Question 10, page 23

Patrice's conjecture is reasonable. The square of an even number will always be even and the square of an odd number will always be odd. This means you will either be adding two even numbers or two odd numbers. Both cases result in an even number.

Question 16, page 24

a. Examples will vary.

$18 = 11 + 7$

$48 = 7 + 41$

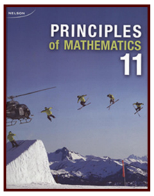
$220 = 197 + 23$

b. A counterexample would need to be an even number greater than two that is not the sum of any pair of prime numbers.

Question 19, page 25

Disagree. $8^2 - 3 = 61$

## Lesson 3.2: Deductive Reasoning




Refer to *Principles of Mathematics 11* pages 27 – 30 for more examples.

- Page 31, #2, 5, 7, 8, 10, 11, 17, and 19.

Question 2, page 31

You can deduce that Austin got a good haircut.

Question 5, page 31

Let $2j$ represent an even integer and let $2k + 1$ represent an odd integer.

The product of an even integer and an odd integer can then be represented as $2j(2k+1)$. This can be rearranged as follows:

$$\begin{aligned}&2j(2k+1)\\&= 2j(2k) + 2j(1)\\&= 4jk + 2j\\&= 2(2jk + j)\end{aligned}$$

2 is a factor of the product so the product is even.

Question 7, page 32

a.

| | | | |
|---|---|---|---|
| Choose a number | 0 | 3 | –4 |
| Multiply by 4 | 0 | 12 | –16 |
| Add 10 | 10 | 22 | –6 |
| Divide by 2 | 5 | 11 | –3 |
| Subtract 5 | 0 | 6 | –8 |
| Divide by 2 | 0 | 3 | –4 |
| Add 3 | 3 | 6 | –1 |

b.

| | |
|---|---|
| Choose a number | $x$ |
| Multiply by 4 | $4x$ |
| Add 10 | $4x + 10$ |
| Divide by 2 | $\frac{4x + 10}{2} = 2x + 5$ |
| Subtract 5 | $2x$ |
| Divide by 2 | $x$ |
| Add 3 | $x + 3$ |

Question 8, page 32

The fact that Adrian's pants are not khaki does not tell you anything about their price (or their comfort level).

Question 10, page 32

Let $2n + 1$ represent an odd integer.

$$\begin{aligned}(2n + 1)^2 &= (2n + 1)(2n + 1)\\ &= 4n^2 + 2n + 2n + 1\\ &= 4n^2 + 4n + 1\\ &= 2(2n^2 + 2n) + 1\end{aligned}$$

$2(2n^2 + 2n)$ has a factor of 2 so it is even. This means $2(2n^2 + 2n) + 1$ must be odd.

Question 11, page 32

Inductive examples:

$$\begin{aligned} 10^2 - 8^2 &= 100 - 64 \\ &= 36 \\ 36 \div 4 &= 9 \end{aligned}$$

$$\begin{aligned} 13^2 - 11^2 &= 169 - 121 \\ &= 48 \\ 48 \div 4 &= 12 \end{aligned}$$

Proof:

Let $n$ represent any number. $n + 2$ then represents a consecutive number of the same type (both are odd or both are even).

$$\begin{aligned} (n-2)^2 - n^2 &= n^2 - 2n - 2n + 4 - n^2 \\ &= -4n + 4 \\ &= 4(-n + 1) \end{aligned}$$

The result has a factor of 4 and so will always be divisible by 4.

Question 17, page 33

Jamie's work is the only one that could be called a proof. Joan and Garnet showed that the conjecture works for specific examples but not for every possibility. Jamie's work showed the conjecture will always work and so proved the conjecture.

Question 19, page 33

$$n^2 + n + 2 = n(n+1) + 2$$

$n(n+1)$ represents the multiplication of two consecutive numbers so one must be even and one must be odd. The product of an even number and an odd number is always even (see question 5). Adding 2 to an even number will always result in another even number so $n(n+1) + 2$ must be even.