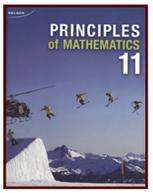
Lesson 2.4: Quadratic Functions and the Real World



Refer to *Principles of Mathematics 11* for the following practice questions.

- Page 347, # 8, 9 and 15
- Page 365, #8 and 13
- Page 378, # 14
- Page 404, #11
- Page 430, #2, 3, and 7

Question 8, page 347

$A(x) = -2x^2 + 50x$ Determine the $x-$ intercepts: $A(x) = -2x(x-25)$ $0 = -2x(x-25)$ $-2x = 0 \qquad x-25 = 0$ $x = 0 \qquad x = 25$	Determine the vertex: $x = \frac{0+25}{2} = \frac{25}{2} = 12.5$ $x = \frac{25}{2}$ or 12.5 $A(12.5) = -2(12.5)^2 + 50(12.5)$ $A(12.5) = -312.5 + 625$ $A(12.5) = 312.5$ $(12.5, 312.5)$	a. The vertex of the graph of the function is $(12.5, 312.5)$. The x-value of 12.5 represents the width of each rectangle when the area is maximized and the y-value of 312.5 m^2 represents the maximum area of the enclosure. b. Domain: $\{x \mid 0 \le x \le 25, x \in R\}$ Range: $\{A(x) \mid 0 \le A(x) \le 312.5, A(x) \in R\}$

Question 9, page 347

Let y = total revenue = (number of games) × (price)
Let x = number of $1 price increases

The new price of the games will be $10 + 1x$ or $10 + x$. Since 70 games are sold at $10/game and the number of games sold will decrease by 5 games every time the price is increased by $1, then $70 - 5x$ represents the number of games Paulette will sell at the new price.

$$y = (70 - 5x)(10 + x)$$

$70 - 5x = 0 \qquad 10 + x = 0$

$-5x = -70 \qquad x = -10$

$\frac{-5}{-5}x = \frac{-70}{-5}$

$x = 14$

The function has zeros at –10 and 14.

$$x = \frac{14 + (-10)}{2} = \frac{4}{2} = 2$$

$x = 2$

$y = (70 - 5x)(10 + x)$

$y = (70 - 5(2))(10 + 2)$

$y = (60)(12)$

$y = 720$

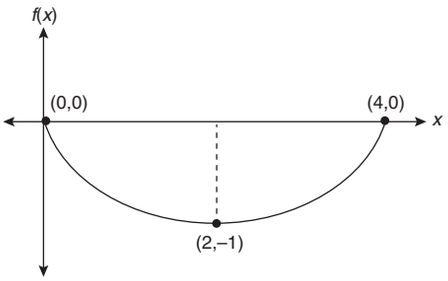
$(2, 720)$

The axis of symmetry is located at $x = 2$ and the coordinates of the vertex of the graph of the function are $(2, 720)$. As such, the games should be sold at a price of $10 + $2 = $12/game.

The maximum revenue will be $720.00.

Question 15, page 349

A graph helps to visualize the given information and to gather additional information. Let x = the length of time the pelican is underwater. Let $f(x)$ = the pelican's depth underwater.



The x-coordinate of the vertex is 2 seconds because it represents the midpoint of the total 4 seconds the pelican is under water.

a. Domain: $\{x \mid 0 \le x \le 4, x \in \mathrm{R}\}$
Range: $\{f(x) \mid -1 \le f(x) \le 0, f(x) \in \mathrm{R}\}$

b. $f(x) = a(x - h)^2 + k$

$0 = a(4 - 2)^2 - 1$

$1 = a(2)^2$

$\frac{1}{4} = \frac{4a}{4}$

$\frac{1}{4} = a$ $\qquad f(x) = \frac{1}{4}(x - 2)^2 - 1$

Question 8, page 365

a. The function is given in vertex form and the vertex is at $(h,k)=(9,8)$. As such, the equation of the axis of symmetry is $x=9$.

b. The maximum height of the ball corresponds to the *y*-coordinate of the vertex, so the maximum height of the ball, when it was hit, was 8 ft.

c. The edge of the court is at coordinates (0,0) and Candice is on the court, 2 ft from the edge. As such, the *x*-coordinate of the ball at that location is 2. Using the function $h(x)=-0.03(x-9)^2+8$, find the height of the ball when $x=2$.

$h(2)=-0.03(2-9)^2+8$
$h(2)=-0.03(-7)^2+8$
$h(2)=-0.03(49)+8$
$h(2)=6.53$

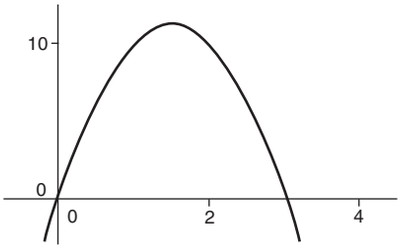
The volleyball ball was 6.53 ft high when it was set.

d. Range: $\{h(x)\mid 6.53\le h(x)\le 8, h(x)\in \mathrm{R}\}$

The ball is at its lowest height of 6.53 feet as it is set by Candice and the ball is at its highest height of 8 feet as Marleen made contact for the attack hit.

Question 13, page 366

13. a) Use technology to graph the function.



Based on the graph, the zeros of the function can be estimated to be 0 and 3.

13. b) The zeros represent the starting point of the water ($t=0$), where the sprinkler is located, and the landing point of the water ($t=3$). The water passes through the entire arc from sprinkler to ground is approximately 3 seconds.

Question 14, page 379

Set one edge of the cross-section of the dish at (0, 0). The dish is 305 m across, so another point (the other edge) will be at (305, 0). Both 0 and 305 are zeros of the function.

$f(x) = a\,(x - 0)(x - 305)$

The axis of symmetry will be halfway between the two zeros.

$$\frac{0 + 305}{2} = 152.5$$

The vertex is on the axis of symmetry, so (152.5, –21.94) is a point on the graph of the function.

$$\begin{aligned} -21.94 &= a(152.5 - 0)(152.5 - 305) \\ 0.000943\ldots &= a \\ f(x) &= 0.000943x(x - 305) \end{aligned}$$

Question 11, page 404

$l = 4 + w$

$$\begin{aligned} A &= lw \\ 117 &= (4 + w)(w) \\ 117 &= 4w + w^2 \\ 0 &= w^2 + 4w - 117 \\ &= (w + 13)(w - 9) \end{aligned}$$

$w + 13 = 0 \qquad w - 9 = 0$

$w = -13 \qquad w = 9$

Ignore the negative solution since w represents a measurement.

$$\begin{aligned} w &= 9 \\ l &= w + 4 \\ &= 9 + 4 = 13 \end{aligned}$$

The garden measures 9 m by 13 m.

Question 2, page 430

a. $V = \pi r^2 h$

$1150 = \pi r^2 (18)$

$0 = 18\pi r^2 - 1150$

$a = 18\pi,\ b = 0,\ c = -1150$

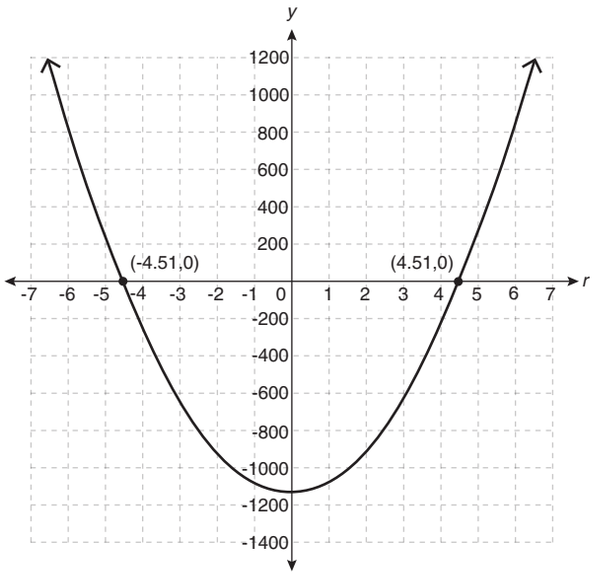
$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r = \frac{0 \pm \sqrt{0^2 - 4(18\pi)(-1150)}}{2(18\pi)}$$

$r = \pm 4.51$

The negative solution is ignored, so the radius is 4.51 cm.

b.



c. Choices and explanations will vary.

Question 3, page 430

$$x + y = 11$$
$$y = 11 - x$$
$$xy = -152$$
$$x(11 - x) = -152$$
$$11x - x^2 = -152$$
$$0 = x^2 - 11x - 152$$
$$0 = (x - 19)(x + 8)$$
$$x = 19, -8$$
$$19 + y = 11$$
$$y = -8$$

One number is 19 and the other is –8.

Question 7, page 431

a. $E(x) = (200 - 5x)(25 + 1x)$

b. She is currently earning $25 \times 200 = 5000$ dollars per week.

$$5000 = (200 - 5x)(25 + x)$$
$$5000 = 5000 - 125x + 200x - 5x^2$$
$$0 = -5x^2 + 75x$$
$$0 = -5x(x - 15)$$
$$x = 0,\ 15$$

She will earn the same amount of money if she increases the price by \$15 to \$40.

$$\frac{0 + 15}{2} = 7.5$$

The maximum will occur on the axis of symmetry of the graph of the function. The two points (where $x = 0$ and $x = 15$) are mirror images because they share a common y-value, so the axis of symmetry will occur halfway between them.

Alexis should increase her price by \$7.5 to \$32.50 in order to maximize her earnings.