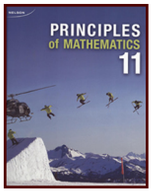
Lesson 3.3: Invalid Reasoning



Refer to *Principles of Mathematics 11* page 36 – 41 and 45 – 48 for more examples.

- Page 42, #1, 3, 6a, and 6b.
- Page 49, #2, 5a, 7, 8, 10, 13, 14, and 18.

Question 1, page 42

a. The structure of the statement reasons correctly, however, the premise that all runners train on a daily basis is invalid because it is not true. This means the conclusion is also invalid.

b. Rectangles are also quadrilaterals with four right angles. This means a quadrilateral with 4 right angles is not enough information to conclude that it is a square.

Question 3, page 42

All of Mickey's steps are reasonable except for dividing both sides by $(a - b)$. This step is flawed because $a - b = 0$ (since a and b are both equal to 1) and dividing by 0 is undefined.

Questions 6a and b, page 43

a. Suppose your house number was 104 and your age was 17.

Write down the number of your street address	104
Multiply by 2	$104 \times 2 = 208$
Add the number of days in a week	$208 + 7 = 215$
Multiply by 50	$215 \times 50 = 10750$
Add your age	$10750 + 17 = 10767$
Subtract the number of days in a year	$10767 - 365 = 10402$
Add 15	$10402 + 15 = 10417$

The final value shows 10417, the house number followed by the age.

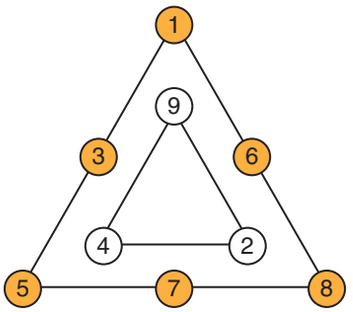
b. There are 365 days in a year. The final expression should be $100n + a$.

Question 2, page 49

The numbers 1 through 9 sum to 45. This total needs to be split so that one group is twice the size of the other group.

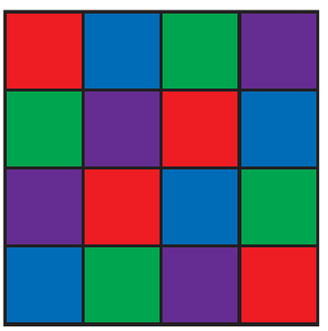
This will work if $\frac{1}{3}$ of the total is on the inside triangle and $\frac{2}{3}$ of the total is on the outside.

$\frac{1}{3} \times 45 = 15$ and $\frac{2}{3} = 30$, so the inside sum must be 15 and the outside sum must be 30. Any combination of values that accomplish this will be a solution. A sample is shown.



Question 5a, page 49

Multiple solutions are possible. A sample is shown.



Question 7, page 49

Note that the difference between terms increases by 1. So the first difference is 5, the next 6, then 7, then 8. The unknown value will be 28 using this pattern.

Question 8, page 49

Both siblings would describe themselves as truth-tellers. This means the sister would not have said that she was a liar and so the brother is a liar.

Question 10, page 50

One strategy is to look at all the possible pairs.

Envelope	Possible pairs
3	(0,3), (1,2)
7	(0,7), (1,6), (2,5), (3,4)
8	(0,8), (1,7), (2,6), (3,5)
13	(4,9), (5,8), (6,7)
14	(5,9), (6,8)

a. 6 must be used in envelope 13 or 14 and so is not available for envelope 8.

b. 9 only shows up twice, in envelopes 13 and 14. Suppose the 9 is in envelope 13. This means (6,8) must be in envelope 14 and so the 6 is used there.

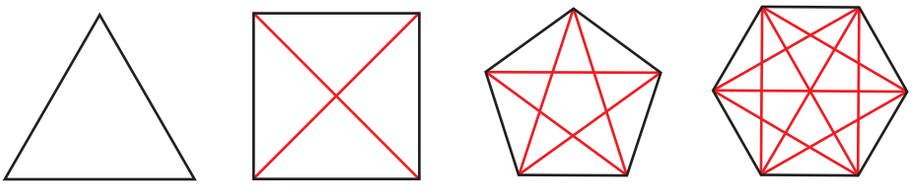
   Alternatively, suppose the 9 is in envelope 14. This means 5 is also in envelope 14. The only possibility left for envelope 13 is (6,7). Again the 6 is used.

Question 13, page 50

Early in the race the order was Tamara, Shreya, Kateri, then Justine.
At the midpoint the order was Shreya, then Tamara, and Justine, followed by Kateri
The final order is Kateri, Shreya, Tamara, then Justine.

Tamara is third.

Question 14, page 50



0 diagonals | 0 + 2 = 2 diagonals | 2 + 3 = 5 diagonals | 5 + 4 = 9 diagonals

Continuing this pattern…

Number of sides	Number of diagonals
3	0
4	0 + 2 = 2
5	2 + 3 = 5
6	5 + 4 = 9
7	9 + 5 = 14
8	14 + 6 = 20
9	20 + 7 = 27
10	27 + 8 = 35

The decagon should have 35 diagonals.

Question 18, page 51

According to the information, when rain occurred, it was for only half the day. Sunshine, however, could have occurred all day.

There were 3 more sunny afternoons than mornings. This suggests there were 3 more rainy mornings than afternoons. Because there are 5 rainy times altogether, you can conclude that it rained on 4 mornings and 1 afternoon (3 more rainy mornings than afternoons). This gives you a total of 4 + 6 = 10 mornings and 9 + 1 = 10 afternoons. The holiday was 10 days long.