

Unit 2: Quadratic Functions Lesson 2.4

Coach's Corner - VII

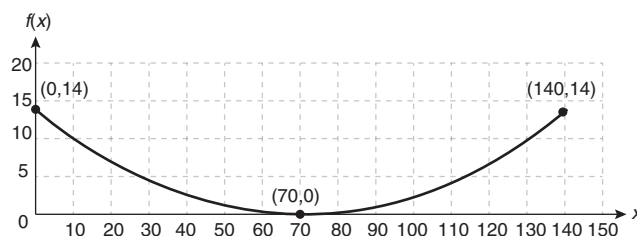


The cross-section of the Calgary Saddledome roof is parabolic. The lowest point of the saddle is 14 metres below the highest point. The length of the Saddledome's roof is 140 metres.



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- a. Sketch a graph of the curve that can be used to represent the roof.



Graphs may vary. A sample is shown.

- b. Determine the equation of a function, in standard form, that models the shape of the roof.

Using the vertex and y -intercept, solve for the a -value, in vertex form, and then expand to standard form.

$$\begin{aligned}
 14 &= a(0 - 70)^2 + 0 \\
 14 &= a(4900) \\
 \frac{14}{4900} &= \frac{4900}{4900}a \\
 \frac{14}{4900} &= a \\
 \frac{1}{350} &= a
 \end{aligned}$$

$$\begin{aligned}
 f(x) &= \frac{1}{350}(x - 70)^2 + 0 \\
 f(x) &= \frac{1}{350}(x - 70)^2 \\
 f(x) &= \frac{1}{350}(x - 70)(x - 70) \\
 f(x) &= \frac{1}{350}(x^2 - 140x + 4900) \\
 f(x) &= \frac{1}{350}x^2 - \frac{140}{350}x + \frac{4900}{350} \\
 f(x) &= \frac{1}{350}x^2 - \frac{2}{5}x + 14
 \end{aligned}$$

- c. State the domain and range of the function.

$$\text{Domain: } \{x \mid 0 \leq x \leq 140, x \in \mathbb{R}\}$$

$$\text{Range: } \{y \mid 0 \leq y \leq 14, y \in \mathbb{R}\}$$

2. Rick sells cotton candy at the Nanton Candy Store. He sells each bag for \$2.00. At this price, he sells 60 bags per day. He did an experiment for a month and found that every \$0.50 increase in price resulted in three fewer sales per day. At what price should Rick sell his cotton candy to maximize his revenue from cotton candy sales?

$$\text{Let } y = \text{total revenue} = (\text{number of bags sold}) \times (\text{price/bag})$$

$$\text{Let } x = \text{number of \$0.50 price increases.}$$

$$\text{Number of bags sold} = 60 - 3x$$

$$\text{Price} = 2.00 + 0.50x$$

$$y = (60 - 3x)(2 + 0.5x)$$

The maximum revenue will occur at the vertex of the graph of the function, which is halfway between the zeros of the function (the axis of symmetry).

$$60 - 3x = 0$$

$$2.00 + 0.5x = 0$$

$$60 = 3x$$

$$0.5x = -2.00$$

$$20 = x$$

$$x = -4.00$$

The axis of symmetry is halfway between 20 and -4.00.

$$\frac{20 + (-4.00)}{2} = 8$$

The axis of symmetry is at $x = 8$, which means the maximum function value (maximum revenue) occurs when $x = 8$. As such, Rick should increase his price 8 times.

$$\text{New price: } 2.00 + 8 \times 0.50 = 6.00$$

Rick should charge \$6.00/bag.