

Lesson 4.4: The Sine Law

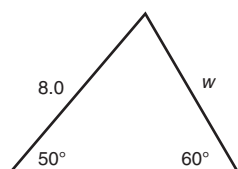
Refer to *Principles of Mathematics 11* pages 131 and 139 for more examples.

- Page 131, #2a, 2d, and 3
- Page 139, #3a, 3d, 4, 6a, 6c, 8a, 9, 12, and 13

Question 2a and d, p. 131

a.

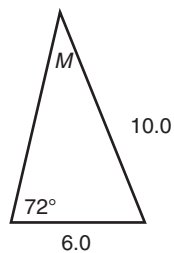
i.



$$\begin{aligned}\text{ii. } \frac{w}{\sin 50^\circ} &= \frac{8.0}{\sin 60^\circ} \\ w &= \frac{8.0 \sin 50^\circ}{\sin 60^\circ} \\ w &= 7.07... \\ w &\doteq 7.1\end{aligned}$$

c.

i.



$$\begin{aligned}\text{ii. } \frac{6.0}{\sin M} &= \frac{10.0}{\sin 72^\circ} \\ \frac{6.0 \sin 72^\circ}{10.0} &= \sin M \\ \sin^{-1}\left(\frac{6.0 \sin 72^\circ}{10.0}\right) &= M \\ 34.79...^\circ &= M \\ 34.8^\circ &\doteq M\end{aligned}$$

Question 3, p. 131

Agree.

$$x \sin Y = y \sin X$$

$$\frac{x \sin Y}{y} = \sin X$$

$$\frac{\sin Y}{y} = \frac{\sin X}{x}$$

Question 3a and d, p. 139

$$\text{a.} \quad D + E + F = 180^\circ$$

$$53^\circ + 68^\circ + F = 180^\circ$$

$$F = 59^\circ$$

$$\frac{d}{\sin D} = \frac{f}{\sin F}$$

$$\frac{d}{\sin 53^\circ} = \frac{22.5}{\sin 59^\circ}$$

$$d = \frac{22.5 \sin 53^\circ}{\sin 59^\circ}$$

$$d = 20.96\dots$$

$$d \doteq 21.0 \text{ cm}$$

$$\text{d.} \quad \frac{\sin N}{n} = \frac{\sin L}{l}$$

$$\frac{\sin \theta}{45.2} = \frac{\sin 29^\circ}{24.4}$$

$$\sin \theta = \frac{45.2 \sin 29^\circ}{24.4}$$

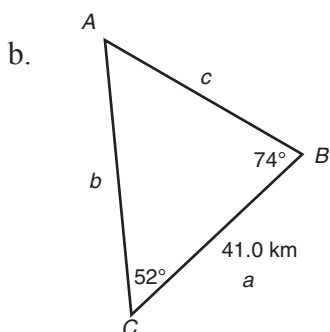
$$\theta = \sin^{-1}\left(\frac{45.2 \sin 29^\circ}{24.4}\right)$$

$$\theta = 63.9\dots^\circ$$

$$\theta \doteq 64^\circ$$

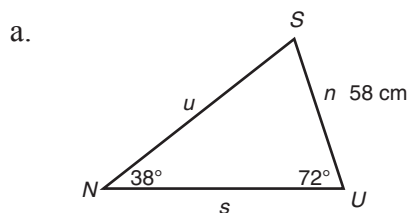
Question 4, p. 139

- a. The angle opposite the lake is 74° , but the angle opposite the 41.0 km is only $180^\circ - 52^\circ - 74^\circ = 54^\circ$, which means the lake must be longer than 41.0 km since it is opposite the largest angle in the triangle.



$$\begin{aligned}
 A + B + C &= 180^\circ \\
 A + 74^\circ + 52^\circ &= 180^\circ \\
 A &= 54^\circ \\
 \frac{a}{\sin A} &= \frac{b}{\sin B} \\
 \frac{41.0}{\sin 54^\circ} &= \frac{b}{\sin 74^\circ} \\
 \frac{41.0 \sin 74^\circ}{\sin 54^\circ} &= b \\
 48.71 \dots &= b \\
 48.7 \text{ km} &\doteq b
 \end{aligned}$$

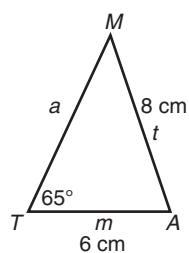
Question 6a and c, p. 140



b.

$$\begin{aligned}
 \frac{u}{\sin U} &= \frac{n}{\sin N} \\
 \frac{u}{\sin 72^\circ} &= \frac{58}{\sin 38^\circ} \\
 u &= \frac{58 \sin 72^\circ}{\sin 38^\circ} \\
 u &= 89.5 \dots \\
 u &\doteq 90 \text{ cm}
 \end{aligned}$$

c.



$$\frac{\sin M}{m} = \frac{\sin T}{t}$$

$$\frac{\sin M}{6} = \frac{\sin 65^\circ}{8}$$

$$\sin M = \frac{6 \sin 65^\circ}{8}$$

$$M = \sin^{-1}\left(\frac{6 \sin 65^\circ}{8}\right)$$

$$M = 42.8\dots^\circ$$

$$M \doteq 43^\circ$$

Question 8a, p. 140

a.

$$\text{i.} \quad \sin N = \frac{n}{m}$$

$$\sin 36.9^\circ = \frac{n}{10}$$

$$10 \sin 36.9^\circ = n$$

$$6.00\dots = n$$

$$6.0 \text{ cm} \doteq n$$

$$\text{ii.} \quad \frac{n}{\sin N} = \frac{m}{\sin M}$$

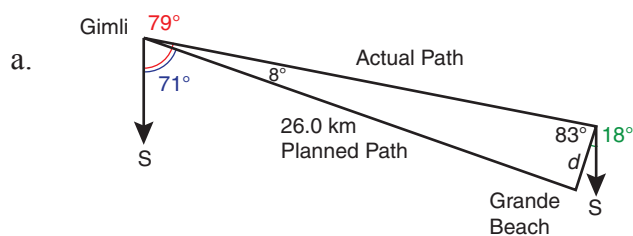
$$\frac{n}{\sin 36.9^\circ} = \frac{10}{\sin 90^\circ}$$

$$n = \frac{10 \sin 36.9^\circ}{\sin 90^\circ}$$

$$n = 6.00\dots$$

$$n \doteq 6.0 \text{ cm}$$

Question 9, p. 140



b.

$$\frac{d}{\sin 8^\circ} = \frac{26.0}{\sin 83^\circ}$$

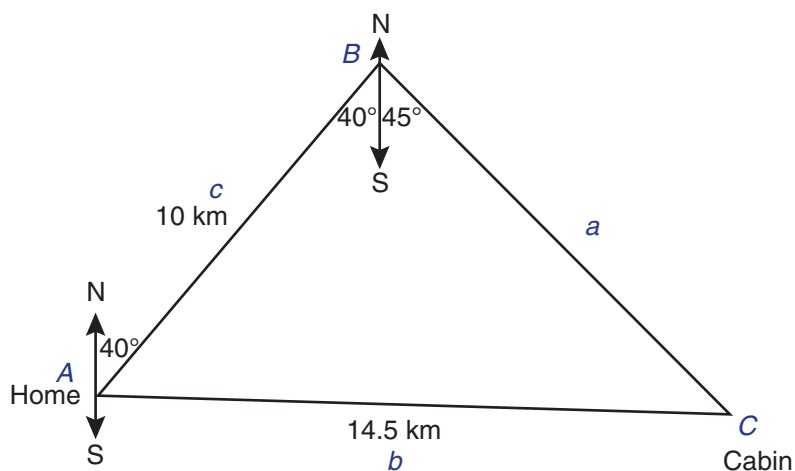
$$d = \frac{26.0 \sin 8^\circ}{\sin 83^\circ}$$

$$d = 3.64\dots$$

$$d \doteq 3.6 \text{ km}$$

The remaining distance is approximately 3.6 km.

Question 12, p. 140



$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin C}{10} = \frac{\sin 85^\circ}{14.5}$$

$$\sin C = \frac{10 \sin 85^\circ}{14.5}$$

$$C = \sin^{-1}\left(\frac{10 \sin 85^\circ}{14.5}\right)$$

$$C = 43.39\dots^\circ$$

$$A + B + C = 180^\circ$$

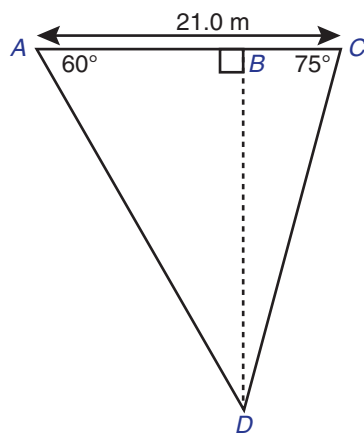
$$A + 85^\circ + 43.39\dots^\circ = 180^\circ$$

$$A = 51.60\dots^\circ$$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} \\ \frac{a}{\sin 51.60\dots^\circ} &= \frac{14.5}{\sin 85^\circ} \\ a &= \frac{14.5 \sin 51.60\dots^\circ}{\sin 85} \\ a &= 11.40\dots \\ a &\doteq 11.4\end{aligned}$$

The second leg of the trip was approximately 11.4 km.

Question 13, p. 141



$$\begin{aligned}\angle A + \angle C + \angle ADC &= 180^\circ \\ 60^\circ + 75^\circ + \angle ADC &= 180^\circ \\ \angle ADC &= 45^\circ\end{aligned}$$

$$\begin{aligned}\frac{AC}{\sin \angle ADC} &= \frac{AD}{\sin \angle C} \\ \frac{21}{\sin 45^\circ} &= \frac{AD}{\sin 75^\circ} \\ \frac{21 \sin 75^\circ}{\sin 45^\circ} &= AD \\ 28.68\dots &= AD\end{aligned}$$

$$\begin{aligned}\sin A &= \frac{BD}{AD} \\ \sin 60^\circ &= \frac{BD}{28.68\dots} \\ (28.68\dots) \sin 60^\circ &= BD \\ 24.84\dots &= BD \\ 24.8 &\doteq BD\end{aligned}$$

The gorge is approximately 24.8 m deep.