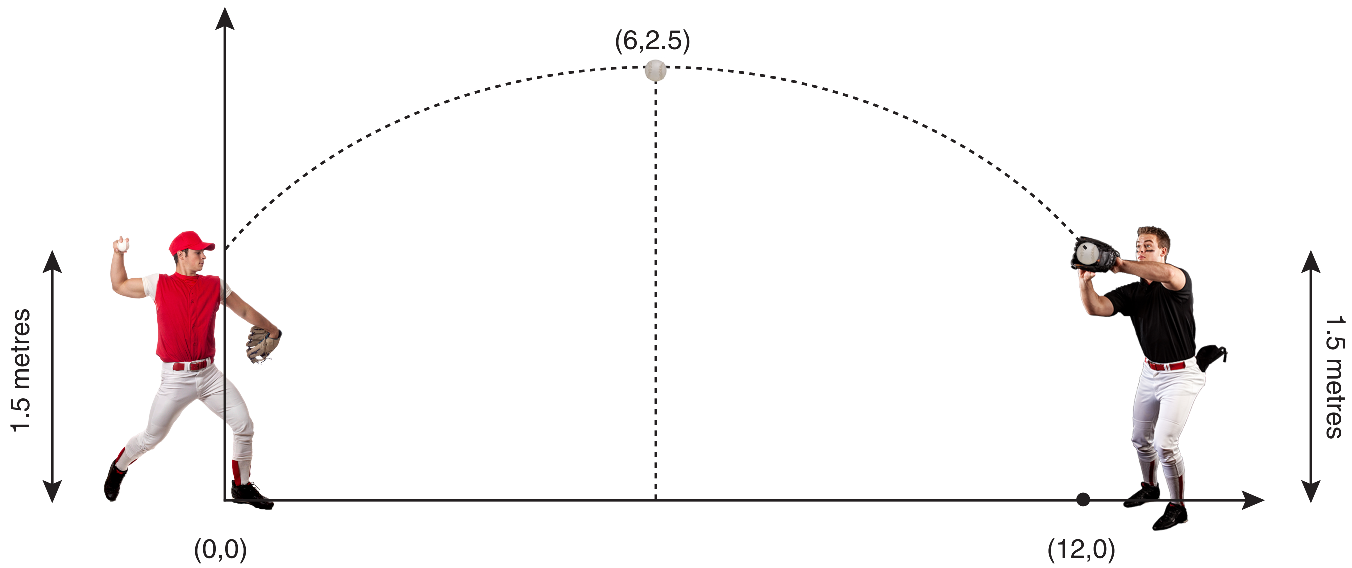
c. State the domain and range of the function.

Domain $\{x \mid 3 \le x \le 15, x \in \mathrm{R}\}$
Range $\{h(x) \mid 0 \le h(x) \le 20, h(x) \in \mathrm{R}\}$

2. Carter and Jordan are warming up for their ball game by playing catch with a softball. They are standing 12 metres from each other and they each catch the softball in their gloves, 1.5 metres above the ground. When Carter throws the ball to Jordan, it reaches a maximum height of 2.5 metres.

- Let x be the horizontal distance, in metres, between Carter and the softball.
- Let $f(x)$ be the height, in metres, of the softball.

a. Sketch the graph that models the flight path of the softball thrown by Carter.



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b. Determine a quadratic function that models the softball's flight path.

The vertex is at (6,2.5) so $(h,k) = (6, 2.5)$.

Use the vertex form of a quadratic function, $f(x) = a(x-h)^2 + k$.

Solve for the a-value using the point (12,1.5).

$$\begin{aligned}1.5 &= a(12-6)^2+2.5\\ 1.5 &= a(6)^2+2.5\\ 1.5-2.5 &= 36a+2.5-2.5\\ -1 &= 36a\\ \frac{-1}{36} &= \frac{36a}{36}\\ -\frac{1}{36} &= a\end{aligned}$$

Substitute the known values into the vertex form, $f(x)=a(x-h)^2+k$.

$$f(x)=-\frac{1}{36}(x-6)^2+2.5$$

Convert all numbers to either decimals or fractions.

$$f(x)=-\frac{1}{36}(x-6)^2+\frac{5}{2}$$

c. State the domain and range of the function.

Domain $\{x \mid 0 \le x \le 12, x \in \mathrm{R}\}$

Range $\{f(x) \mid 1.5 \le f(x) \le 2.5, f(x) \in \mathrm{R}\}$

Lesson 2.4 has shown that quadratic functions can be used to model a variety of contextual problem situations.