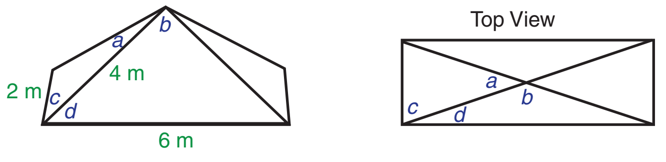
Practice Run

A rectangular pyramid has the measures shown, where all of the slant heights are the same length. When looked at from above, it appears that $a + b = 180^\circ$ and $c + d = 90^\circ$, but neither is true.

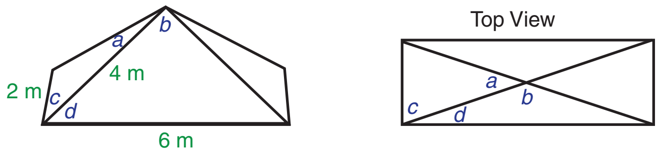


a. Show that $a + b \neq 180^\circ$ and that $b + c \neq 90^\circ$.

b. Explain the discrepancy.

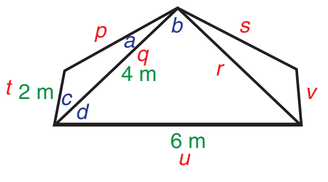
Compare your answers.

A rectangular pyramid has the measures shown, where all of the slant heights are the same length. When looked at from above, it appears that $a + b = 180^\circ$ and $c + d = 90^\circ$, but neither is true.



a. Show that $a + b \neq 180^\circ$ and that $b + c \neq 90^\circ$.

Labelling the sides may make answering this question easier.



Use the cosine law to determine angle c.

$$\begin{aligned} p^2 &= q^2 + t^2 - 2qt\cos c \\ 4^2 &= 4^2 + 2^2 - 2(4)(2)\cos c \\ 16 &= 16 + 4 - 16\cos c \\ -4 &= -16\cos c \\ \frac{1}{4} &= \cos c \\ \cos^{-1}\left(\frac{1}{4}\right) &= c \\ 75.52...^\circ &= c \\ 75.5^\circ &\doteq c \end{aligned}$$

Angle c is part of an isosceles triangle.

$$\begin{aligned} c + c + a &= 180^\circ \\ 75.52... + 75.52... + a &= 180^\circ \\ a &= 28.95...^\circ \\ a &\doteq 29.0^\circ \end{aligned}$$

Use the cosine law to determine angle d.

$$\begin{aligned} r^2 &= q^2 + u^2 - 2uq\cos d \\ 4^2 &= 4^2 + 6^2 - 2(4)(6)\cos d \\ 16 &= 16 + 36 - 48\cos d \\ -36 &= -48\cos d \\ \frac{3}{4} &= \cos d \\ \cos^{-1}\left(\frac{3}{4}\right) &= d \\ 41.40...^\circ &= d \\ 41.4^\circ &\doteq d \end{aligned}$$

It is also the case that angle d is part of an isosceles triangle.

$$\begin{aligned} d + d + b &= 180^\circ \\ 41.40... + 41.40... + b &= 180^\circ \\ b &= 97.18^\circ... \\ b &\doteq 97.2^\circ \end{aligned}$$

$$\begin{aligned} a + b &\doteq 29.0 + 97.2 \\ a + b &\doteq 126.2^\circ \end{aligned}$$

So, $a + b \neq 180^\circ$

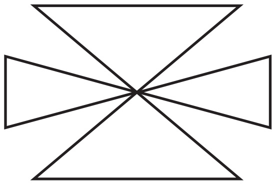
$$\begin{aligned} c + d &\doteq 75.5^\circ + 41.4^\circ \\ c + d &\doteq 116.9^\circ \end{aligned}$$

So, $b + c \neq 90^\circ$

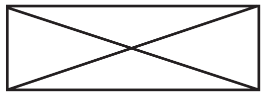
b. Explain the discrepancy.

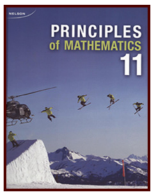
When looking at the pyramid from above, you are looking at each triangle from an angle instead of straight on, which distorts the appearance of the angle measures. If you laid each triangle flat (imagine "unfolding" the pyramid from its base), you would have an image like the one below left. Here you can see the distortion more clearly, especially in comparison with the top view of the pyramid. And, it is clear that $a + b \neq 180^\circ$ and $b + c \neq 90^\circ$.

"unfolded" pyramid



Top View





For further information about solving problems in three dimensions see p. 159 of *Principles of Mathematics 11*.