c. Symptoms of the flu include aches and fever. Julie has a fever and her body aches. Julie has the flu.

Invalid. The flu is not the only illness that includes a fever and body aches as symptoms.

d. Tara lives in Grassy Lake. Grassy Lake is in Alberta. Tara lives in Alberta.

Valid.

Please return to *Unit 3: Logic and Reasoning Lesson 3.2* to continue your training.

Unit 3: Logic and Reasoning Lesson 3.2

Coach's Corner – IV

1. Explain why the sum of the lengths of two sides of a triangle must always be greater than the length of the third side.

   Suppose one side of a triangle was longer than the lengths of the other two combined. It would be impossible to form a closed shape because the two short sides would not be able to touch each other and the ends of the long side at the same time. This means that no side can be longer than the other two combined.

   These sides are too short to touch.

   In addition, if the longest side was the same length as the other two lengths combined, there would be no triangle formed, but rather just overlapping lines.

   As such, to form a triangle, there must be two sides whose combined length is greater than the length of the third side.

2. Arlan was told the following relationship can be used to determine the sum of the first n Natural Numbers: $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$. Arlan wants to prove that this relationship is always true.

   a. Check that the equation works for two different values of n.

   Some possibilities are:
   Let $n = 5$.
   $1 + 2 + 3 + 4 + 5 = 15$ and $\frac{5(5+1)}{2} = 15$

   Let $n = 8$.
   $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$ and $\frac{8(8+1)}{2} = 36$

   b. Why might Arlan find this relationship difficult to prove?

   There is no limit to the values n could take. It would be impossible to check these individually.

   c. Arlan noticed that if he reversed the order of the values and added the terms he would always get the same value. So, for example, if $n = 50$, he could write

Forward	1	2	3	4	…	48	49	50
Reversed	50	49	48	47	…	3	2	1
Sum	51	51	51	51	…	51	51	51

   How many 51s would there be if Arlan wrote out the entire table for 1 to 50?

   There would be fifty 51s.
   How does this example correspond to the expression $\frac{n(n+1)}{2}$?

   The sum of fifty 51s is equivalent to 50(51) and this corresponds to adding the numbers in the list twice. So, to get the sum (once), you can divide by two. This calculation can be written as $\frac{50(51)}{2}$.

d. Using a pattern similar to Arlan's, prove that $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$. It may help to expand $1 + 2 + 3 + \ldots + n$ to $1 + 2 + 3 + \ldots + (n-2) + (n-1) + n$.

Forward	1	2	3	…	$n-2$	$n-1$	n
Reversed	n	$n-1$	$n-2$	…	3	2	1
Sum	$n+1$	$n+1$	$n+1$	…	$n+1$	$n+1$	$n+1$

This sum will contain *n* terms, so twice the sum of the first *n* Natural Numbers is $n(n+1)$. Dividing by two will give the sum of the first *n* numbers: $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$.

3. A list of multiples of 5 is shown.

5, 10, 15, 20, 25, 30, 35, 40, 45

a. Conjecture a divisibility rule for 5. (In other words, how can you tell if a number is divisible by 5 by looking at it?)

If a number ends in 0 or 5, it is divisible by 5.

b. Prove your conjecture.

Any Whole Number can be rewritten in the form $b + c$, where c is the ones value in the given Whole Number. The b value has a zero in the ones place and so is always divisible by 10. Since 5 is a factor of 10, 5 is also a factor of b. This means that if c, the ones value, is divisible by 5, then $b + c$ will be divisible by 5. Only c values of 0 or 5 will make this true.

Please complete *Lesson 3.2 Game On!* located in *Workbook 3A* before proceeding to *Lesson 3.3*.